Congruent Series of Even Powered Number With Respect to the Modulus '144'

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Abstract

*This document introduces **1)** a modulus '144' ,which leaves certain remainder , when divides any even powered number . **2)** Within a certain power the whole number system makes certain congruent families , making a unit. **3)** Within a family the congruent members are placed in particular orders. **4)** There are Seven kinds of such arrangements (unit) of congruent members happen , which repeat in a cyclic order throughout whole even power. **5)** Congruent series of square numbers is useful in determining an odd number to be prime or composite.*

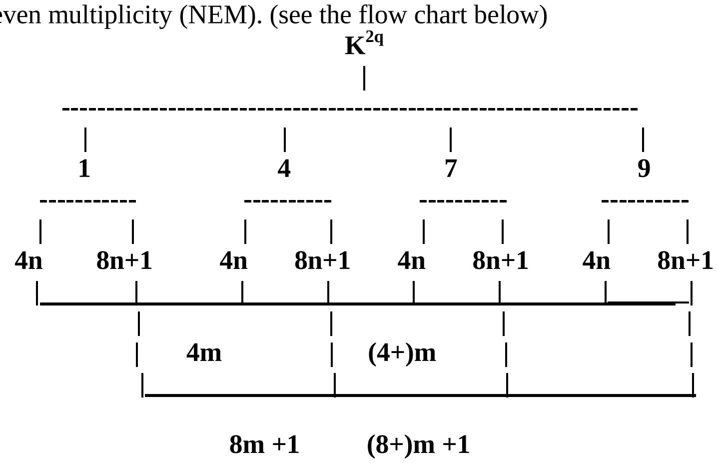
key words - *Congruent series, Unique Remainders, Modulus '144' , Prime numbers , Composite numbers , Remainder families .*

I. INTRODUCTION

Aim of this document is to make the schools to put an eye to the importance of the modulus '144' and researches may be carried out to facilitate needful applications . The congruent families and it's seven unit may be helpful in mathematics world . Unlike principle of Algebra we know , it deals with constant integers , produced by amazing nature of '144'. Here only **1)** One direct application is mentioned , that is easy calculation of remainder when any powered number is divided by '144' or it's factors as well. **2)** The series of square families can be **applicable** to determine a number to be prime or composite ,. though some calculations involve in this method, it is a little efficient than divisibility test . This document deals with +ve integers only .

II. CATAGORIES OF EVEN POWERED NUMBER (K^2Q)

All Perfect even powered numbers in the number system can be divided into sixteen categories in all. This division depends upon two factors. Let the number is of the form K^2q. The main factors is the sum of digits of K^2q into unit digit (SDG) which are four in numbers are '1', '4' , '7' and '9' . Further 'K' can be even or odd . Each even K^2q may be multiple of '4' or greater even multiples (i.e. 4m or (4+)m) & each odd K^2q (8n+1) may be of the form (8m+1) or (8+)m+1. Where 'm' is odd and (+) indicates greater even multiple . Let us name them as nature of even multiplicity (NEM). Hence SDG & NEM are Two characteristics' depending upon which we can categorise a number. Let us see how? Each SDG category can be sub-divided into four types according as nature of even multiplicity (NEM). (see the flow chart below)



As Example SDG 1 category - 127^2 NEM type (8+)m [NEM 126 * NEM 128] , 235^2 NEM 8m , 152^2 NEM (4+)m , 190^2 NEM 4m .

SDG 4 category - 79^2 NEM (8+)m , 61^2 NEM 8m , 56^2 NEM (4+) , 74^2 NEM 4.

SDG 7 category - 49^2 NEM (8+)m , 77^2 NEM 8m , 68^2 NEM (4+) , 58^2 NEM 4.

SDG 9 category - 57^2 NEM (8+)m , 51^2 NEM 8m , 48^2 NEM (4+) , 42^2 NEM 4.

III. SIXTEEN UNIQUE REMAINDERS

It is interesting that each of the above mentioned sixteen categories (K^2q) when divided by '144' leaves an unique remainder. Table I mentions remainders of even powered numbers (power 2 through power 26) [mod '144'] .

Rule I :- All perfect even powered numbers in the number system can be categorise into sixteen types according as particular sum of digit into unit digit and particular nature of even multiplicity. Each category when divided by '144' leaves an unique remainder .

I. TABLE I
Remainders of k^2q [mod '144']
Even powers (2q)---------

2	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*	*22*	*24*	*26*
1	1	1	1	1	1	1	1	1	1	1	1	1
73	1	73	1	73	1	73	1	73	1	73	1	73
64	64	64	64	64	64	64	64	64	64	64	64	64
100	64	64	64	64	64	64	64	64	64	64	64	64
49	97	1	49	97	1	49	97	1	49	97	1	49
121	97	73	49	25	1	121	97	73	49	25	1	121
4	16	64	112	16	64	112	16	64	112	16	64	112
112	16	64	112	16	64	112	16	64	112	16	64	112
25	49	73	97	121	1	25	49	73	97	121	1	25
97	49	1	97	49	1	97	49	1	97	49	1	97
16	112	64	16	112	64	16	112	64	16	112	64	16
52	112	64	16	112	64	16	112	64	16	112	64	16
9	81	9	81	9	81	9	81	9	81	9	81	9
81	81	81	81	81	81	81	81	81	81	81	81	81
0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0

Note that the Remainders include all the squares of integers 0 to 11 and four others 52, 73, 97 and 112.

A. Column wise (rem for a given power) :- Look at above Table I , Ist column mentions Remainders of all numbers even or odd raised to power 2 with respect to the modulus '144' . Second column mentions that of power 4 & subsequently 7th column that of power 14 . The cycle from power 4 to power 14 repeats through out . As a example 115^16 %144 leaves REM '97' & 115^18 %144 leaves REM '73' & so on. Look at the pattern. K^2 is unique where K^16 resembles K^4 & K^26 resembles K^14 & so on. K^2q being the even powered +ve integer where 'K' , 'q' each belongs to 1----infinity , P = 2q%12 , REM of 2q th power resembles that of P th power with the exceptions that if P=0 , P ~ 12 & if P = 2 , P ~ 14 . Hence there are seven different families (patterns) which occur in a particular cyclic order in the whole number system. Let us name them as **2EXP , 4EXP , 6EXP….14EXP** families.

B. Row wise (rem for a given base 'k') **:-**The Header row mentions the even power and the subsequent rows mention corresponding remainders of 'K^2q' % 144 . The first four rows for SDG K = '1' or '8', second four for SDG K = '2' or '7' , third four for SDG K = '4' or '5', and fourth four for SDG K = '3' ,'6' or '9'. We can see clearly that, each of the Remainders (REM) follow a particular sequence. . One can calculate remainders of any powered number divided by '144' or it's factors within a seconds easily. For calculation of REM we don't have to investigate SDG & NEM of 'K^2q' , rather with the help of SDG & NEM of K , obtain REM of square of the number (mod 144) , and thereafter REM of the required powered number (mod 144) . List below mentions corresponding SDGs of K for each SDG of 'K^2'.

SDG K^2	**SDG K**	**Remainders**
========	======	===========
1	1 or 8	1, 73, 64, 100
4	2 or 7	49, 121, 4, 112

7	4 or 5	25, 97, 16, 52
9	3 , 6 or 9	9, 81, 0, 36

***C. Calculation of remainder* :-** For calculation of remainder of any powered **number % 144** ,the base is the **square of that numbe**r. Once we know **SDG & NEM** of K we can know REM of **base%144** and thereafter **REM** of that **powered number%144** following the above pattern **(Table I)** easily. Let us see how to obtain it. Let the number be **K^2q ,** where **'K'** & **'q'** each represent for any +ve integer. With the help of **SDG & NEM of K , REM =** K^2 {mod 144 **)** can be obtained easily (table II & table III) . Use pattern from **Table I** to find out the required Remainder .

II. TABLE II
'K' → EVEN

SDG 'K'	*NEM 'K'*	*REM 'K^2' %144*
1 or 8	2	100
	2+	64
2 or 7	2	4
	2+	112
4 or 5	2	52
	2+	16
3 , 6 or 9	2	36
	2+	0

III. TABLE III
'K' →_ODD

SDG 'K'	**NEM (K+1)*(K-1)**	**REM 'K^2' %144**
1 or 8	8	73
	8+	1
2 or 7	8	121
	8+	49
4 or 5	8	25
	8+	97
3 , 6 or 9	8	9
	8+	81

Example :- **(151^123) % 144 =** Here NEM of 152 is 8 & naturally NEM of 150 is 2. Hence NEM of (k+1)*(k-1) is (8+) & SDG of 151 is '7' , Hence 151^2 % 144 = 49 (48+1 ~ (8+)m + 1). Now base is 49. Look at base 49 **row** . We have to divide even power by '6' if remainder ='2' , REM = '49' , else if remainder ='4' , REM = '97' , else REM = '1' . 122 % 6 = 2 , 151^122 % 144 = 49 , multiply 49 x 151 & divide 144 to obtain the required remainder as '55'

III.. CONGRUENT SERIES OF EVEN POWERED NUMBERS WITH RESPECT TO MODULUS '144'

Another important thing is that we can arrange the whole number system, into series' , which raise to any power are congruent with respect to the modulus "144" within a certain power. Let us take **2EXP** (power '2') as an example.

IV. TABLE IV
SERIES OF K (REM = K^2 % 144)

SDG K SDG K^2	**REM**	**K Series**	**IO**	**PIO**
1 , 8 1	1	1, 17, 55, 71, 73, 127, 143	16, 38, 16, 2	72
	73	19, 35, 37, 53, 91, 107, 109	16, 2, 16, 38	72
	64	8, 28, 44, 64, 80, 100, 116	20, 16	36
	100	10, 26, 46, 62, 82, 98, 118	16, 20	36
	49	7, 25, 47, 65,79,97,119	18, 22, 18, 14	72

2 , 7 **4**	121	11, 29, 43, 61, 83, 101, 115	18, 14, 18, 22	72
	4	2, 34, 38, 70, 74, 106, 110	32, 4	36
	112	16, 20, 52, 56, 88, 92,1 24	4, 32	36
4 , 5 **7**	**25**	5, 13, 59, 67, 77, 85, 131	8, 46, 8, 10	72
	97	23, 31, 41, 49, 95, 103	8, 10, 8, 46	72
	16	4, 32, 40, 68, 76, 104, 112	28, 8	36
	52	14, 22, 50, 58, 86, 94, 112	8, 28	36
3, 6, 9 **9**	**9**	3,21, 27, 45, 51, 69, 75, 93	18, 6	24
	81	9, 15, 33, 39, 57, 63, 81, 87	6, 18	24
	0	12, 24, 36, 48, 60, 72, 84	12	12
	36	6, 18, 30, 42, 54, 66, 78	12	12

A. K^{2q} %144 (q = 1)

Lists of the Remainders are mentioned in the **Table IV**. The whole number system can be summarised into series', whose squares divided by '144' leave particular Remainders within a particular SDG & NEM. The elements of series are in certain increasing orders which repeat in particular period (P.I.O) . **IO** refer to Increasing Order, **PIO** refer to periodic increasing order and **REM** refer to Remainder (i.e. K^2%144) .In above Table, the first seven or eight elements of K series are mentioned.

(Refer to TABLE IV) **Note** that in case of SDG K^2 = 1,4 & 7 , K series of every odd remainder is a combination of series of four AP series with a common difference of 72, which is in a periodic increasing order(PIO). The first consecutive four numbers as the first terms of the AP series and every fifth member of the main series as the consecutive terms of the AP series respectively. Similarly every even remainder is a series of two AP series with a common difference of 36 (PIO) each. The first consecutive two numbers as the first terms of the AP series' and every third member of the main series as the consecutive terms of the AP series' respectively.

Exception is the SDG K^2 = 9 , in case of odd remainders , the common difference is '24', unlike '36', in above mentioned even remainders (SDG K^2 =1,4 & 7) , although number of AP series are two. This is because SDG K^2 =9 has three K's, where as SDG K^2 1, 4 & 7 has two K's each. Even remainders of SDG K^2 = 9 (0, 36) are single AP series with a common difference of 12 each.

Note the difference between series of 3's multiple REM & that of non 3's multiple REM. Within a certain range say range 72 the former has 6 elements & later has 4 elements.

The main K series can be resolved into subseries' according to SDG K(refer Table V) .The series are somewhat simpler than the main K series. Below table V is an example of only one SDG (K^2 = '1')

V. Table v

Sub-series of k (rem = k^2 % 144)

REM	***SDG K***	***K Sub-Series***	***IO***
1	**1**	**1,55,73,127**	**54,18**
	8	**17,71,89,143**	**54,18**
73	**1**	**19,37,91,109**	**18,54**
	8	**35,53,107,125**	**18,54**
64	**1**	**28,64,100,136**	**36**
	8	**8,44,80,116**	**36**
100	**1**	**10,46,82,118**	**36**
	8	**26,62,98,134**	**36**

The main K series of each Remainder can also be resolved into Sub-series as per particular unit number of K. These sub series are useful in testing a number to be prime.

Similarly we can arrange higher powered K^2q (q belongs 4---14) into series' , as above (K^2) . Following the seven power patterns mentioned in **Table I** . As these six power patterns (4EXP…14EXP) repeat in a cyclic order through out up to infinity, the physical order of the series of numbers repeat with power being different.

Rule II :- All perfect even powered numbers of particular <u>sum of digit into unit digit</u> having particular nature of even multiplicity, are congruent with respect to the modulus '144' .

Rule III :- Within a particular even power the whole number system act as members of certain series', which raised to the particular power are congruent with respect to modulus '144'. We can call those series' as EXP families . There are seven unit of such EXP Families are possible.

IV. DETERMINING A NUMBER TO BE PRIME OR COMPOSITE

Congruent series of square numbers can be helpful in testing a number to be prime or composite. Every odd composite no. $2n+1= (n+1)^2 - n^2$ can be expressed in the form $x^2- y^2$ $(x>y+1)$,where $(n+1)^2$ & x^2 are congruent with respect to the modulus '144', so as n^2 & y^2. The congruency can be proved easily. Here we are discussing about the numbers which are not multiple of '3'. Hence either n+1 or n must be 3's multiple .

Proof :- Let (n+1)^2 = 144a + R1 , n^2 = 144b + R2 , x^2 = 144c + R3 & y^2 = 144d + R4.

$(n+1)^2 - n^2 = x^2 – y^2$. Hence R1 - R2 = R3 - R4 (as each of the Remainders are less than 144)

R1 & R2 are opposite in nature (odd or even) so as R3 & R4.

R1 or R2 must be either 9 , 81 (if odd) or 0 , 36 (if even).

Let R1 be 3's multiple. Then R1 is either 9 , 81 or 0 ,36. Let R1 be '9'.

R2 Must be one from 64, 100, 4, 112, 52, 16. In contradiction let R3 be 81.

There is no even Remainder for R4 in the mentioned list to satisfy the relation R1 - R2 = R3 - R4.

So R3 must be '9'. We can proof this for other values of R1 (i.e. 81,0 & 36) also.

Hence x belongs to same series as (n+1) & and y belongs to same series as n .

E) **<u>Procedure to find out value of 'x' or 'y'</u>**

- Obtain Upper and Lower limit of 'x' or 'y'
- Obtain REM of (n+1)^2 or ' n^2 '
- Obtain possible unit nos. of 'x' or 'y' (as per Table V)
- From the series of REM list the elements of possible unit no's, within the limit
- Calculate squares of the numbers obtained above
- If choice is 'x' subtract 2n+1 from the squares
- else if choice is 'y' add 2n+1 with the squares individually
- If any of the result is a perfect square, 2n+1 is composite else 2n+1 is prime

F) Lower limit of 'x' or 'y'

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Lower limit of 'x^2' exceeds 2n + 1 . Lower limit of 'y' can be fix by following procedure.
Obtain square root of 2n+1 rounded to nearest higher no. (p)
$r = p^2 - (2n+1)$. $y^2 = q^2 + 2pq + r$.

F) **Upper limit of 'x' or 'y'**

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Upper limits of 'x' & 'y' .can be fixed by taking help of popular divisibility method of testing a number to be prime, by dividing 2n+1 by '7'. If not divisible, Upper limit of 'x' may be taken equal to {(2n+1)%10 + 10}/2 & that of 'y' is equal to {(2n+1)%10 - 10}/2. For that we need not divide 10 physically. we can drop the unit no. of 2n+1, and round up to next even no. (if 'odd'). In case of 2n+1 of greater digits (five digits or more) we can low the upper limit considerably by replacing '10' with 50 , 100 … at our convenience . In addition we have to perform divisibility test of 2n+1 by all the primes up to the value we replace. Let we call this value DT.

Hence **root over(2n+1) < x < {(2n+1) / DT + DT} / 2** & sqrt **$(q^2 + 2pq+ r)$ < y < {(2n+1) / DT - DT} / 2**

Example 1 -: (DT 10) 2n+1 = 1097 , n+1 = 549 , n = 548 , SDG of 549 is 9 , NEM is 8m , Hence REM of x^2 = 9

3	1097 9	34
6	197 - 256	
	- 59	

p = 34. 'r' = 59. Lower limit of 'x' = 35 and upper limit of 'x' = (110 +10) / 2 = 60. Hence 35 < x < 60 . From the Table V possible unit no. of 'x' must 1 or 9 . From congruent series of REM 9 (Table V) only 51 is the possible value of 'x'. $(51)^2$ = 2601, 2601 – 1097 = 1504 is not a perfect square number. Hence 1097 is a prime number.

Example 2 -: (DT 100) 2n+1 = 42169 , (q^2+ 412 q +267) = y^2', (1 + 412 + 267) < y^2 , n+1 = 21085 n = 21084 , n = (2+)m , REM = '0' . Possible unit numbers of 'y' are 0, 4 & 6 , 27 < y < 161 , possible values of 'y' are 60, 120, 84, 144, 36, 96, 156 . Add squares of these numbers to 2n+1 (84517) one by one to get a perfect square. No perfect square found. Hence 42169 is prime.

Alternatively DL 100 we can take n+1 instead of n (i.e. the non 3's multiple part) for our calculation . n*(n+2) = 8m, SDG $(n+1)^2$ = 4, REM = 121 , possible unit number of 'x' are 3, 5 & 7 , 207 <x < 261 , Possible values of "x" are 245 & 227.

Note that we have to deal with more elements in 3's multiple part than non 3's multiple part . But advantage is that, as this is a constant based method, 3's multiple series are easily accessible (3 in numbers) as compared to others which are 12 in numbers.

V. TABLE V

Unit No 2n+1	*x(odd)*	*x(even)*	*y(odd)*	*y(even)*
1	**1, 5, 9**	**0, 4, 6**	**3, 5, 7**	**0, 2, 8**
3	**3, 7**	**2, 8**	**1, 9**	**4, 6**
7	**1, 9**	**4, 6**	**3, 7**	**2, 8**
9	**3, 5, 7**	**0, 2, 8**	**1, 5, 9**	**0, 4, 6**

Here we are not considering unit no. 5, as it need not be tested to be prime.

V. CONCLUSIONS

Above theory of the modulo '144' will add a great value to the remainder system, Congruent series of square numbers is helpful to determine nature of a odd number (prime or composite) , when added to divisibility test method. The seven patterns of Remainders may be useful in various aspects of the mathematics world as their constituents (congruent families) are repeating in nature . Further researches can be done for suitable applications of congruent families .

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VII. REFERENCES

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