On Pre Generalized ωα-Closed Sets in Topological Spaces

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Abstract

The aim of this paper is to introduce the new class of closed sets called pre generalized ωα-closed (briefly pgωα-closed) sets in topological spaces which is properly placed between the class of pre-closed sets and the class of gp-closed sets and obtained some of their properties Also we define the pgωα-open sets and studied some of their characterizations.

Keywords - *Topological spaces, g-closed sets, ωα- open sets, pgωα--closed sets, pgωα-open sets.*

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I. INTRODUCTION

Njastad [16] and Mashhour et al [14] introduced and studied the concept of α-open (originally called α- sets) and α-closed sets respectively in topological spaces. Levine [9] introduced and investigated the weaker forms of open sets called semi-open sets in 1963. Andrijevic [2] introduced the notion of semi pre-closed set. The concept of generalized closed (briefly g-closed) sets as a generalization of closed set is defined by Levine [10] in 1970. Later on many researchers like Dontchev [7], Sundaram and Sheik John [19] and others introduced and studied the notion of generalized semi pre-closed sets and ω-closed sets in topological spaces respectively. Recently Benchalli et. al. [4] defined and studied the concept of ωα-closed sets in topological spaces.
The aim of this paper is to introduce the new weaker forms of closed sets called pgωα-closed sets and studied the some of their characterizations and also we define the pgωα-open sets and studied some of their properties.

II. PRELIMINARIES

Throughout this paper, the space (X, τ) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), then cl(A), int(A) and A^c denote the closure of A, the interior of A and the compliment of A in X respectively.

Definition 2.1: A subset A of a topological space X is called
regular open [18] if A =int(cl(A)) and regular closed if A = cl(int(A)).
semi-open set [9] if A$\subseteq$ cl(int(A)) and semi-closed set if int(cl(A)) $\subseteq$ A.
pre-open set [14] if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.
α-open set [16] if A $\subseteq$ int(cl(int(A))) and α-closed set if cl(int(cl(A))) $\subseteq$ A.
semi-preopen set [2] (= β-open [1]) if A $\subseteq$ cl(int(cl(A))) and semi-pre closed set [2] (= β-closed [1]) if int(cl(int(A))) $\subseteq$ A.
The intersection of all semi-closed (resp. semi-open) subsets of (X, τ) containing A is called the semi-closure (resp. semi-kernel) of A and by scl(A) (resp. sker(A)). Also the intersection of all pre-closed (resp. semi-pre-closed and α-closed) subsets of (X, τ) containing A is called the pre-closure (resp. semi-preclosure and α-closure) of A and is denoted by pcl(A) (resp. spcl(A) and α-cl(A)).

Definition 2.2: A subset A of a topological space X is called a
generalized closed (briefly g-closed) set [10] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

generalized semi-closed (briefly gs-closed) set[3] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
α-generalized closed (briefly αg-closed) set [12] if αcl(A) ⊆ U whenever A ⊆ U and U is open in X.
generalized α-closed (briefly gα-closed) set[11] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in X.
generalized pre-closed (briefly gp-closed) set [13] if pcl(A) ⊆ U whenever A ⊆ U and U is open in X.
generalized semi-preclosed (briefly gsp-closed) set[7] if spcl(A) ⊆ U whenever A⊆ U and U is open in X.
generalized pre-regular-closed (briefly gpr-closed) set[8] if pcl(A) ⊆ U whenever A ⊆ U and U is regular-open inX.
ω-closed [19] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.

g^*-closed set [20] if cl(A) ⊆ U whenever A ⊆ U and U is g-open set in X.

g^*-pre closed (briefly g^*p-closed) set[21] if pcl(A) ⊆ U whenever A ⊆ U and U is g-open set in X.
ωα-closed [4] if αcl(A) ⊆ U whenever A ⊆ U and U is ω-open in X.
generalized ωα-closed (briefly gωα-closed) set[5] if αcl(A) ⊆ U whenever A ⊆ U and U is ωα-open set in X.
semi pre generalized ωα-closed (briefly spgωα-closed) set[17] if spcl(A) ⊆ U whenever A ⊆ U and U is ωα-open set in X.

III. PRE GENERALIZED ωα - CLOSED SETS IN TOPOLOGICAL SPACES

In this section, we introduce pre generalized ωα-closed (briefly pgωα-closed) sets in topological spaces and obtained some of their properties.

Definition 3.1: A subset A of a topological space (X, τ) is called pre generalized ωα- closed (briefly pgωα-closed) set if pcl(A) ⊆ U whenever A ⊆ U and U is ωα-open in X.

We denote the set of all pgωα-closed sets in (X, τ) by PGωα(X, τ).

Theorem 3.2: Every closed set in X is pgωα-closed set.

Proof: Let A be a closed set in X and G be an ωα-open set in X such that A ⊆ G. Since A is closed, cl(A) = A. but pcl(A) ⊆ cl(A) is always true. So pcl(A) ⊆ G. Thus A is pgωα-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let X= {a, b, c} and τ = {X, ϕ, {a}, {b, c}}. Then the set {a, b} is pgωα-closed but not a closed set in X.

Theorem 3.4: Every preclosed set is pgωα-closed set but not conversely.

Proof: Let A be a pre-closed and G be an ωα-open set in X such that A ⊆ G. Since A is pre-closed, we have pcl(A) = A. So that pcl(A) ⊆ A ⊆ G. Therefore pcl(A) ⊆ G. Hence A is pgωα-closed set.

Example 3.5: Let X= {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Then the set A = {b} is pgωα-closed but not pre-closed set in X.

Theorem 3.6: Every α-closed set is pgωα-closed but not conversely.

Proof: Since every α-closed set is pre-closed and Theorem 3.4, the proof follows.

Example 3.7: In Example 3.3, the subset A = {a, b} is pgωα-closed but not α-closed set in (X, τ).

Theorem 3.8: Every pgωα-closed set is spgωα-closed set but not conversely.

Proof: Let A be a pgωα-closed and G be an ωα-open set in X such that A ⊆ G. As A is pgωα-closed, we have pcl(A) ⊆ G. But spcl(A) ⊆ pcl(A) is always true. So that spcl(A) ⊆ pcl(A) ⊆ G. Therefore spcl(A) ⊆ G. Hence A is spgωα-closed set.

Example 3.9: Let X= {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Then the set A = {a } is spgωα-closed but not pgωα-closed set in X.

Theorem 3.10: Every gωα-closed set is pgωα-closed.

Proof: Let A be gωα-closed set and G be an ωα-open set in X such that A ⊆ G. since A is gωα-closed. We have αcl(A) ⊆ G. But pcl(A) ⊆ αcl(A) is always true. So that pcl(A) ⊆ αcl(A) ⊆ G. Therefore pcl(A) ⊆ G. Hence A is pgωα-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11: In the Example 3.3, the subset A = {a, b} is pgωα-closed set but not gωα-closed set in X.

Theorem 3.12: Every pgωα-closed set is gp-closed but not conversely.

Proof: Let A be a pgωα-closed set in X. Let G be an open set, so that G is ωα-open such that A ⊆ G. Since A is pgωα-closed, pcl(A) ⊆ G. Hence A is gp-closed set.

Example 3.13: Let X= {a, b, c} and τ = {X, ϕ, {a}}. Then the set A = {a, b} is gp-closed but not pgωα-closed set in X.

Theorem 3.14: Every pgωα-closed set is gsp-closed but not conversely.
Proof: Since every gp-closed set is gsp-closed and Theorem 3.14, the proof follows.
Example 3.15: In Example 3.13, the subset A = {a, b} is gsp-closed but not pgωα-closed set in X.
Remark 3.16: The concept of pgωα-closed set is independent of the concept of sets namely g-closed, αg-closed, αgr-closed gpr-closed, g*-closed, g*p-closed, ωα-closed sets as seen from the following examples.
Example 3.17: In Example 3.13, the subset A = {a, b} is g-closed, αg-closed, αgr-closed, ωα-closed but not pgωα-closed set in (X, τ).
Example 3.18: Let X= {a, b, c} and τ = {X, ϕ, {a}, {a, c}}. Then the subset A = {a, b} is g*-closed, g*p-closed, gpr-closed but not pgωα-closed set in X.
Example 3.19: Let X= {a, b, c} and τ = {X, ϕ, {a, b}}. Then the subset A = {b} is pgωα-closed set but not αgr-closed, g*-closed, αg-closed, g*p-closed, gp-closed, gpr-closed, ωα-closed in (X, τ).
Remark 3.20: Union of two pgωα-closed sets need not be a pgωα-closed set as seen from the following example.
Example 3.21: In Example 3.19, the subsets {a} and {b} are pgωα-closed sets but their union {a} ∪ {b} = {a, b} is not a pgωα-closed set in (X, τ).
Theorem 3.22: If a subset A of X is pgωα-closed, then pcl(A) – A does not contain any non-empty ωα-closed set in (X, τ).
Proof: Suppose that A is pgωα-closed set and F be a non-empty ωα-closed subset of pcl(A) – A. Then F ⊆ pcl(A) ∩ (X –F). Since (X – F) is ωα-open and A is pgωα-closed, pcl(A) ⊆ (X – F). Since (X – F) is ωα-open and A is pgωα-closed, Then pcl(A) ⊆ (X – F). Therefore F ⊆ (X – pcl(A)). Then F ⊆ pcl(A) ∩ (X – pcl(A)) = ϕ. That is F = ϕ. Thus pcl(A) – A does not contain any non-empty ωα-closed set in (X, τ).
Theorem 3.23: For an element x ∈ X, the set X – {x} is pgωα-closed or ωα-open.
Proof: Suppose X – {x} is not ωα-open set. Then X is only ωα-open set containing X – {x} and also (X – {x}) ⊆ X. Hence X – {x} is pgωα-closed set in X.
Theorem 3.24: If a subset of a topological space X is pgωα-closed such that A ⊆ B ⊆ pcl(A), then B is also pgωα-closed.
Proof: Let G be an ωα-open set in X such that B ⊆ G, then A ⊆ G. Since A is pgωα-closed, pcl(A) ⊆ G. By hypothesis, pcl(B) ⊆ pcl(pcl(A)) = pcl(A) ⊆ G. Consequently, pcl(B) ⊆ G. Therefore B is also pgωα-closed set in (X, τ).

The converse of the above theorem need not be true as seen from the following example.

Example 3.25: In Example 3.3, the set A = {a} and B = {a, b} such that A and B are pgωα-closed sets but A ⊆ B ⊄ pcl(A).
Theorem 3.25: If A is open and gsp-closed set, then A is pgωα-closed set in X.
Proof: Let A be an open and gsp-closed set in X, Let A ⊆ U and U be a ωα-open in X. Now A ⊆ A. By hypothesis, pcl(A) ⊆ A. That is pcl(A) ⊆ U. Hence A is pgωα-closed in X.
Theorem 3.26: If A is ωα-open and pgωα-closed then A is preclosed in X.
Proof: Let A ⊆ A, where A is ωα-open, Then pcl(A) ⊆ A as A is pgωα-closed in X, But A ⊆ pcl(A) is always true. Therefore A = pcl(A) . Hence A is preclosed in X.
Theorem 3.27: If A is a pgωα-closed set in X and A ⊆ Y ⊆ X, then A is a pgωα-closed set relative to Y.
Proof: Let A ⊆ Y ∩ G, where G is an ωα-open set in X. Then A ⊆ Y and A ⊆ G. Since A is pgωα-closed set in X, so pcl(A) ⊆ G which implies that Y ∩ plc(A) ⊆ Y ∩ G. Hence A is spgωα-closed relative to Y.
Theorem 3.28: If A is both open and g-closed in X, then it is pgωα-closed in X.
Proof: Let A be an open and g-closed set in X. Let A ⊆ U and U be a ωα-open set in X. Now A ⊆ A, By hypothesis, cl(A) ⊆ A, so that pcl(A) ⊆ cl(A) ⊆ A, that is pcl(A) ⊆ A. Thus pcl(A) ⊆ U, Hence A is pgωα-closed in X.
Remark 3.29: If A is both pre-open and pgωα-closed in X, then A need not be g-closed in general as seen from the following example.
Example 3.30: Let X= {a, b, c} and τ = {X, ϕ, {a,bc}}. Then the subset {b} is pre-open and pgωα-closed, but not g-closed.
Definition 3.31[4]: The intersection of all ωα-open subsets of (X, τ) containing A is called the ωα-kernel of A and is denoted by ωα-ker(A).
Theorem 3.32: A subset A of X is pgωα-closed if and only if pcl(A) ⊆ ωα-ker(A).

Proof: Suppose that A is pgωα-closed, pcl(A) ⊆ U whenever A ⊆ U and U is ωα-open. Let x ∈ pcl(A) and suppose x ∉ ωα-ker(A), then there is a ωα-open set U containing A such that x is not in U. Since A is pgωα-closed, pcl(A) ⊆ U. We have x is not in pcl(A), which is contradiction. Hence x ∈ ωα-ker(A) and so pcl(A) ⊆ ωα-ker(A).
Conversely, let pcl(A) ⊆ ωα-ker(A). If U is any ωα-open set containing A, then ωα-ker(A) ⊆ U. That is pcl(A) ⊆ ωα-ker(A) ⊂ U. Therefore A is pgωα-closed in X.
Now we introduce the following.
Definition 3.33: A subset A of a topological space (X, τ) is called pre generalized ωα-open (briefly pgωα-open) set in X if A^c is pgωα-closed in (X, τ).
Theorem 3.34: Every singleton point set in a space is either pgωα-open or ωα-open in X.
Proof: Let X be a topological space. Let x ∈ X. We prove {x} is either pgωα-open or ωα-open, i.e. X\{x} is either pgωα-closed or ωα-open. From Theorem 3.23, we have X\{x} is pgωα-closed or ωα-open. Thus {x} is either pgωα-open or ωα-open in X.
Theorem 3.35: A subset A of a topological space X is pgωα-open, then F ⊆ pint(A) whenever F ⊆ A and F is ωα-closed in (X, τ).
Proof: Assume that A is pgωα-open. Then A^c is pgωα-closed. Let F be a ωα-closed set in X contained in A. Then F^c is ωα-open set containing A^c in (X, τ). Since A^c is pgωα-closed, this implies that pcl(A) ⊆ F^c. Taking complements on both sides, we have F ⊆ pint(A).
Theorem 3.36: If pgωα spint(A) ⊆ B ⊆ A and if A is a pgωα-open, then B is a αg*s-open in (X, τ).
Proof: We have pint(A) ⊆ B ⊆ A. Then A^c ⊆ B^c ⊆ pcl(A^c) and since A^c is pgωα-closed set. By the Theorem 3.24, B^c is pgωα-c osed. Hence B is a pgωα-open.

IV. CONCLUSIONS

In this paper, we have introduced the new class of generalized form of closed sets namely ωα- closed established their relationships with some generalized sets in topological space.

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