A New Property of Triangular Numbers

Rahil Miraj
Class - XI,SaradaVidyapith (H.S.),Sri Ramakrishna Pally, Sonarpur,
Kolkata 700150, West Bengal,India

Abstract:
If a Triangular number (except 1 and 3) is expressed as the sum of its consecutive partitions starting from unit, then among the partitions, one can find at least one pair of numbers, which give the same remainder when divided by the greatest integer not exceeding the square root of that Triangular number.

Keywords: *Triangular number; Partitions; Floor function or greatest integer function; Pigeon Hole Principle.*

I. INTRODUCTION

A Triangular number is the count of total number of objects, which are arranged in an equilateral triangle as in the figure-1 .

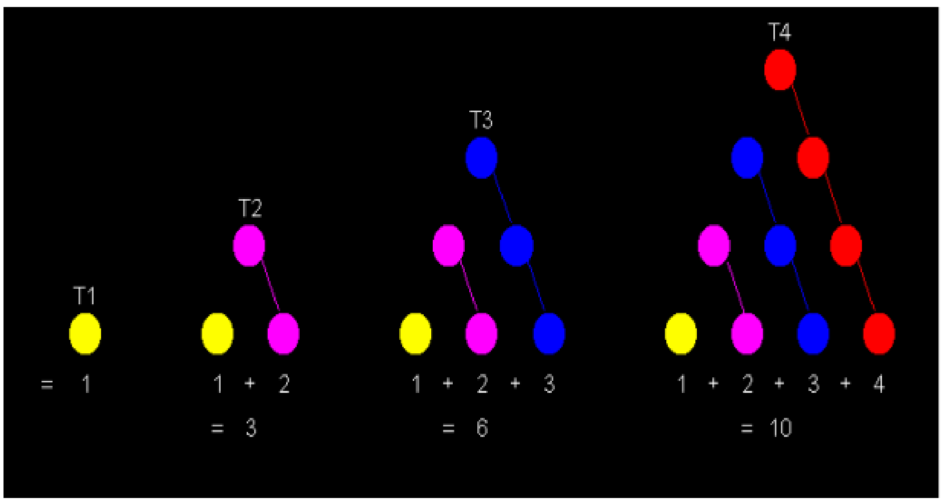

Fig. – 1 : Triangular number (Courtesy Wikipedia [1])

There are many properties of Triangular number are observed in literature. However, most of the properties are related to the addition and square properties [2]. In this article, I am proposing a new property of Triangular number.

II.PROPOSAL

If a Triangular number (except 1 and 3) is expressed as the sum of its consecutive partitions starting from unit, then among the partitions, one can find at least one pair of numbers, which give the same remainder when divided by the greatest integer not exceeding the square root of that Triangular number.

III.EXAMPLES

At first I am providing some examples in support of my proposal.

1. The fifth Triangular number, T_5 is 15.

Then $\lfloor\sqrt{15}\rfloor$ = 3.

Here, 15= 1+2+3+4+5 .

One can find , $5 \equiv 2 \equiv 2 \ (mod\ 3)$.

2. The eighth Triangular number, T_8 is 36.

Then $\lfloor\sqrt{36}\rfloor$ = 6.
Here, 36= 1+2+3+4+5+6+7+8 .
One can get, $7 \equiv 1 \equiv 1 \ (mod\ 6).$
3. The ninth Triangular number, T_9 is 45.
Then $\lfloor\sqrt{45}\rfloor = 6.$
Here, 45= 1+2+3+4+5+6+7+8+9.
One can see, $9 \equiv 3 \equiv 3 \ (mod\ 6).$

IV.GENERAL PROOF

Now, I will provide the general proof.
Proof: At first we can see that

$$T_n = 1 + 2 + 3 + 4 + 5 + ... + n = \frac{n(n+1)}{2}.$$

We know that the greatest integer not exceeding a number is the Floor function or the greatest Integer function of that number ($\lfloor x \rfloor$ is the Floor function or the greatest Integer function of x).
Now, we have to prove that

$$\left\lfloor \sqrt{\frac{n(n+1)}{2}} \right\rfloor < n.$$

We can see that

$$n^2 < n^2 + n < n^2 + 2n + 1,$$

$$or,\ n^2 < n(n+1) < (n+1)^2,$$

$$or,\ \frac{n^2}{2} < \frac{n(n+1)}{2} < \frac{(n+1)^2}{2},$$

$$or,\ \frac{n}{\sqrt{2}} < \sqrt{\frac{n(n+1)}{2}} < \frac{(n+1)}{\sqrt{2}}.$$

Now we have to prove that

$$\frac{(n+1)}{\sqrt{2}} < n,$$

$$or,\ (n+1)^2 < 2n^2,$$

$$or,\ 2n+1 < n^2,$$

$$or,\ 2 < (n-1)^2,$$

which is true for all $n > 2$.

$$Hence,\quad \left\lfloor \sqrt{\frac{n(n+1)}{2}} \right\rfloor < n.$$

Then $\left\lfloor \sqrt{\frac{n(n+1)}{2}} \right\rfloor$ is at most $(n-1)$.

Also, $T_n$can be expressed as the sum of n consecutive positive integers starting from unit.
Hence, by Pigeon Hole Principle, one can say that there are at least one pair of numbers among the partitions, which give the same remainder when divided by $\left\lfloor \sqrt{\frac{n(n+1)}{2}} \right\rfloor$.
Hence the proof.

V. CONCLUSION

Thus apart from some well known properties of Triangular number related to the addition and square properties,in this article, I have proposed a new property related to the remainder of its consecutive partitions starting from unit when divided by the greatest integer not exceeding the square root of that Triangular number.

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REFERENCES

[1] Wikipedia.
[2] David M. Burton, Elementary Number Theory, Macgraw Hill Education, (2016) .