Particle Swarm Optimization (PSO) Inspired Grey Wolf Optimization (GWO) Algorithm

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Abstract

This paper presents a new modified Grey Wolf Optimization (GWO) Algorithm inspired by the Particle Swarm Optimization (PSO) algorithm. The main features of the proposed algorithm called PSO Inspired Grey Wolf Optimization (PSOIGWO) is the integration of global best and inertia weights into the basic GWO algorithm that allows the better searching capability and quicker convergence. The combination of well-established features of PSO into the newly developed GWO algorithm provides an efficient hybrid algorithm which comprises the best features of the both algorithms. Experiments on standard optimization problems show the usefulness of the combined approach and its ability to efficiently and quickly search the solution.

Keywords - *Global Optimization, Particle Swarm Optimization (PSO), Grey Wolf Optimization.*

I. INTRODUCTION

The optimal solution is animportant requirement for many practical problems where the exact solution is either not feasible or difficult to find due to its complexity.The optimization approach is used in many branches of mathematics and engineering such structure designing, aerospace modeling, travelling postman problem etc. since the computation time requiredfor the exact solution finding methods, like branch andbound, dynamic programming, increases exponentially with the size of the instance to solve. The meta-heuristic algorithms are the best alternative of the best solution algorithms as it can provide an acceptable solution with in the required margin without computational complexity.

The meta-heuristic algorithms are not only simple but also have many interesting characteristics such as problem independency, adaptivenessand learning capabilities [2]. Most of the meta-heuristic algorithms uses natural (either physical or bio-intelligence) phenomena's to find the solutions. Examples of the bio-intelligence inspired optimization algorithms are genetic algorithm, ant colony optimization, bee colony optimization, while the physical phenomenon inspired algorithms are water filling algorithm, particle swarm optimization, gravitational search algorithms etc. Although the meta-heuristic algorithms have several advantages but they also have some limitations as solution is not always guaranteed to be optimum the improper initialization could cause completely irrelative solution etc. hence for any meta-heuristic optimization algorithm these problems must be dealt properly. As sated above a number of meta-heuristic algorithms are already available but everyone has its own advantages and limitations which provide space for development of new algorithms one of such algorithm is Grey Wolf Optimization (GWO).

Grey wolf optimizer (GWO) algorithm described by Mirjalili [1] which is modeledfrom the hunting strategy of grey wolves. With results comparable to particle swarm optimization (PSO) and other optimization algorithms with fewer adjustable parameters and low complexity the GWO can be the preferable choice for deployment in practical applications. However like other optimization algorithms the GWO also has some limitations such as it can be easily trapped in the local optima when used with high-dimensional nonlinear objective functions. Furthermore the higher convergence speed of GWO makes it difficult to manage the balance between exploitation and exploration [3].

To resolve these limitations this paper presents a PSO inspired GWO optimization algorithm which applies the exploration and convergence techniques used with PSO. To validate the proposed algorithm it is tested with some well-known optimization problems and simulation results shows the superiority of the proposed algorithm. The rest of the paper is arranges as the second section presents a brief literature review of GWO and its derivatives. In third section PSO and GWO algorithms are explained, while the fourth section describes the proposed algorithm, followed by the simulation results and conclusion in fifth and sixth sections respectively.

II. LITERATURE REVIEW

The GWO algorithm is firstly proposed by the Mirjalili [1] the algorithm applies the hunting strategy followed by the grey wolves, after that many modifications have been proposed to overcome the shortcomings of the algorithm some of them are discussed in this section. Wen Long et al. [3] proposed the use of a time-varying function of decreasing linearly for changing the value of vector $\vec{a}$ which balances the exploration and exploitationabilities of the GWO. Furthermore the good-point-set method is employed for generating the initial population which enhances the global convergence of the algorithm. Aijun Zhu et al. [7] presented a hybrid GWO which utilizes the DE's strong searching abilityto update the previous best positions of Alpha, Beta andDelta wolves, the position updating in such way makes GWO prone to the stagnation. Another modification of GWO is proposed by Narinder Singh et al. [8] which modifies the position update equation of standard GWO algorithm. The presented modification uses the mean of wolf position vectors for the estimation of movement direction of wolves. The use of exponential function for the decaying the value of vector $\vec{a}$ is presented by Nitin Mittal et al. [9] the use of exponential decay function improves the exploitation and exploration capability of the algorithm.A Genetic Algorithm (GA) based initial population generation approach for GWO is presented by Qiang Li et al. [10], the proper initialization leads to greater possibility in finding global optimum. As with other meta-heuristic algorithms the GSO also requires proper initial value settings of variables to achieve the best results. Since these values depends upon problem under consideration and must be estimated on the basis of objective function characteristics to address this problem E. Emary et al [11] presented a reinforcement learning and neural network based approach EGWO (Experienced GWO) which evaluates the right parameters values for the algorithm. In their model the exploration rate of each wolf estimated bywolf's own experienceand the current environment ofthe search space. The experience is storedin the form of neural network that maps agentstates to corresponding actions. The Powell local optimization based GWO algorithm PGWO is presented by Sen Zhang et al. [12]. This proposal uses Powell's [16] conjugate direction method, is an algorithm used for finding a local minimum ofafunction. The Powell's algorithm work with non-differentiable functions, and it takes no derivatives, this makes it suitable choice for deciding the direction of movement of wolf.

III.GREY WOLF OPTIMIZATION (GWO)

The Grey Wolf Optimization (GWO) was proposed by Mirjalili et al [4]. The GWO is inspired by the social structure and hunting behavior of grey wolves. The experimental results demonstrated its capabilities and excellent performance in solving many classical engineering design problems, such as spring tension, welded beam etc. [7].

The GWO technique considers the finding optimal solution problem as hunting of prey by grey wolves. The prey is equivalent to optimal solution. As the grey wolves hunting strategy involves three steps encircling prey, hunting, and attacking prey it also uses these approaches to find the optimal solution. The grey wolves strictly follows social hierarchy of leadership. In the hierarchy the group is led by the alpha (α) wolf, which remains at the top of the hierarchy. After alpha the second level of wolves are called beta (β) wolf similarly the third and fourth level wolves are called delta (δ) and omega (ω) respectively. The alpha wolf is followed by all (beta, delta and omega) wolves, while the beta wolves are followed by delta and omega, and delta wolves are followed by only omega. Since the omega remains in the lowest level they does not have any followers.

Now as the hunting is guided by alpha, beta and delta wolves and rest (omega) wolves just follow them. The movement of all the population in the optimization problem guided by the top three best solutions and these solutions are named as alpha, beta and delta respectively the rest of solutions are considered as omega.

A. Encircling the Prey

the first step of hunting is to encircle prey. The encircling process of grey wolves is equivalent to encircling the optimum solution by all population and it is given by:

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_{prey}\,(i) - \vec{X}_{wolf}\,(i)\right| \qquad (1)$$

$$\vec{X}_{wolf}\,(i+1) = \vec{X}_{prey}\,(t) - \vec{A} \cdot \vec{D} \qquad (2)$$

Here i represents the current iteration number, $\vec{A}$ and $\vec{C}$ are the coefficient vectors, $\vec{X}_{Prey}$ and $\vec{X}_{wolf}$ are the position vectors of prey and wolf respectively. The coefficient vectors $\vec{A}$ and $\vec{C}$ are calculated as follows:

$$A = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \qquad (3)$$

$$\vec{C} = 2\vec{r}_2 \qquad (4)$$

Here the values of vector $\vec{a}$ is linearly decreased from 2 to 0 with the iterations and $\vec{r}_1, \vec{r}_2$ are random vectors bounded within the interval of [0, 1].

B. Hunting

In real hunting scenario the position of prey is known but in optimization problem the optimum solution is not known hence a rough estimation of optimum location is estimated by the alpha, beta and delta solutions knowing that they have the best knowledge of solution. The position update of wolves is done as follows:

$$\vec{X}_{wolf}\,(i+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (5)$$

where the$\vec{X}_1, \vec{X}_2$ *and* $\vec{X}_3$ are estimated as:

$$\vec{X}_1 = \vec{X}_\alpha - A_1 \cdot (D_\alpha), \quad (6)$$
$$D_\alpha = \left|C_1 \cdot X_\alpha - X_{wolf}\right|,$$

$$\vec{X}_2 = \vec{X}_\beta - A_2 \cdot (D_\beta), \quad (7)$$
$$D_\beta = \left|C_1 \cdot X_\beta - X_{wolf}\right|,$$

$$\vec{X}_3 = \vec{X}_\delta - A_3 \cdot (D_\delta), \quad (8)$$
$$D_\delta = \left|C_1 \cdot X_\delta - X_{wolf}\right|,$$

The equations 6, 7 and 8 assumes the location of prey (optimum solution) is the location of α, β and δ respectively, then the mean location of prey is estimated by equation 5, and this is the location where the wolf (population) should move to get the prey (optimum).

C. Attacking

As the grey wolf start tightening their grip to prey the movement of prey becomes more and more smaller so as the movement of wolves, and at last the prey stops moving and wolf perform final attack. This scenario is simulated in mathematical model by decreasing the values of vector $\vec{a}$ linearly from 2 to 0 with every iteration (as shown in equation 3), which limits the movements of prey (optimum location) and wolf (population locations) and finally it gets the prey (optimum).

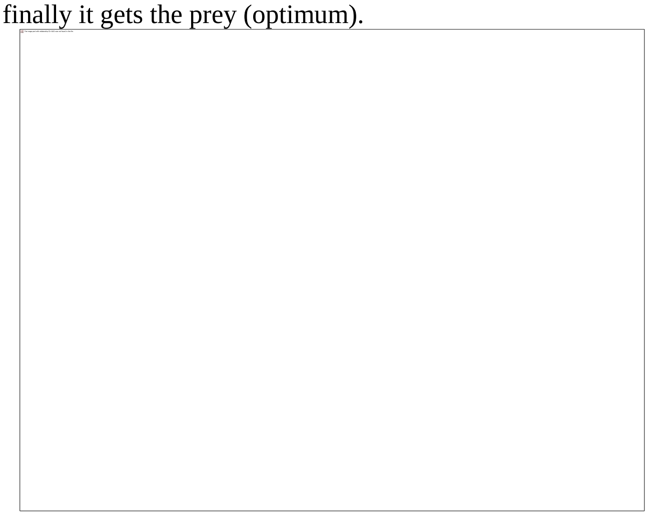


Figure 1: the position updating process in GWO as presented by Mirjalili et al [4].

IV. PARTICLE SWARM OPTIMIZATION (PSO)

This algorithm is firstly proposed by the James Kennedy et al. [18]. This algorithms was based on collaborative social behavior observed in some of the species of animals and insects like bird flocks searching for corn and fish schooling. In the PSO the solutions are presented by particles these particles are the points in the search space. The particle positions are evolved to find the optimal solutions similarly as the bird flocks searches for corn.

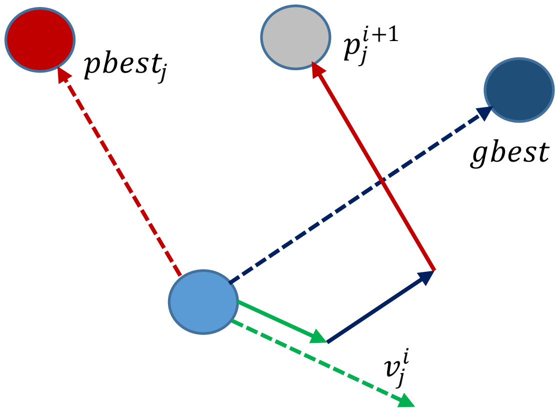


Figure 2: movement of particle in PSO [19].

The important properties of particles are they all knows the best solution found by particle itself up to current iteration as well as the best of all particles solutions found till current iteration, these positions are known as $pbest$ (particle's best) and $gbest$ (global best). The positions of particles are evolved using these two values till the solution found, the complete process can be described in following steps:

D. Initialization

firstly the particles are randomly positioned all over the search space. For example let there be n number of total particles whose location can be defined as $\{p_1, p_2, p_3, \ldots . p_n\}$.

E. Finding the fitness

each of the generated particles are evaluated for the provided objective function, let for the j^{th} particle p_j the locations and fitness values till i^{th} generation be $P_j^i = \{p_j^1, p_j^2, p_j^3, \ldots\ldots, p_j^{i-1}, p_j^i\}$, and $F_j^i = \{f_j^1, f_j^2, f_j^3, \ldots\ldots, f_j^{i-1}, f_j^i\}$ respectively. So the particle p_j will remember it location related to the best value of F_j^i which can be defined as $pbest_j$. Similarly it will also remember the values of best location related to the best values of $\{pbest_1, pbest_2, pbest_3, \ldots\ldots, pbest_n\}$ which is named as $gbest$.

Location Update: now each particle updates their location using their $pbest$ and $gbest$ as follows:

$$p_j^{i+1} = p_j^i + v_j^{i+1} \tag{9}$$

$$v_j^{i+1} = \omega v_j^i + c_1 r_1 \left(pbest_j - p_j^i\right) + c_2 r_2 (gbest - p_j^i) \tag{10}$$

$$\omega = (\omega_{max} - \omega_{min}) \left(\frac{iter_{max} - i}{iter_{max}}\right) \tag{11}$$

Where the r_1 andr_2 are the random variable within the range [0, 1], and c_1 and c_2 are the trust coefficient which defines the weightage $pbest$ and $gbest$ in the of movement ofparticle. The ω represents the inertia of the particle this controls the exploration capabilities of the algorithm.

These three parameters are problem dependent and can be fine-tuned depending upon the nature of the problem.

F. Termination

the process from step 2 to 4 are repeated until the specified terminating criteria found.

V. PARTICLE SWARM OPTIMIZATION INSPIRED GREY WOLF OPTIMIZATION (PSOIGWO)

Looking into the both GWO and PSO algorithms it can be seen that GWO uses α, β and δ wolf positions to find the solution location (equation 5) and then it updates positions of all the wolves, while the PSO uses $gbest, pbest$ and inertia (ω) (equation 10).

The involvement of inertia in PSO increases the exploration capability, while the knowledge of $gbest$ and $pbest$ keeps track on best locations of the particles encountered which increases its capabilities of both exploitation and exploration.

The proposed algorithm uses equivalents of these parameters to improve the performance of GWO as follows: The position of the wolves in GWO modified using

$$\vec{X}_{wolf}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (12)$$

In proposed algorithm this is considered as the equivalent to $gbest$ as they actually represents the mean of best three.

Next the $pbest$ and inertia (ω) are estimated in same way as in the standard PSO algorithm.So the position estimation equation for the PSO inspired GWO is defined as follows:

$$\vec{X}_j^{i+1} = f_d \cdot \omega \vec{X}_j^i + c_1 \cdot f_d \cdot r_1\left(\overrightarrow{pbest_j}\right) + c_2(1 - f_d \cdot r_2)(\overrightarrow{gbest}) \quad (13)$$

Since c_1 and c_2 are set to 1, the above equation can be re written as:

$$\vec{X}_j^{i+1} = f_d \cdot \omega \vec{X}_j^i + f_d \cdot r_1\left(\overrightarrow{pbest_j}\right) + (1 - f_d \cdot r_2)(\overrightarrow{gbest}) \quad (14)$$

Where the f_d (decay factor) and $gbest$ are given by

$$f_d = \left(\frac{a}{2}\right)^2 \quad (15)$$

$$gbest = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (16)$$

wherer_1 and r_2 are the random variables in the range [-1, 1] unlike the PSO where it remains in the range [0, 1]. The a is GWO linearly decreasing variable from 2 to 0.

The application of f_d decays the impact of inertia, $pbest$ and randomness in $gbest$ these all terms are adopted PSO features. Hence it can be said that the algorithm initially uses the PSO features for exploration of search space and then gradually shifts to GWO for convergence.

The comparison of the algorithm in pseudo code is provided in table 1.

Table 1: pseudo codes for PSO, GWO and PSOIGWO.

PSO:	GWO:	PSOIGWO:
Initialize the particle population positions $P_i(i = 1,2, \dots, n)$ and velocities $V_i(i = 1,2, \dots, n)$. Initialize the ω, c_1 andc_2. ***while***$(t <$ Max number of iterations) ***for*** each particle Update the position of	Initialize the grey wolf population$X_i(i = 1,2, \dots, n)$. Initialize a, A and C. Calculate the fitness of each search agent. X_α=the best search agent. X_β=the best search agent. X_δ=the best search agent.	Initialize the grey wolf population$X_i(i = 1,2, \dots, n)$. Initialize a, A, C, ω andf_d. Calculate the fitness of each search agent. X_α=the best search agent. X_β=the best search agent. X_δ=the best search agent. ***while*** $(t <$ Max number of iterations)

current search agent by equations:

$p_j^{i+1} = p_j^i + v_j^{i+1}$

$v_j^{i+1} = \omega v_j^i + c_1 r_1 (pbest_j - p_j^i) + c_2 r_2 (gbest - p_j^i)$

end for

Update ω using equation:

$\omega = (\omega_{max} - \omega_{min}) \left(\frac{iter_{max} - i}{iter_{max}}\right)$

Calculate the fitness of all particles.

Calculate the $pbest$ for each particle.

Calculate the $gbest$.

$t = t + 1$

end while

return $gbest$.

while ($t <$ Max number of iterations)

for each search agent

Update the position of current search agent by equations:

$\vec{D} = |\vec{C} \cdot \vec{X}_{prey}(i) - \vec{X}_{wolf}(i)|$

$\vec{X}_{wolf}(i+1) = \vec{X}_{prey}(t) - \vec{A} \cdot \vec{D}$

$\vec{X}_{wolf}(i+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$

end for

Update a, A and C using equations:

$A = 2\vec{a} \cdot \vec{r}_1 - \vec{a}$

$\vec{C} = 2\vec{r}_2$

Calculate the fitness of all search agents.

Update X_α, X_β and X_δ.

$t = t + 1$

end while

return X_α.

for each search agent

Update the position of current search agent by equations:

$\vec{X}_j^{i+1} = f_d \cdot \omega \vec{X}_j^i + f_d \cdot r_1 (\overrightarrow{pbest_j}) + (1 - f_d \cdot r_2)(\overrightarrow{gbest})$

$\vec{D} = |\vec{C} \cdot \vec{X}_{prey}(i) - \vec{X}_{wolf}(i)|$

$\vec{X}_{wolf}(i+1) = \vec{X}_{prey}(t) - \vec{A} \cdot \vec{D}$

$\vec{X}_{wolf}(i+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$

end for

Update a, A, C, ω and d_f using equations:

$A = 2\vec{a} \cdot \vec{r}_1 - \vec{a}$

$\vec{C} = 2\vec{r}_2$

$\omega = (\omega_{max} - \omega_{min}) \left(\frac{iter_{max} - i}{iter_{max}}\right)$

$f_d = \left(\frac{a}{2}\right)^2$

Calculate the fitness of all search agents.

Update X_α, X_β and X_δ.

$t = t + 1$

end while

return X_α.

VI. SIMULATION RESULTS

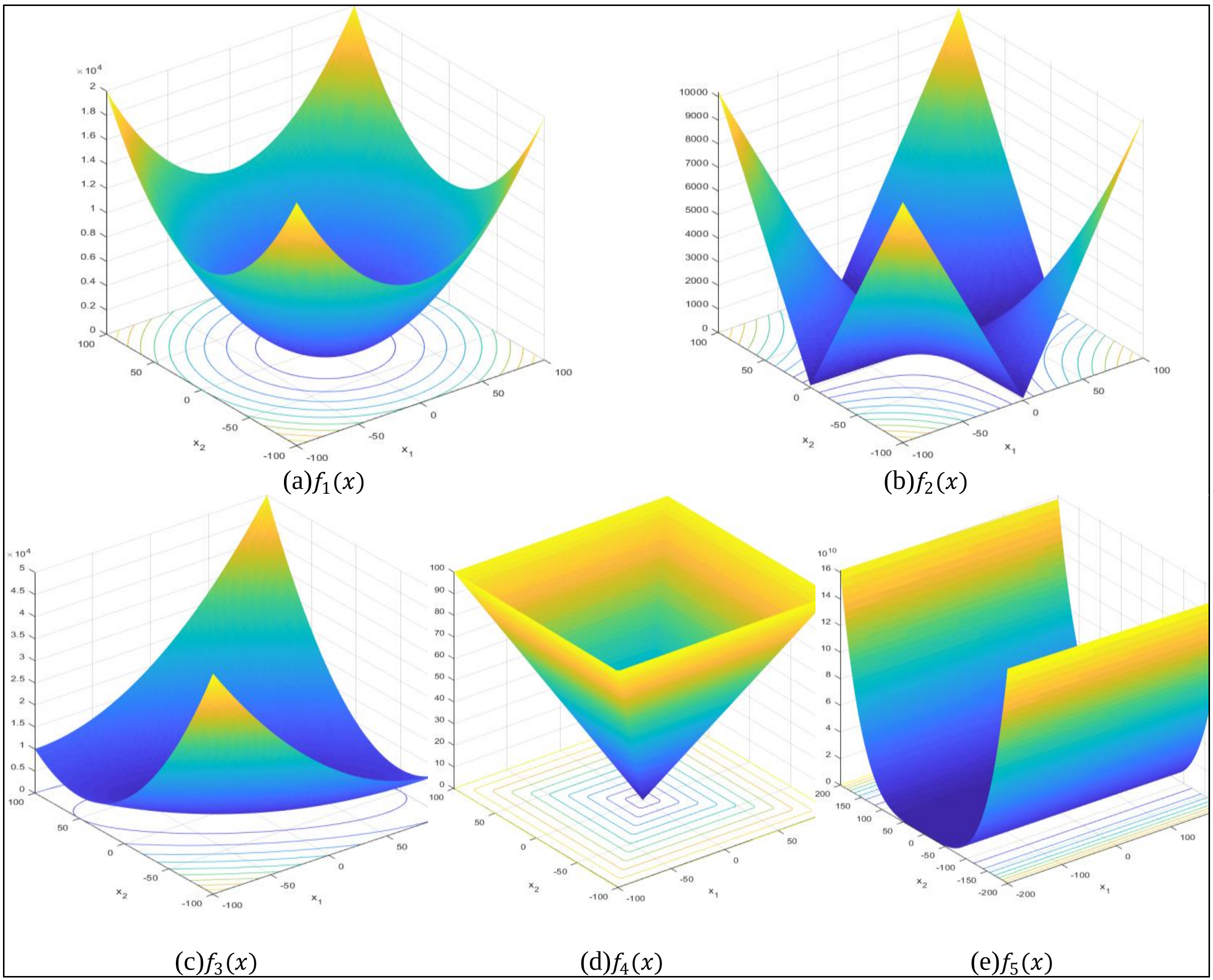
To evaluate the capabilities of the proposed PSOIGWO algorithm, is tested against 23 classical and popular benchmark test problems listed in [7, 19]. The test functions can be divided into three different group's unimodal functions, multimodal functions and fixed-dimensionmultimodal functions. The details of the functions and their plots are provided in table 2-4 and figure 3-5 respectively. To evaluate the performance of the proposed algorithm four parameters named best, worst, average and standard deviation are used. These performance parameters are obtained for each benchmark test function by repeatedly evaluating them for 50 times. To evaluate the comparative performance of the proposed algorithm these results are compared with the GWO, PSO, GA and DE algorithms. For the comparison following configurations are used each algorithm.

Table 2: Algorithm configurations used for comparison.

Algorithm	Parameter's Name	Parameter's Value
PSOIGWO	c_1	1
	c_2	1
	ω_{max}	0.8
	ω_{min}	0.2
	Papulation Size	25
	Maximum Iterations	500
GWO	*Papulation Size*	25
	Maximum Iterations	500
HGWO	*scaling factor F*	0.5
	Crossover Prob P_c	0.2
	Papulation Size	25
	Maximum Iterations	500
PSO	c_1	2
	c_2	2
	ω_{max}	0.9
	ω_{min}	0.1
	V_{max}	6

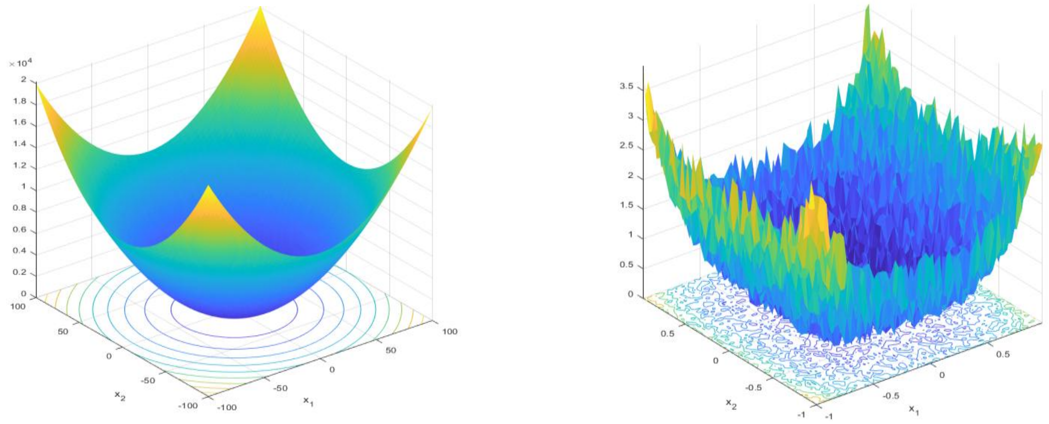
Table 3: List of the Unimodal Functions.

Function Definition	Variables and Their Limits	Exact Solution
$f_1(x) = \sum_{i=1}^{n} x_i^2$	$-100 \le x_i \le 100, n = 30$	$\min(f_1) = 0$
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	$-10 \le x_i \le 10, n = 30$	$\min(f_2) = 0$
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2$	$-100 \le x_i \le 100, n = 30$	$\min(f_3) = 0$
$f_4(x) = \max_i \{\lvert x_i \rvert, 1 \le i \le n\}$	$-100 \le x_i \le 100, n = 30$	$\min(f_4) = 0$
$f_5(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2 \right]$	$-30 \le x_i \le 30, n = 30$	$\min(f_5) = 0$
$f_6(x) = \sum_{i=1}^{n} (x_i + 0.5)^2$	$-100 \le x_i \le 100, n = 30$	$\min(f_6) = 0$
$f_7(x) = \sum_{i=1}^{n} i x_i^4 + random[0,1)$	$-1.28 \le x_i \le 1.28, n = 30$	$\min(f_7) = 0$



(a) $f_1(x)$ (b) $f_2(x)$

(c) $f_3(x)$ (d) $f_4(x)$ (e) $f_5(x)$

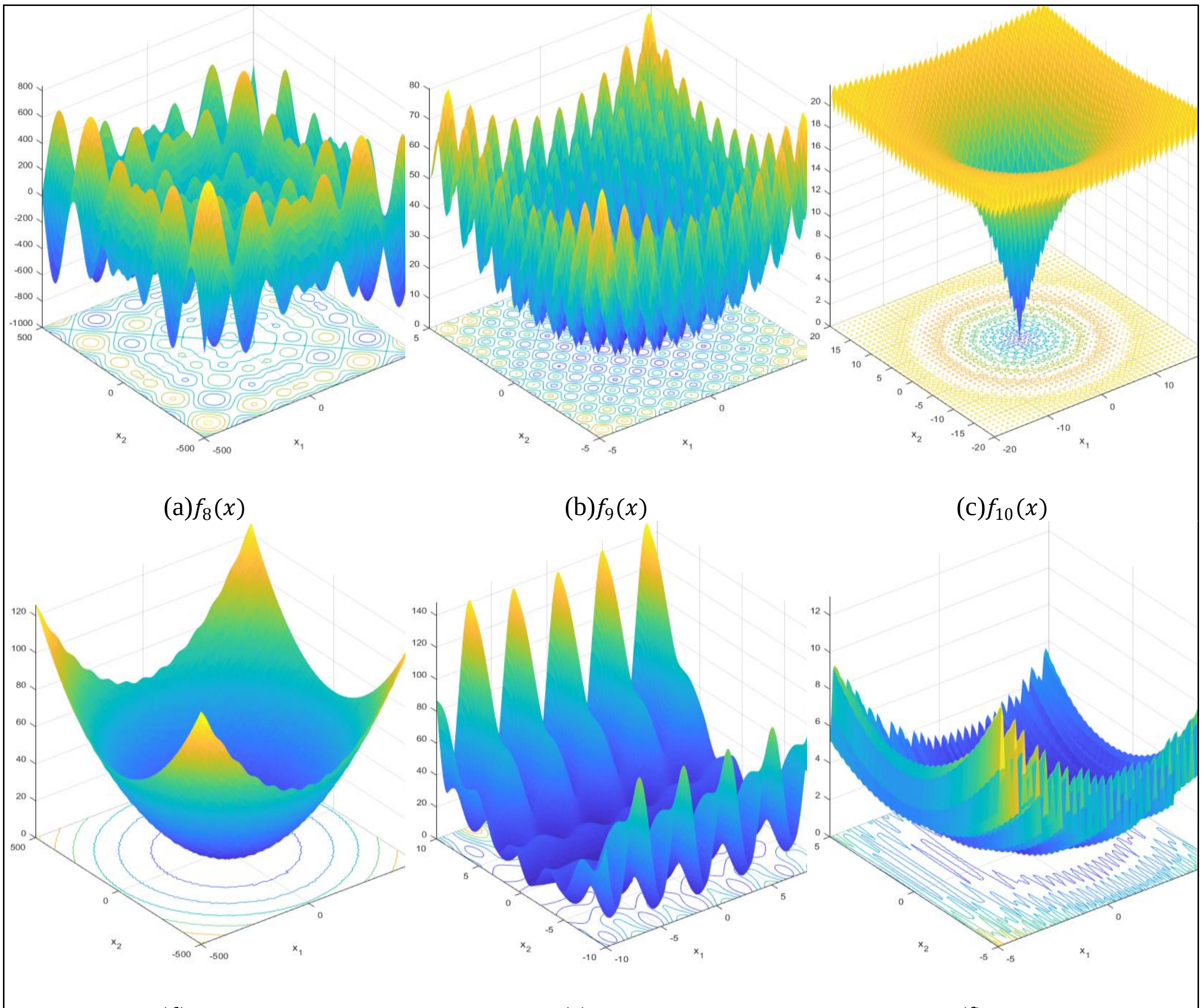


(f) $f_6(x)$ (g) $f_7(x)$

Figure 3: showing the plots of unimodal functions presented in table 2.

Table 4: List of the Multimodal Functions.

Function Definition	Variables and Their Limits	Exact Solution
$f_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	$-500 \le x_i \le 500, n = 30$	$\min(f_8) = -4189.829 \times n$
$f_9(x) = \sum_{i=1}^{n} \lvert x_i^2 - 10\cos(2\pi x_i) + 10 \rvert,$	$-5.12 \le x_i \le 5.12, n = 30$	$\min(f_9) = 0$
$f_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)\right) + 20 + e,$	$-32 \le x_i \le 32, n = 30$	$\min(f_{10}) = 0$
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1,$	$-600 \le x_i \le 600, n = 30$	$\min(f_{11}) = 0$
$f_{12}(x) = \frac{\pi}{n}\{10\sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1}) + (y_n - 1)^2]\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	$y_i = 1 + \frac{x_i + 1}{4},$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \le x_i \le x \\ k(-x_i - a)^m, & x_i < -a \end{cases}$ $-50 \le x_i \le 50, n = 30$	$\min(f_{12}) = 0$
$f_{13}(x) = 0.1\left\{\sin^2(3\pi x_i) + \sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i)] + (x_i + 1)^2[1 + \sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4),$	$-50 \le x_i \le 50, n = 30$	$\min(f_{13}) = 0$



(a)$f_8(x)$ (b)$f_9(x)$ (c)$f_{10}(x)$

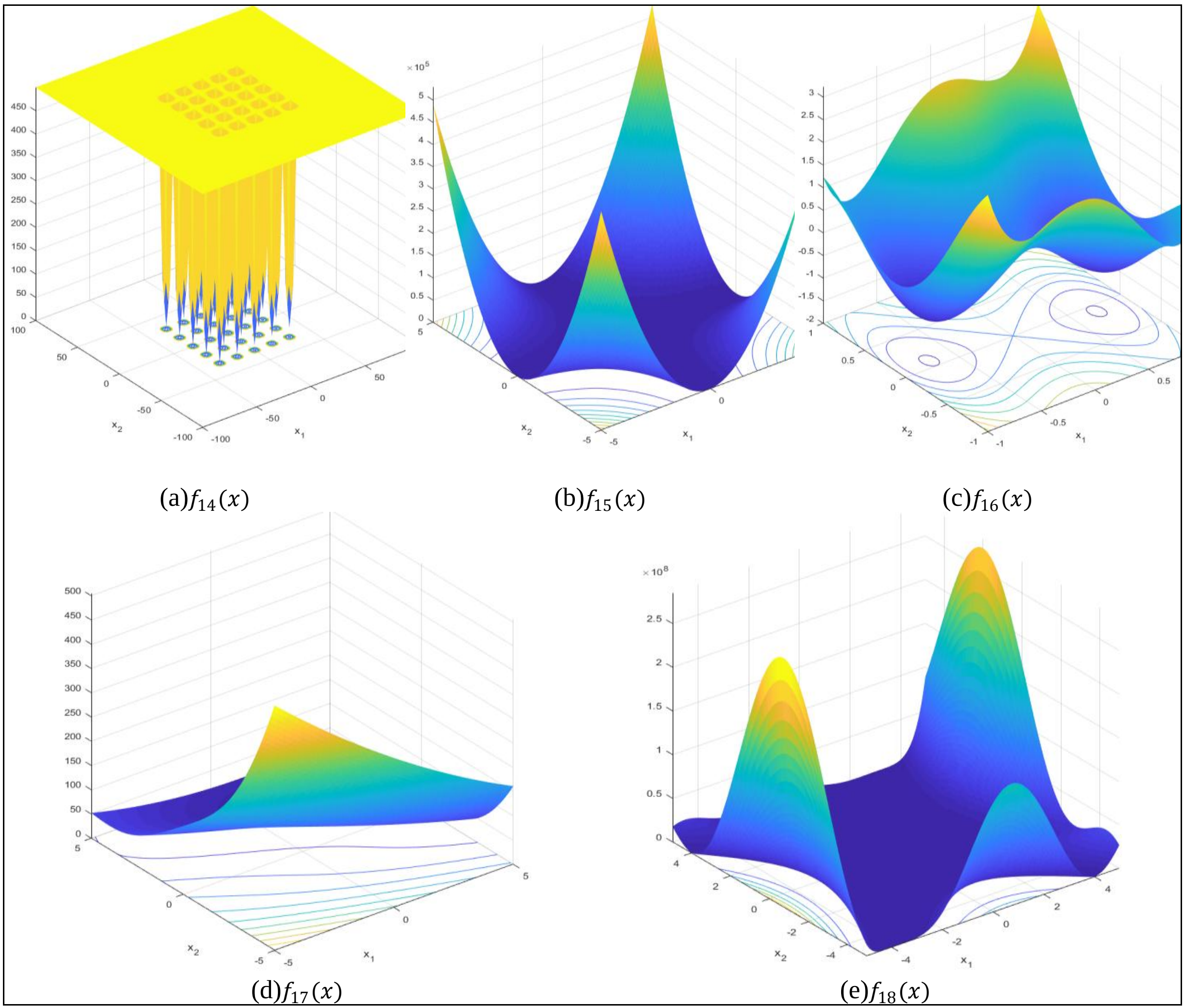
(d)$f_{11}(x)$ (e)$f_{12}(x)$ (f)$f_{13}(x)$

Figure 4: showing the plots of multimodal functions presented in table 3.

Table 5: List of the Fixed Dimension Multimodal Functions.

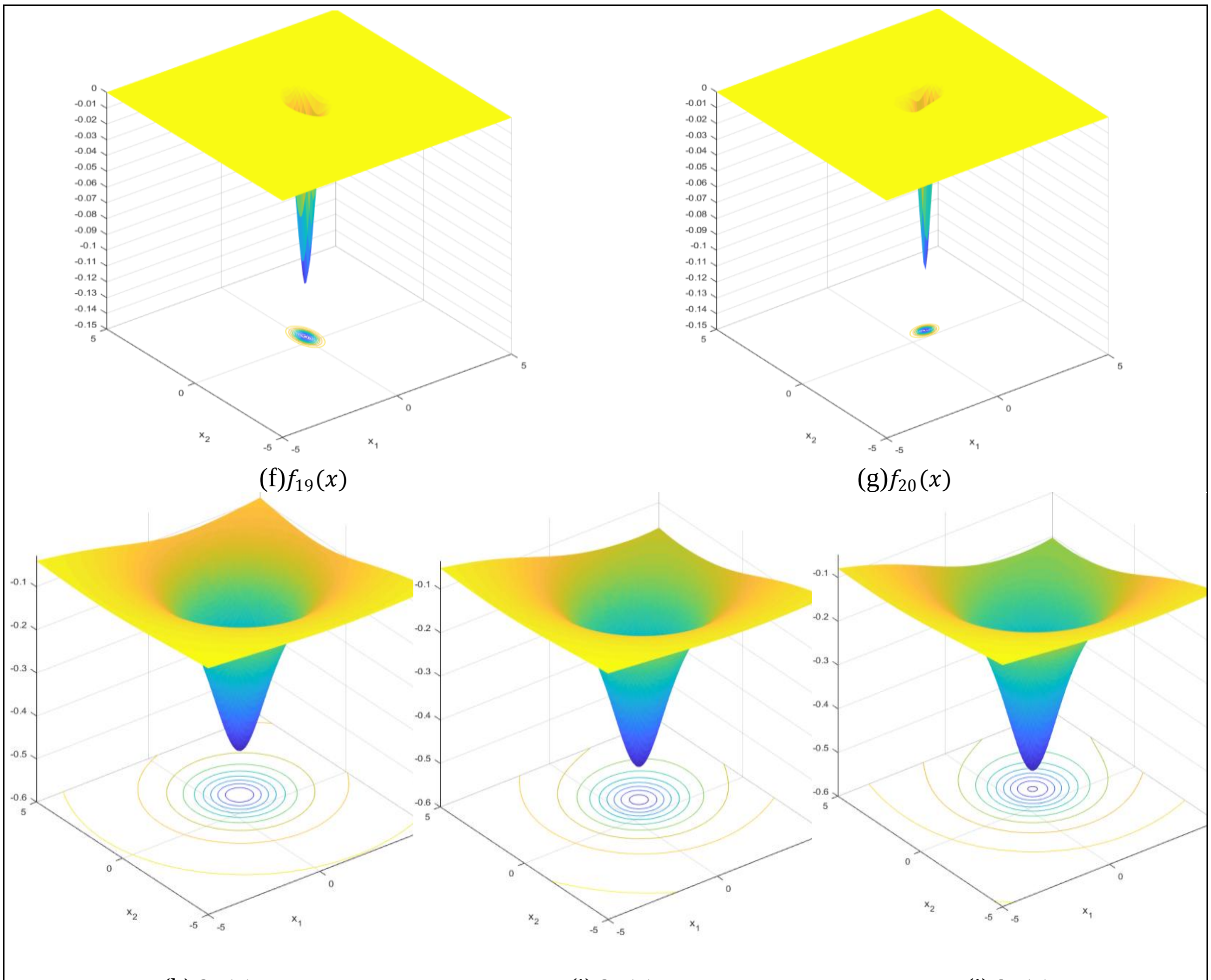
Function Definition	Variables and Their Limits	Exact Solution
$f_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{\sum_{i=1}^{n}(x_i - a_j)^2}\right)^{-1}$	$-65 \le x_i \le 65, n = 2$	$\min(f_{14}) = 1$
$f_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	$-5 \le x_i \le 5, n = 4$	$\min(f_{15}) = 3$
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4,$	$-5 \le x_i \le 5, n = 2$	$\min(f_{16}) = -1.0316$
$f_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2} + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10,$	$-5 \le x_i \le 5, n = 2$	$\min(f_{17}) = -0.398$

$f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)][30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)],$	$-2 \leq x_i \leq 2, n = 2$	$\min(f_{18}) = 3$
$f_{19}(x) = \sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{n} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	$1 \leq x_i \leq 3, n = 3$	$\min(f_{19}) = 3.86$
$f_{20}(x) = \sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{n} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	$0 \leq x_i \leq 1, n = 6$	$\min(f_{20}) = -3.32$
$f_{21}(x) = -\sum_{i=1}^{5}[(x - a_i)(x - a_i)^T + c_i]^{-1}$	$0 \leq x_i \leq 10, n = 4$	$\min(f_{21}) = -10.1532$
$f_{22}(x) = -\sum_{i=1}^{7}[(x - a_i)(x - a_i)^T + c_i]^{-1}$	$0 \leq x_i \leq 10, n = 4$	$\min(f_{22}) = -10.4028$
$f_{23}(x) = -\sum_{i=1}^{10}[(x - a_i)(x - a_i)^T + c_i]^{-1}$	$0 \leq x_i \leq 10, n = 4$	$\min(f_{23}) = -10.5363$



(a)$f_{14}(x)$ (b)$f_{15}(x)$ (c)$f_{16}(x)$

(d)$f_{17}(x)$ (e)$f_{18}(x)$



(f)$f_{19}(x)$ (g)$f_{20}(x)$

(h)$f_{21}(x)$ (i)$f_{22}(x)$ (j)$f_{23}(x)$

Figure 5: showing the plots of multimodal functions presented in table 4.

Table 6: Evaluation and Comparison of Best Results.

Unimodal Functions						
function	Exact Solution	PSOIGWO	GWO	PSO	GA	DE
$f_1(x)$	0	5.73312E-81	1.19156E-27	1.15737E-03	1.08515E-01	6.41676E-02
$f_2(x)$	0	2.29022E-43	3.86384E-16	1.89361E+00	2.17493E+00	2.28411E-03
$f_3(x)$	0	1.63830E-54	1.38199E-07	6.82600E+04	4.87321E+00	2.44008E+02
$f_4(x)$	0	3.37323E-35	1.13011E-07	1.21886E+02	1.24059E+00	1.64606E+01
$f_5(x)$	0	2.58802E+01	2.59425E+01	1.31698E+03	2.10619E+00	2.73052E+02
$f_6(x)$	0	5.22899E-01	1.50290E-04	1.18912E-03	2.41504E-01	4.58609E-01
$f_7(x)$	0	8.72565E-05	4.40180E-04	5.10906E+00	7.02064E-01	8.97338E-02
Multimodal Functions.						
		PSOIGWO	GWO	PSO	GA	DE
$f_8(x)$	-12569.487	-1.02004E+04	-7.43762E+03	-5.75454E+2	-5.58382E+02	-1.04188E+04
$f_9(x)$	0	0.00000E+00	5.68434E-14	8.47039E+01	1.01643E+01	3.04228E+01
$f_{10}(x)$	0	4.44089E-15	1.18128E-13	2.09643E+01	1.53522E+00	1.86102E+00
$f_{11}(x)$	0	0.00000E+00	0.00000E+00	1.74743E-03	7.17464E-03	3.98328E-02
$f_{12}(x)$	0	2.45736E-02	1.33638E-02	3.22578E+00	1.05966E-02	4.17999E-01
$f_{13}(x)$	0	5.57171E-01	2.47707E-01	3.87094E+00	1.27219E-02	2.96557E+00
Fixed Dimension Multimodal Functions.						
		PSOIGWO	GWO	PSO	GA	DE
$f_{14}(x)$	1	9.98004E-01	9.98004E-01	9.98004E-01	1.07632E+01	9.98004E-01
$f_{15}(x)$	0.00030	3.07611E-04	3.07601E-04	1.19064E-03	7.28637E-04	3.07486E-04
$f_{16}(x)$	-1.0316	-1.03163E+00	-1.03163E+00	-1.03163E+00	-1.03163E+00	-1.03163E+00

$f_{17}(x)$	0.398	3.97887E-01	3.97887E-01	3.97887E-01	3.97887E-01	3.97887E-01
$f_{18}(x)$	3	3.00000E+00	3.00000E+00	3.00000E+00	3.00000E+00	3.00000E+00
$f_{19}(x)$	-3.86	-3.86278E+00	-3.86278E+00	0.00000E+00	-3.86278E+00	-3.86278E+00
$f_{20}(x)$	-3.32	-3.32199E+00	-3.32199E+00	0.00000E+00	-3.32200E+00	-3.32200E+00
$f_{21}(x)$	-10.1532	-1.01530E+01	-1.01530E+01	-1.01532E+01	-1.01532E+01	-1.01532E+01
$f_{22}(x)$	-10.4028	-1.04028E+01	-1.04027E+01	-1.04029E+01	-1.04029E+01	-1.04029E+01
$f_{23}(x)$	-10.5363	-1.05360E+01	-1.05360E+01	-1.05364E+01	-1.05364E+01	-1.05364E+01

Table 6: Evaluation and Comparison of Worst Results.

Unimodal Functions						
		PSOIGWO	GWO	PSO	GA	DE
$f_1(x)$	0	5.97424E-74	1.82667E-24	7.71682E+00	3.63538E+00	1.81294E+03
$f_2(x)$	0	2.94281E-38	7.64921E-15	7.75558E+40	1.00655E+01	6.19411E+00
$f_3(x)$	0	2.70203E-44	1.33103E-02	6.65502E+06	1.47210E+02	2.34668E+03
$f_4(x)$	0	4.41039E-30	1.12640E-05	8.04105E+02	2.95263E+00	5.71032E+01
$f_5(x)$	0	2.88005E+01	2.87683E+01	6.54249E+07	4.06518E+02	1.64963E+06
$f_6(x)$	0	2.53379E+00	1.76372E+00	1.61236E+02	1.76519E+01	7.47995E+02
$f_7(x)$	0	3.74774E-03	6.38262E-03	3.30940E+05	1.66857E+01	4.70505E-01
Multimodal Functions.						
		PSOIGWO	GWO	PSO	GA	DE
$f_8(x)$	-12569.487	-3.89086E+03	-3.07866E+03	-1.10083E+02	-4.12515E+02	-5.47418E+03
$f_9(x)$	0	0.00000E+00	2.44096E+01	9.97946E+02	6.32870E+01	1.92381E+02
$f_{10}(x)$	0	7.99361E-15	3.70370E-13	2.13342E+01	3.06378E+00	1.33922E+01
$f_{11}(x)$	0	0.00000E+00	3.10384E-02	7.67146E-01	2.73973E-01	9.99982E+00
$f_{12}(x)$	0	2.01877E-01	5.76287E-01	7.12867E+08	7.06768E-01	7.45479E+05
$f_{13}(x)$	0	2.41035E+00	1.47133E+00	1.41027E+05	6.88525E-01	2.87328E+06
Fixed Dimension Multimodal Functions.						
		PSOIGWO	GWO	PSO	GA	DE
$f_{14}(x)$	1	1.07632E+01	1.26705E+01	1.55038E+01	1.45631E+01	1.07632E+01
$f_{15}(x)$	0.00030	2.03634E-02	2.03634E-02	2.18017E-03	2.30309E-02	2.03633E-02
$f_{16}(x)$	-1.0316	-1.03163E+00	-1.03163E+00	-2.15464E-01	-2.15464E-01	-1.03163E+00
$f_{17}(x)$	0.398	3.97957E-01	3.97914E-01	3.97887E-01	3.97887E-01	3.97887E-01
$f_{18}(x)$	3	3.00080E+00	8.40001E+01	8.40000E+01	8.40000E+01	3.00000E+00
$f_{19}(x)$	-3.86	-3.85681E+00	-3.85489E+00	0.00000E+00	-1.00082E+00	-3.86278E+00
$f_{20}(x)$	-3.32	-3.13847E+00	-3.02235E+00	0.00000E+00	-3.20074E+00	-3.18514E+00
$f_{21}(x)$	-10.1532	-3.41617E+00	-2.63013E+00	-2.63047E+00	-2.63047E+00	-2.63047E+00
$f_{22}(x)$	-10.4028	-5.08760E+00	-5.08762E+00	-1.83759E+00	-1.83759E+00	-2.76590E+00
$f_{23}(x)$	-10.5363	-3.83531E+00	-2.42160E+00	-1.67655E+00	-1.67655E+00	-2.42734E+00

Table 8: Evaluation and Comparison of Average Results.

Unimodal Functions						
function	Exact Solution	PSOIGWO	GWO	PSO	GA	DE
$f_1(x)$	0	3.43514E-75	1.48218E-25	7.43253E-01	9.05963E-01	1.54308E+02
$f_2(x)$	0	1.10540E-39	1.45688E-15	1.55115E+39	5.91828E+00	5.12678E-01
$f_3(x)$	0	2.11222E-45	4.84967E-04	1.24860E+06	2.21582E+01	1.12947E+03
$f_4(x)$	0	1.09587E-31	2.03761E-06	2.80495E+02	2.01666E+00	3.02199E+01
$f_5(x)$	0	2.76209E+01	2.72722E+01	1.43070E+06	9.94700E+01	1.16563E+05
$f_6(x)$	0	1.58236E+00	8.87844E-01	7.08462E+00	3.07052E+00	1.10022E+02
$f_7(x)$	0	9.33567E-04	2.43732E-03	1.05588E+04	2.20326E+00	2.35599E-01
Multimodal Functions						
		PSOIGWO	GWO	PSO	GA	DE
$f_8(x)$	-12569.487	-8.16029E+03	-5.76905E+03	-1.48439E+71	-5.17081E+02	-8.61955E+03
$f_9(x)$	0	0.00000E+00	3.11343E+00	3.10710E+02	3.16062E+01	9.94517E+01
$f_{10}(x)$	0	4.51195E-15	2.05027E-13	2.11487E+01	2.28089E+00	4.57833E+00
$f_{11}(x)$	0	0.00000E+00	5.96285E-03	6.86429E-02	6.80370E-02	1.86448E+00
$f_{12}(x)$	0	9.34140E-02	6.51778E-02	1.59101E+07	2.86477E-01	3.15298E+04

$f_{13}(x)$	0	1.18332E+00	8.22504E-01	3.64029E+03	1.78862E-01	3.06647E+05
Fixed Dimension Multimodal Functions						
		PSOIGWO	GWO	PSO	GA	DE
$f_{14}(x)$	1	2.33542E+00	5.51611E+00	2.98717E+00	1.21745E+01	1.39101E+00
$f_{15}(x)$	0.00030	3.66615E-03	4.06751E-03	1.79814E-03	2.56545E-03	1.87705E-03
$f_{16}(x)$	-1.0316	-1.03163E+00	-1.03163E+00	-9.01042E-01	-1.01531E+00	-1.03163E+00
$f_{17}(x)$	0.398	3.97894E-01	3.97890E-01	3.97887E-01	3.97887E-01	3.97887E-01
$f_{18}(x)$	3	3.00009E+00	4.62007E+00	1.70400E+01	8.40000E+00	3.00000E+00
$f_{19}(x)$	-3.86	-3.86211E+00	-3.86106E+00	0.00000E+00	-3.64008E+00	-3.86278E+00
$f_{20}(x)$	-3.32	-3.25338E+00	-3.27757E+00	0.00000E+00	-3.25764E+00	-3.24786E+00
$f_{21}(x)$	-10.1532	-7.92965E+00	-9.51189E+00	-5.32892E+00	-5.54997E+00	-8.13711E+00
$f_{22}(x)$	-10.4028	-9.23216E+00	-1.00818E+01	-5.48814E+00	-4.96282E+00	-8.66517E+00
$f_{23}(x)$	-10.5363	-9.31027E+00	-1.03718E+01	-4.14082E+00	-4.19933E+00	-9.66556E+00

Table 9: Evaluation and Comparison of Standard DeviationResults.

Unimodal Functions					
	PSOIGWO	GWO	PSO	GA	DE
$f_1(x)$	1.13396E-74	2.69266E-25	1.66374E+00	8.13535E-01	2.95330E+02
$f_2(x)$	4.36546E-39	1.32875E-15	1.09680E+40	2.04803E+00	1.02999E+00
$f_3(x)$	5.96944E-45	1.92228E-03	1.86078E+06	2.87813E+01	5.42933E+02
$f_4(x)$	6.27561E-31	1.80660E-06	1.03376E+02	5.03672E-01	8.22575E+00
$f_5(x)$	8.21158E-01	7.93564E-01	9.23711E+06	8.13696E+01	2.67122E+05
$f_6(x)$	4.74129E-01	4.26683E-01	3.17071E+01	3.20128E+00	1.78668E+02
$f_7(x)$	7.54238E-04	1.38837E-03	4.98531E+04	2.49356E+00	8.34265E-02
Multimodal Functions					
	PSOIGWO	GWO	PSO	GA	DE
$f_8(x)$	1.51016E+03	9.35468E+02	8.14214E+71	4.23001E+01	1.12532E+03
$f_9(x)$	0.00000E+00	6.06477E+00	1.96301E+02	9.65077E+00	3.90375E+01
$f_{10}(x)$	5.02430E-16	6.45630E-14	9.33279E-02	3.32551E-01	2.18245E+00
$f_{11}(x)$	0.00000E+00	9.33413E-03	1.32577E-01	5.48832E-02	2.25803E+00
$f_{12}(x)$	4.02251E-02	7.88044E-02	1.01249E+08	1.71622E-01	1.09763E+05
$f_{13}(x)$	3.37832E-01	2.52039E-01	1.99611E+04	1.61148E-01	6.02454E+05
Fixed Dimension Multimodal Functions					
	PSOIGWO	GWO	PSO	GA	DE
$f_{14}(x)$	2.66955E+00	4.55991E+00	2.94990E+00	8.86815E-01	1.59533E+00
$f_{15}(x)$	7.36411E-03	7.71527E-03	1.19158E-04	4.07410E-03	4.11825E-03
$f_{16}(x)$	3.44073E-08	3.47382E-08	3.02249E-01	1.15423E-01	2.37376E-16
$f_{17}(x)$	1.34584E-05	5.19163E-06	4.06345E-09	3.15912E-09	3.36448E-16
$f_{18}(x)$	1.55692E-04	1.14551E+01	2.99966E+01	1.44321E+01	2.87453E-15
$f_{19}(x)$	1.30359E-03	2.71072E-03	0.00000E+00	6.07517E-01	3.14018E-15
$f_{20}(x)$	6.84498E-02	8.09129E-02	0.00000E+00	6.00007E-02	5.86834E-02
$f_{21}(x)$	2.59848E+00	1.97063E+00	2.80483E+00	2.36077E+00	2.78313E+00
$f_{22}(x)$	2.22344E+00	1.27457E+00	3.13909E+00	2.45880E+00	2.98448E+00
$f_{23}(x)$	2.33557E+00	1.14728E+00	2.77786E+00	2.41495E+00	2.39071E+00

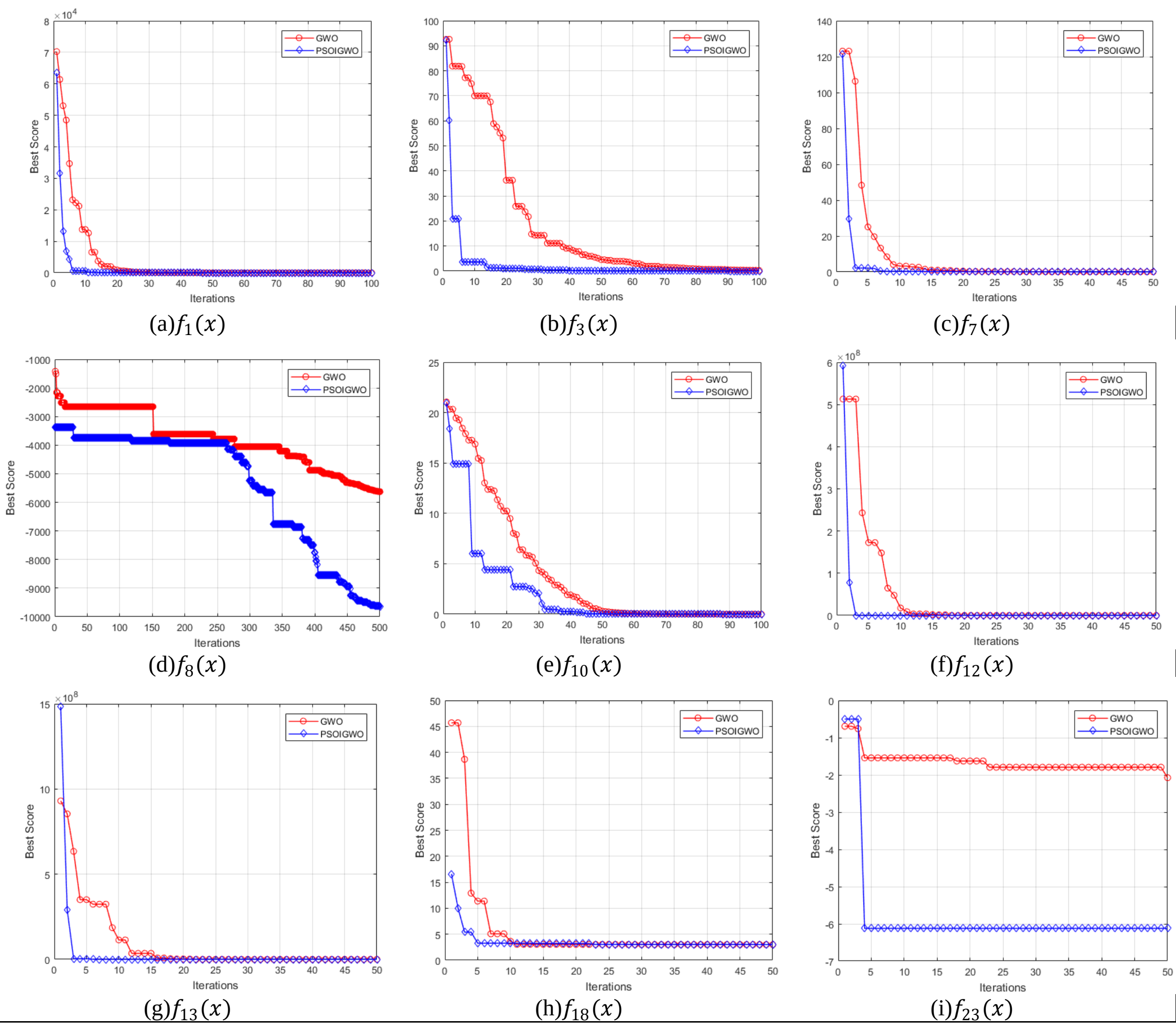


Figure 6: Convergence curve for different functions.

From Table VI it can be seen that the PSOIGWO, compared to standard GWO, PSO,GA and DE algorithms, provide best values for five Unimodal functions$f_1(x), f_2(x), f_3(x), f_4(x), f_7(x)$, one Multimodal function $f_{10}(x)$ it outperforms all other algorithms by large margin, while for Fixed Dimension MultimodalFunctions it provides the same best results obtained by other algorithm. Even for the remaining functions in Unimodal and Multimodal group it remains competitive.

Comparing for worst results from Table VII the PSOIGWOprovide best values for six Unimodal functions $f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_7(x)$, four Multimodal function $f_9(x), f_{10}(x), f_{11}(x), f_{12}(x)$ it outperforms all other algorithms by large margin, while for Fixed Dimension MultimodalFunctions it provides the same best results obtained by other algorithm. For the remaining functions in all groups it remains competitive.

Comparing for average results from Table VIII the PSOIGWOprovide best values for five Unimodal functions $f_1(x), f_2(x), f_3(x), f_4(x), f_7(x)$, four Multimodal function $f_8(x), f_9(x), f_{10}(x), f_{11}(x)$ and six FixedDimension MultimodalFunctions$f_{16}(x), f_{17}(x), f_{18}(x), f_{19}(x), f_{22}(x), f_{23}(x)$, it outperforms all other algorithms by large margin, while for remainingfunctions either it provides the same best results obtained by other algorithm or at least remains competitive.

Comparing for standard deviation results from Table IX the PSOIGWOprovide the lowest deviation for five Unimodal functions$f_1(x), f_2(x), f_3(x), f_4(x), f_7(x)$, four Multimodal function $f_8(x), f_9(x), f_{10}(x), f_{11}(x)$ it outperforms all other algorithms by large margin, while for six Fixed Dimension MultimodalFunctions $f_{14}(x), f_{15}(x), f_{16}(x), f_{18}(x), f_{19}(x), f_{20}(x)$ it provides the second lowest deviation. For remainingfunctions either it provides the same best results obtained by other algorithm or at least remains competitive.

VII. CONCLUSION

The proposed PSOIGWO algorithm utilizes the exploration capabilities of PSO and convergence capability GWO to achieve the best of both. The comparative analysis of the algorithm for the 23 standard test benchmark function shows that for most of Unimodal functions and Multimodal functions it outperforms the other algorithms with large margin, while for Fixed Dimension Multimodalfunctions it provides the same best results obtained by other algorithm. The analysis also shows that for most cases the PSOIGWO remains first or second best. The algorithm also shows best stability as for five Unimodal functions, four Multimodal functions and six Fixed Dimension Multimodalfunctions it gives the lowest deviation. The algorithm also shows quick convergence than the standard GWO algorithm as shown in figure 6.

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