Occasionally Weak Compatible Mappings and Common Fixed Point Theorem in Intuitinistic Fuzzy Metric Space

Rajesh Shrivastava[1], Arihant Jain[2] and Amit Kumar Gupta[1]
[1]Department of Mathematics, Govt. Science and Commerce College,Benazir, Bhopal (M.P.) India
[2]Department of Mathematics, Shri Guru Sandipani Girls' Institute of Professional Studies, Ujjain (M.P.) 456 550 India
[1]Department of Mathematics,Govt. Science and Commerce College,Benazir, Bhopal(M.P.) India

Abstract

The object of this paper we use the concept of occasionally weak-compatible mapping and prove a common fixed point theorem in Intuitionistic Fuzzy Metric space.

Keywords: *Intuitionistic Fuzzy Metric space, Common fixed points, Compatible maps, Weak compatibility andoccasionally weak-compatible mapping.*

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I. INTRODUCTION

In 1965 the concept of fuzzy sets was defined by Zadeh [16] since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, George and Veermani [4] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions.Pant [9,10,11] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets, In 2004, Park [12] defined the concept of intuitionistic fuzzy metric space with the help of continuous t- norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] using the notion of intuitionistic fuzzy sets and defined the concept of intuitionistic fuzzy metric space with the help of continuous t–norms and continuous t-conorms as a generalization of fuzzy metric space which is introduced by Kramosil and Michalek [7]. Turkoglu et al. [14] gave generalization of Jungck's [5]. Common fixed point theorem in intuitionistic fuzzy metric spaces. They first create the concept of weakly commuting and R-weaklycommuting mappings in intuitionistic fuzzy metric spaces. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the converse is not true. After that, many authors proved common fixed point theorems using different mappings in such spaces. Al-Thagafi and N. Shahzad [2] introduced the concept of occasionally weakly compatible mappings which is more general than the concept of weakly compatible mappings.The aimed of this paper presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in intuitionistic fuzzy metric spaces.

II. PRELIMINARIES

Definition (2.1)[12]: A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition (2.2)[12]: A binary operation ◊: [0, 1] × [0, 1]→[0, 1] is continuous t-conorm if ◊ is satisfying the following conditions:

(i) ◊ is commutative and associative;

(ii) ◊ is continuous;

(iii) a ◊ 0 = a for all a ∊ [0, 1];

(iv) a ◊ b ≥ c ◊ d whenever a ≤ c and b ≤ d for all a, b, c, d∊ [0, 1].

Definition (2.3)[1]: A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on X^2× (0, ∞) satisfying the following conditions:-

(i) M(x, y, t) + N(x, y, t) ≤ 1, for all x, y ∊ X and t > 0;

(ii) M(x, y, 0) = 0, for all x, y ∊ X;

(iii) M(x, y, t) = 1, for all x, y ∊ X and t > 0 if and only if x = y;

(iv) M(x, y, t) = M(y, x, t), for all x, y ∊ X and t > 0;

(v) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s), for all x, y, z ∊ X and s, t > 0;

(vi) For all x, y ∊ X, M(x, y,·): [0, ∞) → [0, 1] is continuous;

(vii) $\lim_{t\to\infty} M(x_{,}y_{,t}) = 1$, for all x, y∊ X and t > 0;

(viii) N(x, y, 0) = 1 for all x, y ∊ X;

(ix) N(x, y, t) = 0 for all x, y∊ X and t > 0 if and only if x = y;

(x) N(x, y, t) = N(y, x, t) for all x, y∊ X and t > 0;

(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∊ X and s, t > 0;

(xii) For all x, y ∊ X, N(x, y, ·) : [0, ∞)→[0, 1] is continuous;

(xiii) $\lim_{t\to\infty} N(x_{,}y_{,t}) = 0$,for all x, y in X;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark [2.1]:Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as:

x ◊ y = 1- ((1-x) * (1-y)) , for all x, y∊ X

Remark [2.2]:In intuitionistic fuzzy metric space (X, M, N, *, ◊), M(x, y , .) is non-decreasing and N(x, y, ·) is non-increasing for all x, y ∊ X.

Example [2.1]:Let (X, d) be a metric space, define t-norm a * b=Min {a, b} and t- conorm

a ◊ b = Max{a b} and for all x,y ∊ X and t > 0

$$M_d(x,y,t) = \frac{t}{t+d(x,y)} \quad , N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric

Definition (2.4)[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(a) A Sequence $\{x_n\}$ in X is said to be Cauchy sequence if for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_{n,t}) = 1, \lim_{n\to\infty} N(x_{n+p}, x_{n,t}) = 0$$

(b)) A Sequence $\{x_n\}$in X is said to be Convergent to a point x∊ X if , for all t >0,

$$\lim_{n\to\infty} M(x_{n+p}, x_{n,t}) = 1, \qquad \lim_{n\to\infty} N(x_{n+p}, x_{n,t}) = 0$$

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi) of definition (2.3), respectively.

Definition (2.5)[1]: An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition (2.6)[13]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. Then the maps A and B are said to be compatible if, for all t > 0,

$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$, $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$

Whenever $\{x_n\}$ is a sequence in X such that- $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$, for some x∊ X.

Definition (2.7)[6]: Two self-maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weak compatible if they commute at their coincidence points. i.e. Ax = Bx for some x in X, then

$$ABx = BAx.$$

Definition (2.8)[6]: Let (X, M, N, *,◊) be an intuitionistic fuzzy metric space, A and B be self maps in X, Then a point x in X is called a coincidence point of A and B iff Ax = Bx. In this case y = Ax = Bx is called a point of coincidence of A and B.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition (2.9)[[2]: Two self mappings A and B of a intuitionistic fuzzy metric space (X, M, N, ∗, ◊) are said to be occasionally weakly compatible(owc) iff there is a point x in X which is coincidence point of A and B at which A and B commute.

Lemma (2.1)[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X , if there exit a number k∊(0, 1) such that-

$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$

$N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

For all t > 0 and n=1, 2, 3………., then $\{y_n\}$ is a Cauchy sequence in X.

***Lemma (2.2)[15]*:** Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if there exists a number k∊(0, 1)

$$M(x, y, kt) \geq M(x, y, t) \quad \text{and} \quad N(x, y, kt) \leq N(x, y, t), \text{ then } x=y.$$

III. MAIN RESULT

Theorem3.1Let (X, M, N, *, ◊) bea complete Intuitionistic Fuzzy Metric Spaces with continuous t-norm andletA,B,S and T be self-mappings of X. Let the pairs {A, S} and {B, T}be owc. If thereexistsq ∈ (0, 1) suchthat-

$M(Ax, By, qt) \geq \min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t),$
$M(Ax,Ty,t), M(By,Sx,t)\}$……………… (1)

And

$N(Ax, By, qt) \leq \max\{ N(Sx, Ty, t), N(Sx, Ax, t), N(By, Ty, t),$

$N(Ax,Ty,t), N(By,Sx,t) \}$………………(2)

for allx, y∈Xand for all t > 0,then thereexists a unique pointw∈Xsuch that Aw = Sw = w and a unique point z∈ X such that Bz = T z = z. Moreover,z=w,so thatthereisauniquecommonfixedpointofA,B,Sand T.

Proof:Let the pairs (A, S) and (B, T)be owc, so there are points x, y∈X such that Ax=Sx and By =Ty. We claim that Ax=By If not, by inequality (1) & (2) –

$M(Ax,By,qt) \geq \min\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t), M(Ax,Ty,t),M(By,Sx,t)\}$

$=\min\{M(Ax,By,t),M(Ax,Ax,t),M(By,By,t),M(Ax,By,t),M(By,Ax,t)\}$

$= M(Ax, By, t)$

And

$N(Ax,By,qt) \leq \max\{N(Sx,Ty,t), N(Sx,Ax,t), N(By,Ty,t), N(Ax,Ty,t),N(By,Sx,t)\}$

$=\max\{N(Ax,By,t), N(Ax,Ax,t),N(By,By,t), N(Ax,By,t),N(By,Ax,t)\}$

$= N(Ax, By, t)$.

Therefore Ax = By, i.e. Ax = Sx = By = Ty. Suppose that there is another point z such that Az = Sz then by (1) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 2.14 w is the only common fixed point of A and S. Similarly there is a unique point z ∈ X such that z = Bz = Tz.

Assume that $w \neq z$, we have

$M(w, z, qt) = M(Aw, Bz, qt)$

$\geq \min\{M(Sw,Tz,t),M(Sw,Az,t),M(Bz,Tz,t), M(Aw,Tz,t),M(Bz,Sw,t)\}$

$=\min\{M(w,z,t),M(w,z,t),M(z,z,t), M(w,z,t),M(z,w,t)\}$

$= M(w, z, t)$

And

$N(w, z, qt) = N(Aw, Bz, qt)$

$\leq \max\{N(Sw,Tz,t),N(Sw,Az,t),N(Bz,Tz,t), N(Aw,Tz,t),N(Bz,Sw,t)\}$

$=\max\{N(w,z,t),N(w,z,t),N(z,z,t), N(w,z,t),N(z,w,t)\}$

$= N(w, z, t)$

Therefore we have z = wby Lemma 2.14 and z is a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (1) &(2).

Theorem3.2Let (X, M, N, *,◊) bea complete Intuitionistic Fuzzy Metric Spaces andletA,B,S and T be self-mappings of X. Let the pairs (A, S) and (B, T)be owc. If there exists q ∈ (0, 1) suchthat-

$M(Ax, By, qt) \geq \phi(\min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax,Ty,t),M(By,Sx,t)\})$

And

$N(Ax, By, qt) \leq \Psi(\max\{N(Sx, Ty, t), N(Sx, Ax, t), N(By, Ty, t), N(Ax,Ty,t),N(By,Sx,t)\})$

for all x, y ∈ X and $\phi: [0, 1] \to [0, 1]$ and $\Psi : [0, 1] \to [0, 1]$ such that and $\phi(t) > t$, $\Psi(t) < t$ for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

Proof:The proof follows from Theorem 3.1.

Theorem3.3Let (X, M, N, *, ◊) be a complete Intuitionistic Fuzzy Metric Spaces andletA,B,S and T be self-mappings of X. Let the pairs (A, S) and (B, T) be owc. If there existsq $\in$ (0, 1) suchthat

$M(Ax, By, qt) \geq \phi(\min\{ (M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t),$

$M(Ax,Ty,t),M(By,Sx,t)\})$…………..(3)

And

$N(Ax, By, qt) \leq \Psi(\min\{ (N(Sx, Ty, t), N(Sx, Ax, t), N(By, Ty, t),$

$N(Ax,Ty,t),N(By,Sx,t)\})$…………..(4)

for all x, y $\in$ X andϕ: $[0, 1]^5 \rightarrow [0, 1]$ and $\Psi : [0, 1]^5 \rightarrow [0, 1]$ Such that$\phi(t, 1, 1, t, t) > t$ for all$0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

Proof: Let the pairs (A, S) and (B, T)are owc, so there are points x, y$\in$X such that Ax=Sxand By = Ty . We claim that Ax$\neq$ By by inequality (3) & (4) –

$M(Ax, By, qt) \geq \phi(\min\{ (M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t),$

$M(Ax,Ty,t),M(By,Sx,t)\})$

$= \phi(\min\{ (M(Ax, By, t), M(Ax, Ax, t), M(By, By, t),$

$M(Ax,By,t),M(By,Ax,t)\})$

$= \phi(\min\{ (M(Ax, By, t), 1, 1, M(Ax,By,t),M(By,Ax,t)\})$

$> M(Ax, By, t),$

And

$N(Ax, By, qt) \leq \Psi(\max\{ (N(Sx, Ty, t), N(Sx, Ax, t), N(By, Ty, t),$

$N(Ax,Ty,t),N(By,Sx,t)\})$

$= \Psi(\max\{ (N(Ax, By, t), N(Ax, Ax, t), N(By, By, t),$

$N(Ax,By,t),N(By,Ax,t)\})$

$= \Psi(\max\{ (N(Ax, By, t), 1, 1,N(Ax,By,t),N(By,Ax,t)\})$

$< N (Ax, By, t),$

a contradiction, therefore Ax = By, i.e. Ax = Sx = By = T y.Suppose that there is a another point z such that Az = Sz then by (3) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = T x is the unique point of coincidence of A and T . By Lemma2.14 *w* is a unique common fixed point of A and S. Similarly there is a unique point z$\in$ X such that z = Bz = T z. Thus z is a common fixed point of A, B, S and T . The uniqueness of the fixed point holds from (3).

Theorem3.4Let (X, M, N, *, ◊) be a Complete Intuitionistic Fuzzy Metric Spaces andletA,B,S and T be self-mappings of X. Let the pairs {A, S} and {B, T } be owc. If there exists q $\in$ (0, 1) , for all x, y $\in$ X and t > 0,

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$……….(5)

And

$N(Ax, By, qt) \leq N(Sx, Ty, t) * N(Ax, Sx, t) * N(By, Ty, t) * N(Ax,Ty,t)$………..(6)

then there exists a unique common fixed point of A, B, S and T .

Proof: Let the pairs (A, S) and (B, T)are owc, there are points x, y∈X such that Ax=Sxand By = Ty . We claim that Ax = By by inequality (5) & (6)–

We have

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$

$= M(Ax, Ty, t) * M(Ax, Ax, t) * M(By, By, t) * M(Ax, Ty, t)$

$= M(Ax, Ty, t) * 1 * 1 * M(Ax, Ty, t)$

$\geq M(Ax, Ty, t)$

And

$N(Ax, By, qt) \leq N(Sx, Ty, t) * N(Ax, Sx, t) * N(By, Ty, t) * N(Ax, Ty, t)$

$= N(Ax, Ty, t) * N(Ax, Ax, t) * N(By, By, t) * N(Ax, Ty, t)$

$= N(Ax, Ty, t) * 1 * 1 * N(Ax, Ty, t)$

$\leq N(Ax, Ty, t)$

Thus We have Ax = By, i.e. Ax = Sx = By = T y.Suppose that there is a another point z such that Az = Sz then by (5)&(6) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S . Similarly there is a unique point z ∈Xsuch that z = Bz = Tz. Thus w is a common fixed point of A, B,S and T.

Corollary 3.5: Let (X, M, N, *, ◊) be Complete Intuitionistic Fuzzy Metric Spaces andletA,B,S and T be self-mappings of X. Let the pairs {A, S} and {B, T}be owc. If there exists q ∈ (0, 1) , for all x, y ∈ X and t > 0,

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) *$

$M(By, Sx, 2t) * M(Ax, Ty, t) \ldots\ldots\ldots (7)$

And

$N(Ax, By, qt) \leq N(Sx, Ty, t) * N(Ax, Sx, t) * N(By, Ty, t) *$

$N(By, Sx, 2t) * N(Ax, Ty, t) \ldots\ldots\ldots (8)$

Then there exists a unique common fixed point A, B, S, and T.

Proof: We have

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) *$

$M(By, Sx, 2t) * M(Ax, Ty, t)$

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Sx, Ty, t) *$

$M(Ty, By, t) * M(Ax, Ty, t)$

$\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$

And

$N(Ax, By, qt) \leq N(Sx, Ty, t) * N(Ax, Sx, t) * N(By, Ty, t) * N(By, Sx, 2t) *$

$N(Ax, Ty, t)$

N(Ax, By, qt) ≤ N(Sx, Ty, t) ∗N(Ax, Sx, t) ∗N(By, Ty, t) ∗N(Sx, Ty, t) ∗

N(Ty,By,t)∗N(Ax, Ty, t)

≤ N(Sx, Ty, t) ∗N(Ax, Sx, t) ∗N(By, Ty, t) ∗N(Ax, Ty, t)

And therefore from theorem (3.4), A, B, S and T have a common fixed point.

IV. CONCLUSION

In view of theorem3.1 is generalization of the result. The pairs of self maps has been restricted to occasionally weak compatible mapping.

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VI. REFERENCES

[1] Alaca, C. , Turkoglu, D. and Yildiz, C. : Fixed points in intuitionistic fuzzy metric spaces ,Smallerit Choas, Solitons & Fractals, c 29(5)(2006), 1073-1078.

[2] Al-Thagafi, M. A. and Shahzad, N.: Generalized I-nonexpansive selfmaps and invariant Approximations, Acta. Math. Sinica, 24(5)(2008), 867-876.

[3] Atanassov, K. : Intuitionistic Fuzzy sets, Fuzzy sets and system, 20(1986) 87- 96.

[4] George, A. and Veeramani, P. : On some result in fuzzy metric space, Fuzzy Sets Systems 64 (1994), 395-399.

[5] Jungck ,G. : Commuting mappings and fixed points, Amer. Math. Monthly, 83(1976), 261- 263.

[6] Jungck, G. : Compatible mappings and common fixed points, Internat. J. Math. Sci. 9(1986), 771-779.

[7] Kramosil, I. and Michalek, J.: Fuzzy metric and Statistical metric spaces, Kybernetica, 11 (1975) 326-334.

[8] Pant, B. D. and Chouhan, S. : fixed point theorem for occasionally weakly compatible mapping in Menger Space, Mathematiqki Vesnikcs (Accepted for Publication).

[9] Pant R.P., "Common fixed points of four mappings", Bull. Cal. Math.Soc. 90, (1998),281-286.

[10] Pant R.P.,"Common fixed point theorems for contractive maps", J. Math. Anal. Appl.226 (1998), 251-258.

[11] Pant R. P.,Jha K.,"A remark on common fixed points of four mappings in a fuzzy metric space", J. Fuzzy Math. 12(2) (2004), 433-437.

[12] J Park, H. : Intuitionistic fuzzy metric spaces, chaos, Solitions & Fractals 22(2004), 1039- 1046.

[13] Park, J. S. and S. Kim, S. Y. : Common fixed point and example in intuitionistic fuzzy metric space, J. K. I. I. S. 18(2008), NO. 4, 524-529.

[14] Turkoglu,D. , Alaca, C., Cho, Y.J. and Yildiz, C. : Common fixed point theorems in intuitionistic fuzzy metric spaces, J. Appl. Math. & Computing, 22(2006),411-424.

[15] Turkoglu, D., Alaca,C. and Yildiz, C, : Compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39(2006), 671-684.

[16] Zadeh, L. A.: Fuzzy sets, Inform. and Control 8(1965), 338-353.