Energy of Complete Fuzzy Labeling Graph through Fuzzy Complete Matching

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Abstract

The matching is a set of non-adjacent edges. In this paper, a new concept of energy for matching in complete fuzzy labeling graph is introduced. A graph is said to be a complete fuzzy labeling graph if it has every pair of adjacent vertices of the fuzzy graph. Here, we also defined the adjacency matrix of complete fuzzy labeling graph through fuzzy complete matching. The energy is defined as the average of the absolute values of the adjacency matrix.

Keywords: *Complete fuzzy labeling graph, complete matching, energy of graph, Spectrum and adjacency matrix.*

I. INTRODUCTION

Many real world systems can be modeled using graphs. Graph represents the connections between the entities in these systems. The foundation for graph theory was laid in 1735 by Euler when he solved the Konigsberg bridge problem. Fuzzy graphs are generalization of graphs.

Fuzzy graphs are countered in fuzzy set theory. A fuzzy set was defined by L. A. Zadeh in 1965.Every element in the universal set is assigned a grade of membership ,a value in [0,1].The elements in the universal set along with their grades of membership form a fuzzy set.

In 1965 Fuzzy relations on a set was first defined by Zadeh [14].The nature of values discussed in energy of graph is never an odd integer [3] and energy of a graph is never the square root of an odd integer [13].Study on energy of different graphs such as regular, non-regular, circulate and random graphs [4, 5, 6] by I.Gutman and shao.In [1] Anjali Narayanan and Sunil Mathew introduced the concept of the energy of fuzzy graph.

A matching is a set of edges which are non-adjacent. Energy of complete fuzzy labeling graph is the average of the absolute Eigen values of adjacency matrix. Here our results consider for complete fuzzy labeling graph.

II. PRELIMINARIES

Definition: 2.1

A graph with n vertices in which every pair of distinct vertices is joined by a line is called **complete graph** on n vertices. It is denoted by $K_{n.}$

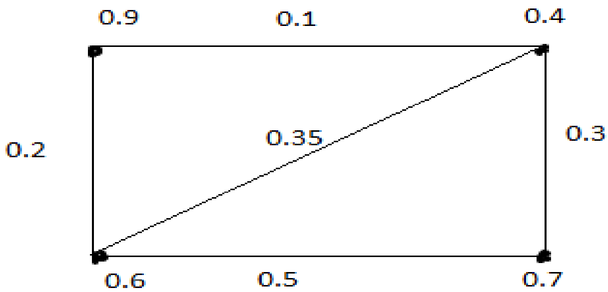
Definition 2.2

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$. A **fuzzy graph** $G = (\alpha, \beta)$ is a pair of functions $\alpha : V \rightarrow [0, 1]$ and $\beta: V \times V \rightarrow [0,1]$ where for all $u, v \in V$, we have $\beta(u,v) \leq \min\{\alpha(u), \alpha(v)\}$.

Definition 2.3

A graph $G = (\alpha, \beta)$ is said to be a ***fuzzy labeling graph*** if $\alpha: V \rightarrow [0, 1]$ and $\beta: V \times V \rightarrow [0, 1]$ is a bijective such that the membership value of edges and vertices are distinct and $\beta(u, v) < \min\{\alpha(u), \alpha(v)\}$ for all $u, v \in V$.

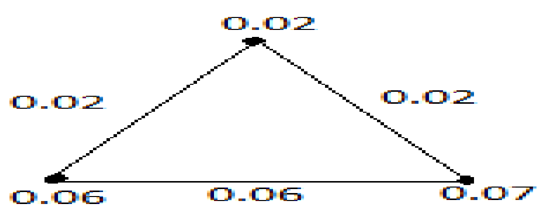
Example: 2.4



Definition: 2.5.

A fuzzy graph G = (α, β) is said to be ***complete*** *if* β (u, v) = min {α (u), α (v)} for all u, v ∈ V and every pair of vertices are adjacent. It is denoted by K_n [FLG].

Example 2.6

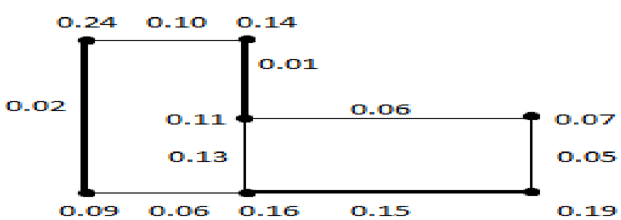


III. MAIN RESULTS

Definition 3.1

A subset M of β ($v_{i,\ v\ i+1}$), $1 \leq i \leq n$ is called a ***matching*** in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be matched under M.

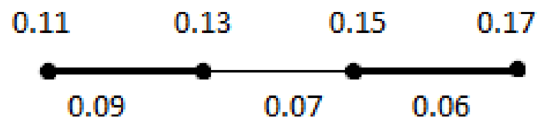
Example: 3.2



Definition 3.3

If every vertex of fuzzy graph is M-saturated then the matching is said to be ***complete***. *It is denoted by* C_{M}.

Example



Definition 3.4

The **adjacency matrix** of complete fuzzy labeling graph through complete matching is defined by

$$A = \begin{cases} \beta(u, v) & \text{if } (u, v) \in C_M. \\ 0 & \text{otherwise} \end{cases}$$

Definition 3.5

If x is a real number then the **floor of x**, $\lfloor x \rfloor$ rounds x-down and **ceiling of x** , $\lceil x \rceil$ rounds x –up.

Definition 3.6

The **energy** of a complete fuzzy labeling graph is the average of the absolute Eigen values of the adjacency matrix and **spectrum** of a graph is the set of all Eigen values.

Definition 3.7

A ***bipartite fuzzy graph*** consists of two node sets V and U with $|V| \geq m$ and $|U| \geq n$ such that $\beta(v, u_i) > 0$ and $\beta(u_i, u_{i+1}) = 0$, $1 \leq i \leq n$ and $m, n \in N$. It is denoted by $(f-g)_{m,n}$.

Definition 3.8

A complete bipartite fuzzy graph is a bipartite fuzzy graph in which all the vertices in vertex set V adjacent to all the vertices in the vertex set U.

Theorem: 3.9

Let K_n [FLG] be a complete fuzzy labeling graph with $|V|$ = n vertices and $\beta = n(n-1)/2$ edges. If n = 2, 4… then the number of edges in the complete matching is $|V|/2$.

Proof.

Let K_n [FLG] be a complete fuzzy labeling graph with $|V|$ = n vertices and $\beta = n(n-1)/2$ edges. Then the sum of the degrees of all vertices is n (n-1).

If every vertex is adjacent to every other vertex then the complete matching form in different ways.

The number of complete matching in K_2[FLG] is 1 and also the number of lines in the complete matching is $|V|/2 = 1$.

Similarly, the number of complete matching in K_4[FLG] is3 and also the number of lines in the complete matching is $|V|/2 = 2$.

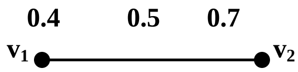
In general, the number of complete matching in K_{2n} [FLG] is $(2n)!\, /\, n!\, 2^n$.The number of complete matching in K_{2n} [FLG] is 1 and also the number of lines in the complete matching is $|V|/2 = 2n/2$.

Form the adjacency matrix for different complete matching in K_{2n} [FLG] . Find the Eigen values by the determinant of adjacency matrix $\lambda_1, \lambda_2 \ldots \lambda_n$ (say).

The energy of Kn [FLG] is $E(G) = \sum_{i=1}^{n} |\lambda i| / n$. Using this formula, to find the energy.

Therefore we get all Eigen values and energies for all complete matching in K_{2n} [FLG].

Example: 3.10



In K $_2$[FLG] , there exists only one complete matching. The adjacency matrix is

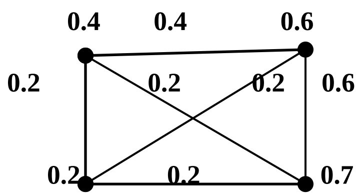
$$A = \begin{array}{c} \begin{array}{cc} v_1 & v_2 \end{array} \\ \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} \end{array}$$

The Eigen values are 0.5, - 0.5.Then the energy is

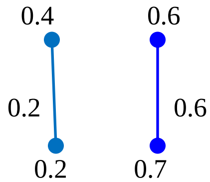
$$E(K_2[FLG]) = |0.5| + |-0.5| /2$$
$$= 1.0/2$$
$$= 0.5$$

The spectrum is $\{0.5, -0.5\}$.

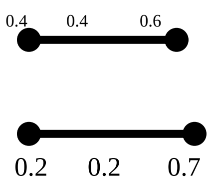
Example: 3.11



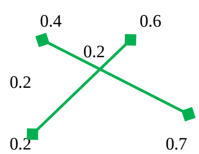
If n = 4, In K $_4$[FLG], there exists three types of complete matching. They are



C $_{M1}$



C $_{M2}$



C $_{M3}$

The adjacency matrices are,

$$A_1 = \begin{bmatrix} 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0.2 \\ 0.4 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0.2 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0.2 & 0 & 0 \end{bmatrix}$$

The Eigen values of A_1 are 0.6477,-0.6477, 0.9052,-0.9052.The energy is 3.1058/4 = 0.77646.
Similarly we can find Eigen values and Energy for remaining adjacency matrices.

Theorem: 3.12

Let K_n [FLG] be a complete fuzzy labeling graph with $|V|$ = n vertices and β= n (n-1) / 2 edges. If n = 1, 3… then the number of edges in the matching is $\lfloor |V| /2 \rfloor$.

Proof.

Let K_n [FLG] be a complete fuzzy labeling graph with $|V|$ = n vertices and β= n (n-1) / 2 edges. Then the sum of the degrees of all vertices is n (n-1).

If n=1.3, every vertex is adjacent to every other vertex then matching exists but the complete matching does not exists.

But the number of lines in the matching is $\lfloor |V| /2 \rfloor$. In K_1[FLG] , There is no lines in matching. ($\lfloor |1| /2 \rfloor = 0$).

Similarly, In K_3 [FLG] there are only one line in the matching. Hence $\lfloor |3| /2 \rfloor = 1$).

In general, the number of lines in matching of K_{2n-1}[FLG] is $\lfloor |V| /2 \rfloor$.But the complete matching not exists for K_{2n-1}[FLG].

Note:

For odd number of vertices, we can't form the adjacency matrix.

Theorem: 3.13

In a bipartite fuzzy complete graph with 2 vertices $K_{1,1}$contains only one complete matching with one edge.

Proof

A bipartite fuzzy complete graph with 2 vertices $K_{1,1}$ is same as K_2 [FLG] . The vertex set can be partition into two subsets U and V, here $|U| = 1$ and $|V| = 1$.

There exists only one complete matching. The number of vertices in the complete matching is $|V|$ or $|U|$ =1.

The adjacency matrix is defined with respect to the complete matching. Here row and column corresponds to number of vertices in the vertex set U and V.

To find the Eigen values by the determinant of adjacency matrix λ_1, λ_2.

Theorem: 3.14

In a bipartite fuzzy complete graph with 4 vertices $k_{2,2}$ contain two complete matching with two edges.

Proof

The vertex set can be partition into two subsets U and V, here $|U| = 2$ and $|V| = 2$. Since the graph is complete, every vertex in U is adjacent to every vertex in V.

There exists only two complete matching. The number of vertices in the complete matching is $|V|$ or $|U| = 2$.

The adjacency matrix is defined with respect to the complete matching .Here row and column corresponds to number of vertices in the vertex set U and V.

To find the Eigen values by the determinant of adjacency matrix.

IV. CONCLUSION

In this paper, the Eigen values and energy of complete fuzzy labeling graph and complete bipartite fuzzy labeling graph are defined and found. Some theorems related to the concept are discussed and verify the results through examples. In future we extend this concept to some special graph like hyper graph, regular graph etc.

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