(τ_1, τ_2) - rg**b Closed Sets in Bitopological Spaces

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Abstract

In this paper, the concept of regular generalized star star b - closed sets are extended to bitopological spaces are obtained.

Key words: *(τ_1, τ_2) - rg**b closed sets, τ_2 - b - closed set, (τ_1, τ_2) - g* - closed sets.*

I. INTRODUCTION

In 1970, Levine [16] introduced the concept of generalized closed sets and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms.

Later in 1985, Fukutake [5] gave a new type of generalized closed set in bitopological spaces. Ahmad Al - Omari and Md. Noorani [1] made an analytical study and gave the concept of generalized b-closed set in topological spaces. Indirani and Banupriya [9] introduce a new class of closed sets called regular generalized star star b-closed sets.

II. PRELIMINARIES

Throughout this paper (X, τ_1, τ_2) represents a bitopological space in which no separation axioms are assumed, if A is a subset of a topological space X with a topology τ, with then the closure of A is denoted by τ - cl(A) or cl(A), the interior of A is denoted by τ - int(A) or int(A), semi - closure and pre - closure of A is denoted by τ - scl(A) or scl(A) and pcl(A) or pcl(A) respectively, semi - interior of A is denoted by τ - sint(A) or sint(A) and the complement of A is denoted by A^c.

2.1 Definition

A subset A of a topological space (X, τ) is called:

1) An α - open set if $A \subseteq$ int(cl(int(A))).
2) A semi - open set if $A \subseteq$ cl(int(A)).
3) A pre - open set if $A \subseteq$ int(cl(A)).
4) A semi - pre - open set or β - open set if $A \subseteq$ cl(int(cl(A))).
5) A regular open set if A = int(cl(A)).
6) A b - open set if $A \subseteq$ int(cl(A)) $\cup$ cl(int(A)).

2.2 Definition

Let (X, τ) a topological space and A be a subset of X, then A is called:

1) A generalized closed set if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X. It is also denoted as g - closed set.
2) A generalized α - closed set if αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is α - open in X. It is also denoted as gα - closed set.
3) An α - generalized closed set if αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X. It is also denoted as αg - closed set.

4) A generalized b - closed set if bcl(A) ⊆ U whenever A ⊆ U and U is open in X. It is also denoted as gb -closed closed set.
5) Semi generalized closed set if scl(A) ⊆ U whenever A ⊆ U and U is semi - open in X. It is also denoted as sg - closed set.
6) A generalized semi - closed set if scl(A) ⊆ U whenever A ⊆ U and U is open in X. It is also denoted as gs - closed set.
7) w - closed set if cl(A) ⊆ U whenever A ⊆ U and U is semi - open in X.
8) A weakly generalized closed set if cl(int(A)) ⊆ U and U is open in X. It is also denoted as wg - closed set.
9) A semi - generalized b - closed set if bcl(A) ⊆ U whenever A ⊆ U and U is semi-open in X. . It is also denoted as sgb - closed set.
10) A strongly generalized closed set if cl(A) ⊆ U whenever A ⊆ U and U is g - open in X. It is also denoted as g* - closed set.
11) A generalized gαb - closed set if bcl(A) ⊆ U whenever A ⊆ U and U is α - open in X. It is also denoted as gαb - closed set.
12) A regular generalized b - closed set if bcl(A) ⊆ U whenever A ⊆ U and U is regular - open in X. It is also denoted as rgb - closed set.

2.3 Definition

A subset A of a bitopological space (X, τ_1, τ_2) is called a

1) (τ_1,τ_2) - pre - open if A ⊆ τ_1 - int(τ_2 - cl(A)).
2) (τ_1,τ_2) - semi - open if A ⊆ τ_2 - cl(τ_1 - int(A)).
3) (τ_1,τ_2) - α - open if A ⊆ τ_1 - int(τ_2 - cl(τ_1 - int(A)).
4) (τ_1,τ_2) - regular - open if A = τ_1 - int(τ_2 - cl(A)).

2.4 Definition

A subset A of a bitopological space (X, τ_1, τ_2) is called a

1) (τ_1, τ_2) - g - closed if τ_2 - cl(A) ⊆ U whenever A ⊆ U and U ϵ $\tau_{1.}$
2) (τ_1, τ_2) - gs - closed if τ_2 - scl(A) ⊆ U whenever A ⊆ U and U ϵ $\tau_{1.}$
3) (τ_1, τ_2) - weakly generalized closed ((τ_1,τ_2) - wg - closed) sets if τ_2 - cl(τ_1 - int(A)) ⊆ U whenever A ⊆ U and U is τ_1 - open in X.
4) (τ_1, τ_2) - w - closed if τ_2 - cl(A) ⊆ U whenever A ⊆ U and U is semi - open in $\tau_{1.}$
5) (τ_1, τ_2) - g* - closed if τ_2 - cl(A) ⊆ U whenever A ⊆ U and U is τ_1 - g open in X.
6) (τ_1, τ_2) - αg - closed if τ_2 - αcl(A) ⊆ U whenever A ⊆ U and U is τ_1 - open in X.
7) (τ_1, τ_2) - gα - closed if τ_2 - αcl(A) ⊆ U whenever A ⊆ U and U is τ_1 - α - open in X.
8) (τ_1, τ_2) - g*p - closed if τ_2 - pcl(A) ⊆ U whenever A ⊆ U and U is τ_1 - g - open in X.
9) (τ_1, τ_2) - rg - closed if τ_2 - cl(A) ⊆ U whenever A ⊆ U and U is τ_1 - regular open in X.
10) (τ_1, τ_2) - rg** - closed if τ_2 - cl(τ_1 - int(A)) ⊆ U whenever A ⊆ U and U is (τ_1, τ_2) - regular open in X.
11) (τ_1, τ_2) - rw - closed if τ_2 - cl(A) ⊆ U whenever A ⊆ U and U is τ_1 - regular - semi - open in X.
12) (τ_1, τ_2) - regular weakly generalized closed((τ_1, τ_2) - wg - closed) sets if τ_2 - cl (τ_1 - int(A)) ⊆ U whenever A ⊆ U and U is τ_1 - regular open in X.

III (T_1, T_2) - RG**B CLOSED SETS IN BITOPOLOGICAL SPACES

In this section (τ_1, τ_2) - rg**b closed sets in bitopological spaces are introduced and some of their properties are studied.

3.1 Definition

Let i, j ϵ {1, 2} be fixed integers. A subset A of a topological spaces (X, τ_1 ,τ_2) is called regular generalized star star b closed set if τ_2 - rg*bcl(A) ⊆ U whenever A ⊆ U and U is τ_1 - open in (X, τ_1).The family of all (τ_1 ,τ_2) - rg**b closed sets in bitopological space (X, τ_1, τ_2) is denoted by D*rg**b(τ_1, τ_2).

3.2 Remark

By setting $\tau_1 = \tau_2$ in ***definition 3.1*** (τ_1, τ_2) - rg**b closed set is a rg**b closed set.

3.3 Proposition

If A is a τ_2 - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Let A be any τ_2 - closed set. Therefore τ_2 - cl(A) = A and U be any τ_1 - open set containing A. Since τ_2 - rg*bcl(A) $\subseteq \tau_2$ - cl(A) $\subseteq$ U then τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

Example 3.4

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed but not τ_2 - closed.

3.5 Proposition

If A is a τ_2 - b - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Let A be any τ_2 - b - closed set in (X, τ_1, τ_2) such that A $\subseteq$ U and U is τ_1 - open set. Since A is a τ_2 - b - closed which implies that τ_2 - rg*bcl (A) $\subseteq \tau_2$ - cl(A) $\subseteq$ U then τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.6 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {a, b} is (τ_1, τ_2) - rg**b closed but not τ_2 - b - closed.

3.7 Proposition

If A is τ_2 - α closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Let A be any τ_2 - α closed set in (X, τ_1, τ_2) such that A $\subseteq$ U and U is τ_1 - open set. Since every α - closed set is rg*b - closed set and A is a τ_2 - α - closed set, it is true that τ_2 - rg*bcl(A) $\subseteq \tau_2$ - αcl(A) $\subseteq \tau_2$ - cl(A) = A $\subseteq$ U then τ_2 - rg*bcl(A) $\subseteq$ U whenever A $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from following example.

3.8 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed set but not in τ_2 - α - closed set.

3.9 Proposition

If A is a (τ_1, τ_2) - g - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Let A be any (τ_1, τ_2) - g - closed set in (X, τ_1, τ_2) such that A $\subseteq$ U and U is τ_1 - open set. Since A is a (τ_1, τ_2) - g - closed which implies that τ_2 - rg*bcl (A) $\subseteq$ τ_2 - cl (A) $\subseteq$ U then τ_2 - rg*bcl (A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.10 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {b} is (τ_1, τ_2) - rg**b closed set but not in (τ_1, τ_2) - g - closed set.

3.11 Proposition

If A is a (τ_1, τ_2) - g* - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Let A be any (τ_1, τ_2) - g* - closed set in (X, τ_1, τ_2) such that A $\subseteq$ U and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - g* - closed which implies that τ_2 - rg*bcl(A) $\subseteq$ τ_2 - cl(A) $\subseteq$ U then τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.12 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {b}, {c} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - g* - closed.

3.13 Proposition

If A is a (τ_1, τ_2) - g*p - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - g*p - closed set in (X, τ_1, τ_2) such that A $\subseteq$ U and U be any τ_1 - open set. Since A is a (τ_1, τ_2) - g*p - closed set, we have τ_2 - pcl(A) $\subseteq$ U, τ_2 - rg*bcl(A) $\subseteq$ τ_2 - pcl(A) $\subseteq$ U then τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.14 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {b} is (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - g*p - closed.

3.15 Proposition

If A is a (τ_1, τ_2) - gb - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - gb - closed set in (X, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1 - open set. Since A is a (τ_1, τ_2) - gb - closed set, we have τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.16 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {b} is (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - gb - closed.

3.17 Proposition

If A is a (τ_1, τ_2) - αg - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - αg - closed set in (X, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1 - open set. Since A is a (τ_1, τ_2) - αg - closed set, then τ_2 - rg*bcl (A) $\subseteq$ τ_2 - αcl(A) $\subseteq$ U. Therefore τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.18 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {b} is (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - αg - closed.

3.19 Proposition

If A is a (τ_1, τ_2) - gα - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - gα - closed set in (X, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - gα - closed set, then τ_2 - rg*bcl(A) $\subseteq$ τ_2 - αcl(A) $\subseteq$ U. Therefore τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following examples.

3.20 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - gα - closed.

3.21 Proposition

If A is a (τ_1, τ_2) - gαb - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - gαb - closed set in (X, τ_1, τ_2) such that A ⊆ U and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - gαb - closed set, then τ_2 - rg*bcl(A) ⊆ τ_2 - αbcl(A) ⊆ U. Therefore τ_2 - rg*bcl(A) ⊆ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.22 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {a, b} is (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - gαb - closed.

3.23 Proposition

If A is a (τ_1, τ_2) - gs - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - gs - closed set in (X, τ_1, τ_2) such that A ⊆ U and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - gs - closed set, then τ_2 - rg*bcl(A) ⊆ τ_2 - scl(A) ⊆ U. Therefore τ_2 - rg*bcl(A) ⊆ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.24 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a, b}, {a, c} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - gs - closed.

3.25 Proposition

If A is a (τ_1, τ_2) - sg - closed subset of (x, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - sg - closed set in (x, τ_1, τ_2) such that A ⊆ U and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - sg - closed set, then τ_2 - rg*bcl(A) ⊆ τ_2 - scl(A) ⊆U. Therefore τ_2 - rg*bcl(A) ⊆ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.26 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {a, b} is (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - sg - closed.

3.27 Proposition

If A is a (τ_1, τ_2) - rg - closed subset of (x, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - rg - closed set in (x, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1- open set containing A. Since A is a (τ_1, τ_2) - sg - closed set, then τ_2 - rg*bcl(A) $\subseteq \tau_2$ - cl(A) $\subseteq$ U. Therefore τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.28 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - rg - closed.

3.29 Proposition

If A is a (τ_1, τ_2) - w - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - w - closed set in (X, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - w - closed set, then τ_2 - rg*bcl(A) $\subseteq \tau_2$ - cl(A) $\subseteq$ U. Therefore τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.30 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - w - closed.

3.31 Proposition

If A is a (τ_1, τ_2) - rwg - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - rwg - closed set in (X, τ_1, τ_2) such that $A \subseteq U$ and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - rwg - closed set, then τ_2 - rg*bcl(A) $\subseteq \tau_2$ - cl(τ_1 - int(A)) $\subseteq$ U. Therefore τ_2 - rg*bcl(A) $\subseteq$ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.32 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The sets {a}, {b} are (τ_1, τ_2) - rg**b closed but not (τ_1, τ_2) - rwg - closed.

3.33 Proposition

If A is a (τ_1, τ_2) - wg - closed subset of (X, τ_1, τ_2) then A is (τ_1, τ_2) - rg**b closed set.

Proof

Assume that A is (τ_1, τ_2) - wg - closed set in (X, τ_1, τ_2) such that A ⊆ U and U be any τ_1 - open set containing A. Since A is a (τ_1, τ_2) - rwg - closed set, then τ_2 - rg*bcl(A) ⊆ τ_2 - cl(τ_1 - int(A)) ⊆ U. Therefore τ_2 - rg*bcl(A) ⊆ U. Hence A is (τ_1, τ_2) - rg**b closed set.

The converse of this proposition need not be true as seen from the following example.

3.34 Example

Consider the topological space X = {a, b, c} and with topologies τ_1 = {X, ϕ, {b}, {c}, {b, c}} and

τ_2 = {X, ϕ, {a}, {b}, {a, b}}.

The set {b} is (τ_1, τ_2) - rg**b closed set but not (τ_1, τ_2) - wg - closed set.

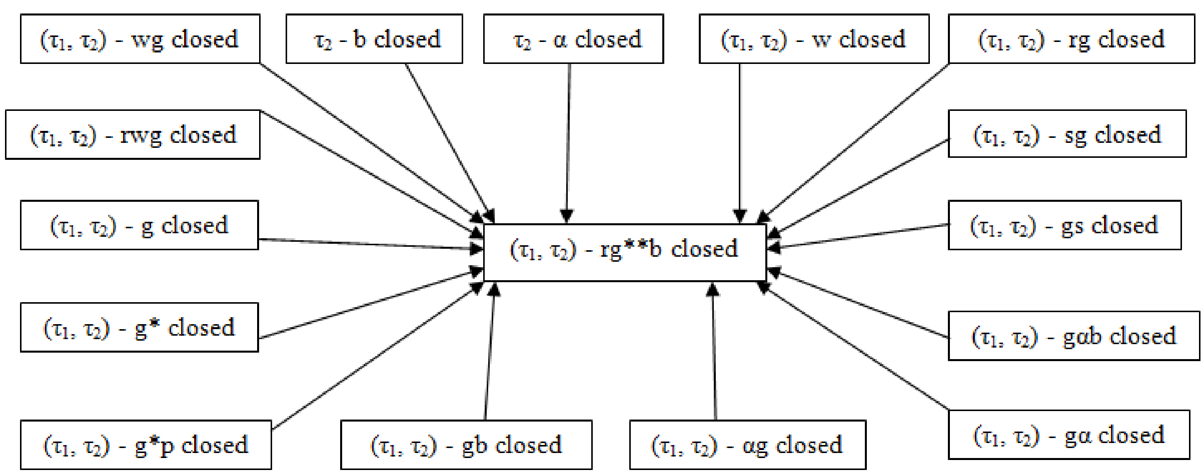

Fig 3.1.1 Pictorial Representation of the above results.

IV. CONCLUSION

In this paper, (τ_1, τ_2) - rg**b closed sets in bitopological spaces were introduced and extended to be investigated. The concept of (τ_1, τ_2) - rg**b closed sets can be extended further to other topological spaces namely ideal topology, soft topology and so on and their properties can be investigated.

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