Strong Perfect Domination in Graphs

Govindalakshmi T.S.[1],Meena.N[2]
[1]*Department of Mathematics,Manonmaniam Sundaranar University College, Puliangudi – 627 855.*
Affiliated to ManonmaniamSundaranarUniversity, Abishekapatti, Tirunelveli -627 012, TamilNadu, India.
[2]*P.G. and Research Department of Mathematics, The M.D.T. Hindu College, Tirunelveli-10*
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu,India.

Abstract

Let G be a simple graph. A subset S $\subseteq$ V(G) is called a strong (weak) perfect dominating set of G if $|N_s(u) \cap S| = 1(|N_w(u) \cap S| = 1$ for every $u \in V(G) - S$ where $N_s(u) = \{v \in V(G)/ \deg v \geq \deg u\}$ $(N_w(u) = \{v \in V(G)/ \deg v \leq \deg u\}$. The minimum cardinality of a strong (weak) perfect dominating set G is called the strong (weak) perfect domination number and is denoted by $\gamma_{sp}(G)$ $(\gamma_{wp}(G))$. In this paper strong perfect domination number of some standard graphs and their middle graphs are determined.

Keywords - *Dominating set, perfect dominating set, strong dominating set, strong perfect dominating set, strong perfect domination number.*

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I. INTRODUCTION

By a graph, it is meant that a finite, undirected graph without loops and multiple edges. Let G be a graph with vertex V and edge set E. Let p = |V (G)| and q = |E (G)|. The minimum and maximum degrees of vertices in G are denoted by $\delta(G)$ and $\Delta(G)$ respectively. A dominating set D of G is a subset of V (G) such that every vertex in V - D is adjacent to one vertex in D. A dominating set of G of minimum cardinality is a minimum dominating set of G and it cardinality is the domination number of G. It is denoted by $\gamma(G)$. A dominating set S is a perfect dominating set of G if $|N(v) \cap S| = 1$ for each $v \in V- S$. Every graph G has at least the trivial perfect dominating set consisting of all vertices in V. Minimum cardinality of the perfect dominating set of G is the perfect domination number of G and it is denoted by $\gamma_p(G)$. Motivated by this definition, the strong perfect domination in graph is defined. In this paper strong perfect domination number of standard graphs and their middle graphs are determined.

Definition 1.1: The middle graph M(G) of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in whichtwo vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 1.2: The wheel W_n is defined to be the graph K_1+C_{n-1}, $n \geq 4$

Definition 1.3: The helm H_n is the graph obtained from the wheel W_n with n spokes by adding n pendant edges at each vertex on the wheel's rim.

Definition 1.4: A flower is a graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Definition 1.5: Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.

II. STRONG PERFECT DOMINATION IN GRAPHS

Definition 2.1: A subset $S \subseteq V(G)$ is called a strong (weak) perfect dominating set of G if $|N_s(u) \cap S| = 1(|N_w(u) \cap S| = 1$ for every $u \in V(G) - S$ where $N_s(u) = \{v \in V(G)/ \deg v \geq \deg u\}$ $(N_w(u) = \{v \in V(G)/\deg v \leq \deg u\})$. The minimum cardinality of a strong(weak) perfect dominating set G is called the strong(weak) perfect domination number and is denoted by$\gamma_{sp}(G)(\gamma_{wp}(G))$.

Remark 2.2: The maximum cardinality of a strong (weak) perfect dominating set of G is called the strong (weak) perfect domination number and is denoted by $\Gamma_{sp}(G)$ $(\Gamma_{wp}(G))$.

Example 2.3: Consider the following graphs G.

v_2v_3

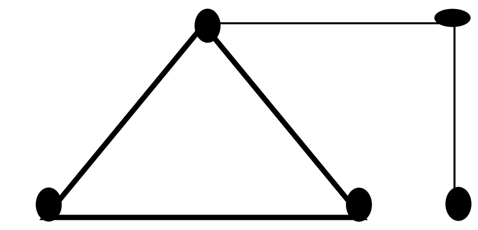

Fig. 1

Let S = $\{v_2,v_3\}$, $|N_s(v_1)\cap S| = |\{v_2,v_4\}\cap S| = |\{v_2\}| = 1$, $|N_s(v_4)\cap S| = |\{v_1,v_2\}\cap S| = |\{v_2\}| = 1$, $|N_s(v_5)\cap S| = |\{v_3\}\cap S| = |\{v_3\}| = 1$. Therefore S is a strong perfect dominating set of G. T = $\{v_2,v_5\}$, $|N_s(v_1)\cap S| = |N_s(v_3)\cap S| = |N_s(v_4)\cap S| = \{v_2\} = 1$. T is also a strong perfect dominating set of G. Therefore $\gamma_{sp}(G) \leq 2$. G has no full degree vertex. Therefore $\gamma_{sp}(G) \geq 2$. Hence $\gamma_{sp}(G) = 2$.

Remark 2.4: $1 \leq \gamma_{sp}(G) \leq n$ where $n = |V(G)|$. For: If G has a full degree vertex then $\gamma_{sp}(G) = 1$ and $\gamma_{sp}(K_{m,n}) = m + n$, $m,n \geq 2$, $m \neq n$.

Remark 2.5: $\gamma(G) \leq \gamma_s(G) \leq \gamma_{sp}(G)$.

Remark 2.6: A strong dominating set of a graph G need not be strong perfect dominating set of G.

Example: Consider the following graph G

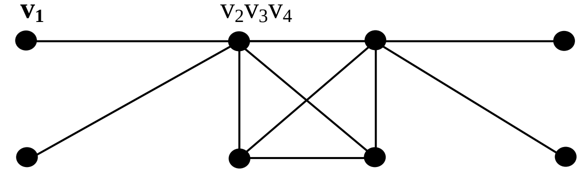

Fig.2

Let S = $\{v_2, v_3\}$ is a strong dominating set but it is not a strong perfect dominating set. Since $|N_s(v_5)\cap S| = |N_s(v_6)\cap S| = |\{v_2,v_3\}| = 2 \neq 1$. Hence S is not a strong perfect dominating set of G. $T_1 = \{v_2,v_4,v_7\}$, $T_2 = \{v_1,v_3,v_8\}$ and $T_3 = \{v_2,v_3,v_5,v_6\}$ are strong perfect dominating sets of G. $\gamma_{sp}(G) = 3$ and $\Gamma_{sp}(G) = 4$. Therefore $\gamma_s(G) < \gamma_{sp}(G)$.

Theorem 2.7: For any path P_m

Then $\gamma_{sp}(P_m) = \begin{cases} n \text{ if } m = 3n, n\epsilon N \\ n+1 \text{ if } m = 3n+1, n\epsilon N \\ n+2 \text{ if } m = 3n+2, n\epsilon N \end{cases}$

Proof:Case (i): Let G $=P_{3n}$, $n \in N$. Let $v_1, v_2, v_3, \ldots, v_{3n}$ be the vertices of G. $\{v_2,v_5,\ldots,v_{3n-1}\}$ is a strong perfectdominating set of G. $\gamma_{sp}(G) \leq n$ for all $n \in N$. Since $n = \gamma_s(G) \leq \gamma_{sp}(G)$. Therefore $\gamma_{sp}(P_{3n}) = n$ for all $n \in N$

Case(ii): Let $G=P_{3n+1}, n\in N$. Let $v_1,v_2,\ldots,v_{3n+1}$ be the vertices of G. $\{v_2,v_5,v_8,\ldots,v_{3n-1},v_{3n+1}\}$, $\{v_1,v_3,v_6,\ldots,v_{3n}\}$, $\{v_2,v_3,v_6,\ldots,v_{3n}\}$ are some strong perfect dominating sets of P_{3n+1}. $\gamma_{sp}(P_{3n+1}) \leq n+1$ for all $n \in N$. Since $n+1 = \gamma_s(P_{3n+1}) \leq \gamma_{sp}(P_{3n+1})$. Therefore $\gamma_{sp}(P_{3n+1}) = n+1$.

Case(iii):

Let $G=P_{3n+2}$, $n\in N$. Let $V(G)=\{v_1,v_2,\ldots v_{3n},v_{3n+1},v_{3n+2}\}$. $\{v_1,v_3,v_6,\ldots,v_{3n},v_{3n+2}\}$, $\{v_2,v_5,v_8,\ldots,v_{3n-1},v_{3n},v_{3n+1}\}$, and $\{v_2,v_5,v_8,\ldots,v_{3n-1},v_{3n},v_{3n+2}\}$ are some strong perfect dominating sets of P_{3n+2}. Since $n+2 = \gamma_s(P_{3n+2}) \le \gamma_{sp}(P_{3n+2})$. Hence $\gamma_{sp}(P_{3n+2}) = n+2$.

Theorem 2.8: For any cycle C_m

Then $\gamma_{sp}(C_m) = \begin{cases} n \ if \ m = 3n \\ n+1 \ if \ m = 3n+1 \\ n+2 \ if \ m = 3n+2 \end{cases}$

Proof:

Case(i): Let $G = C_{3n}$, $n \in N$. Let $V(G) = \{v_1,v_2,\ldots,v_{3n}\}$. $\{v_1,v_4,v_7,\ldots,v_{3n-2}\}$, $\{v_2,v_5,v_8,\ldots,v_{3n-1}\}$ and $\{v_3,v_6,v_9,\ldots,v_{3n}\}$ are strong perfect dominating sets of C_{3n}. $\gamma_{sp}(C_{3n}) \le n$ for all $n \in N$. It is verified that there is no strong perfect dominating set of n-1 vertices. Therefore $\gamma_{sp(G)} \ge n$. Hence $\gamma_{sp}(C_{3n}) = n$ for all $n \in N$.

Case(ii): Let $G= C_{3n+1}$, $n \in N$. Let $V(G)= \{v_1,v_2,\ldots,v_{3n+1}\}$. $\{v_1,v_2,v_5,\ldots,v_{3n-1}\}$, $\{v_2,v_3,v_6,\ldots,v_{3n}\}$, $\{v_3,v_4,v_7,\ldots,v_{3n+1}\}$ are some strongperfect dominating sets of C_{3n+1}. Therefore $\gamma_{sp}(C_{3n+1}) \le n+1$, $n \in N$. It is verified that there is no strong perfect dominating set of n vertices. $\gamma_{sp}(C_{3n+1}) \ge n+1$. Hence $\gamma_{sp}(C_{3n+1}) = n+1$.

Case(iii): Let $G = C_{3n+2}$, $n \in N$. Let $V(G) = \{v_1,v_2,\ldots,v_{3n+2}\}$. $\{v_1,v_2,v_3,v_6,\ldots,v_{3n}\}$, $\{v_2,v_3,v_4,v_7,\ldots,v_{3n+1}\}$, $\{v_3,v_4,v_5,v_8,\ldots,v_{3n+2}\}$ are some strongperfectdominating sets of C_{3n+2}. $\gamma_{sp}(C_{3n+2}) \le n+2$ for all $n \in N$. It is verified that there is no strong perfect dominating set of **n**+1 vertices. $\gamma_{sp}(C_{3n+2}) \ge n+2$. Hence $\gamma_{sp}(C_{3n+2}) = n+2$.

Theorem 2.9: Let G be a connected graph with n vertices. If $\Delta(G) = n-2$ and G contains exactly two maximum degree vertices which are non-adjacent then $\gamma_{sp}(G) = n$

Proof: Let u and v be two non-adjacent vertices in G such that deg u = deg v = $\Delta(G) = n-2$. Let S be a strong perfect dominating set of G. S must contain u and v. Also the vertices other than u and v must belong to S. Since otherwise $|Ns(w)\cap S| \ge 2$, where $w \in V(G)$, $w \ne u,v$, a contradiction. Hence $\gamma_{sp}(G) = n$.

Remark 2.10:

1. $\gamma_{sp}(K_n) = 1$, $n \in N$.
2. $\gamma_{sp}(K_{1,n}) = 1$, $n \in N$.
3. $\gamma_{sp}(W_n) = 1$, $n \in N$.

Theorem 2.11: $\gamma_{sp}(D_{r,s}) = 2$. $r,s \in N$

Proof: Let $G = D_{r,s}$. Let u and v be the central vertices of G. Let $u_1,u_2,\ldots,u_r$ be the vertices adjacent with u and $v_1,v_2,\ldots,v_s$ be the vertices adjacent with v. Clearly {u , v} is the unique strong perfect dominating set of G. Therefore $\gamma_{sp}(G) = 2$.

Theorem 2.12: $\gamma_{sp}(K_{m,n}) = \begin{cases} 2 \ if \ m = n \\ m+n \ if \ m \ne n \end{cases}$

Proof: Let $G = K_{m,n}$.

Let $V(G) =\{v_1,v_2,\ldots,v_m,u_1,u_2,\ldots,u_n\}$.

Case(i): Let m = n. $\{v_i,u_j\}$ $1 \le i,j \le n$ is a strong perfect dominating set of G. Hence $\gamma_{sp}(G) \le 2$. Since G has no full degree vertex, $\gamma_{sp}(G) \ge 2$. Hence $\gamma_{sp}(G) = 2$.

Case(ii): Let $m \ne n$. Without loss of generality let $m < n$, $degv_i > degu_j$, $1 \le i \le m$, $1 \le j \le n$. Each v_i strongly dominates all u_j's, $1 \le j \le m$ and v_i's are mutually non adjacent. Let S be a strong perfect dominating set of G. Then S contains all v_i, $1 \le i \le m$. Some $|N_s(u_j)\cap S| \ge 2$, all u_j's belong to S. Hence $\gamma_{sp}(K_{m,n}) = m+n$.

Theorem 2.13: Let G be a connected graph with $|V(G)| = n$ then $\gamma_{sp}(G \odot K_1) = n$

Proof: Let $V(G) = \{v_1,v_2,\ldots,v_n\}$. Let $u_1,u_2,\ldots,u_n$ be the vertices such that v_i and u_i are adjacent. Let S be a γ_{sp} – set of G. Let $|S| = m$, $m \le n$. The vertices in S strongly unique dominate the pendant vertices u_i's. Remaining n – m pendant vertices together with vertices of S form a strong perfect dominating set of $G\odot K_1$. Hence $\gamma_{sp}(G\odot K_1) = m + n - m = n$.

Theorem 2.14: $\gamma_{sp}(H_n) = n+1, n \geq 3$.

Proof: Let $V(H_n) = \{v,v_1,v_2,\ldots,v_n,u_1,u_2,\ldots,u_n\}$.$E(H_n) = \{vv_i / 1 \leq i \leq n\} \cup \{vu_i / 1 \leq i \leq n\} \cup \{v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{v_nv_1\} \cup \{v_iu_i / 1 \leq i \leq n\}$. $\{v\}\cup\{u_1,u_2,\ldots,u_n\}$ is the unique strong perfect dominating set of H_n. Hence $\gamma_{sp}(H_n) = n+1$.

Theorem 2.15: Let G be a connected graph $\gamma_{sp}(G) = 1$ if and only if G contain atleast one full degree vertex.

III. STRONG PERFECT DOMINATION IN SOME MIDDLE GRAPH

Theorem3.1: Let P_m be the path on m vertices, $m \geq 2$.

Then$\gamma_{sp}(M(P_m))$

$$= \begin{cases} 2n-1 \; if m = 3n, n \geq 2 \\ 2n+1 \; if m = 3n+1 \; or \; 3n+2, n \geq 1 \end{cases}$$

Proof: Let $V(P_m) = \{v_1,v_2,\ldots,v_m\}$ and $E(P_m) = \{e_i/e_i = v_iv_{i+1}, 1 \leq i \leq m-1\}$

Case(i): Let $m = 3n$, $n \geq 2$ $V(M(P_{3n})) = \{v_1,v_2,\ldots,v_{3n},e_1,e_2,\ldots,e_{3n-1}\}$. Each e_i is adjacent with v_i and v_{i+1}, $1\leq i \leq 3n-1$. Their vertices v_i's are mutually non adjacent. Also e_j is adjacent with e_{j+1},$1 \leq j \leq 3n-2$.The sub graph induces by $e_1,e_2,\ldots,e_{3n-1}$ is a path $P_{3n-1.}$ $S = \{e_1,e_3,e_6,\ldots,e_{3n-3},e_{3n-1},v_5,v_8,\ldots,v_{3n-4}\}$ is the unique strong perfect dominating set of $M(P_{3n})$.$|S| = 2n-1$. Hence $\gamma_{sp}(M(P_{3n})) = 2n-1$, $n \geq 2$.

Case(ii): Let $m = 3n+1$, $n \geq 1$. $V(M(P_{3n+1})) = \{v_1,v_2,\ldots,v_{3n+1},e_1,e_2,\ldots,e_{3n}\}$.Each e_i is adjacent with v_i and v_{i+1}, $1 \leq i \leq 3n$ and each e_j is adjacent with e_{j+1}, $1 \leq j\leq 3n+1$. $T = \{e_2,e_5,e_8,\ldots e_{3n-1},v_1,v_4,v_7,\ldots v_{3n+1}\}$ is the unique strong perfect dominating set of $M(P_{3n+1})$.$|T| = 2n+1$.

Case(iii): Let $m = 3n+2$, $n \geq 1$. $V(M(P_{3n+2})) = \{v_1,v_2,\ldots v_{3n+2},e_1,e_2,\ldots e_{3n+1}\}$. Each e_i is adjacent with v_i and v_{i+1},$1\leq i \leq 3n+1$ and each e_j is adjacent with e_{j+1},$1\leq j\leq 3n$. Any strong perfect dominating set must contain either e_1 or e_2. Since if both e_1 and e_2 belong to any strongperfect dominating set S then $|N_s(v_2)\cap S| = |\{e_1,e_2\}| = 2 > 1$, a contradiction, Hence no other strong perfect dominating set without e_1 and e_2 exists. $S_1 = \{e_1,e_3,e_6,\ldots\ldots\ldots,e_{3n-3},e_{3n},v_5,v_8,\ldots,v_{3n-1},v_{3n+2}\}$ $S_2 = \{e_2,e_5,e_8,\ldots\ldots,e_{3n-1},e_{3n+1},v_1,v_4,v_7,\ldots,v_{3n-2}\}$ are strongperfect dominating sets of $M(P_{3n+2})$, $n \geq 1$. $|S_1| = 2n+1$ and $|S_2| = 2n+1$. Therefore $\gamma_{sp}(M(P_{3n+2}) \leq 2n+1$, $n \geq 1$. It is verified that no other strong perfect dominating set of 2n vertices exist. Hence $\gamma_{sp}(M(P_{3n+2})) \geq 2n+1$. Therefore $\gamma_{sp}(M(P_{3n+2}) = 2n+1$, $n \geq 1$

Theorem 3.2: LetC_m be the cycle on m vertices then$\gamma_{sp}(M(C_m)) = \begin{cases} 2n \; if \; m = 3n \\ 2n+2 \; if \; m = 3n+1 \\ 2n+4 \; if \; m = 3n+2 \end{cases}$

Proof: Let $G=C_m$, $m \geq 3$. Let $V(G)=\{v_1,v_2,\ldots,v_m\}$. Let $E(G)=\{e_i/e_i=v_iv_{i+1}, 1\leq i \leq m-1\} \cup\{e_m= v_1v_m\}$. $V(M(G)) = \{v_1,v_2,\ldots,v_{3n},e_1,e_2,\ldots,e_{3n}\}$.Each e_i is adjacent with e_{i+1},$1\leq i\leq m-1$ and e_1 and e_m are adjacent. Each v_i is adjacent with e_i and e_{i+1},$1\leq i \leq m-1$.The vertex v_m is adjacent with e_1 and e_m,$\deg e_i = 4 = \Delta(G)$,$1\leq i \leq m$ and $\deg v_i = 2$,$1\leq i \leq m$.

Case(i): Let $m = 3n$, $n \geq 1$.The sub graph induced by $e_1,e_2,\ldots,e_{3n}$ is C_{3n}.Let $S = \{e_1,e_4,e_7,\ldots,e_{3n-2}\}$.Each e_i in S strongly dominates $e_{i-1},e_{i+1},v_i,v_{i-1}$,$4 \leq i \leq 3n-2$ and e_1 strongly dominates e_2,e_{3n},v_{3n} and v_1.Since the vertices $v_2,v_5,\ldots,v_{3n-1}$ is not dominated by any vertex of S and they are not adjacent with one another. They together with S form a γ_{sp} – set of M(G).Similarly $S_2= \{e_2,e_5,e_8,\ldots\ldots\ldots,e_{3n-1},v_3,v_6,\ldots,v_{3n}\}$ and $S_3 = \{e_3,e_6,e_9,\ldots,e_{3n},v_1,v_4,v_7,\ldots,v_{3n-2}\}$ are the γ_{sp} - sets of M(G). $|S_i| = 2n$, i = 1,2 and 3. Therefore $\gamma_{sp}(M(C_{3n})) \leq 2n$. It is verified that no other strong perfect dominating set of 2n - 1 vertex exists. Hence $\gamma_{sp}(M(C_{3n})) \geq 2n$. Therefore $\gamma_{sp}(M(C_{3n})) = 2n$.

Case(ii): Let $m = 3n+1$,$n\geq1$.The sub graph induced by $e_1,e_2,\ldots,e_{3n+1}$ is C_{3n+1}. Let $S = \{e_1,e_2,e_5,\ldots\ldots\ldots,e_{3n-1},v_1\}$. Each e_i in S strongly dominates $e_{i-1},e_{i+1},v_i,v_{i-1}$, $5 \leq i \leq 3n-1$ and e_1,e_2 strongly dominate e_3,e_{3n+1},v_3 and v_{3n+1}. Since the vertices $v_4,v_7,\ldots,v_{3n}$ are not dominated by any vertex of S and they are not adjacent with one another. They together with S form aγ_{sp} – set of $M(C_{3n+1})$. i.e) $S_1 = \{e_1,e_2,e_5,\ldots,e_{3n-1},v_1,v_4,v_7,\ldots,v_{3n}\}$. $|S_1| = 2n+2$.Therefore $\gamma_{sp}(M(C_{3n+1})) \leq 2n+2$. It is verified that no other strong perfect dominating set of 2n+1 vertex exists. Hence $\gamma_{sp}M(C_{3n+1}) \geq 2n+2$. Therefore$\gamma_{sp}M(C_{3n+1}) = 2n+2$.

Case(iii): Let $m = 3n+2, n \geq 1$. The subgraph induced by $e_1, e_2, \ldots, e_{3n+2}$ is C_{3n+2}. Let $S = \{e_1, e_2, e_3, e_6, \ldots, e_{3n}, v_1, v_2\}$. Each e_i in S strongly dominates $e_{i-1}, e_{i+1}, v_i, v_{i-1}$, $6 \leq i \leq 3n$ and e_1, e_2, e_3, are strongly dominate $e_4, e_{3n+2}, v_1, v_{3n+1}$. Since the vertices $v_4, v_7, \ldots, v_{3n+1}$ are not dominated by any of S and they are not adjacent with one another. They together with S form a γ_{sp}- set of $M(C_{3n+2})$. i.e) $S_1 = \{e_1, e_2, e_3, e_6, \ldots, e_{3n}, v_1, v_2, v_4, v_7, \ldots, v_{3n+1}\}$. $|S_1| = 2n+4$. Therefore $\gamma_{sp}(M(C_{3n+2})) \leq 2n+4$. It is verified that no other strong perfect dominating set of 2n+3 vertices exists.Therefore$\gamma_{sp}M(C_{3n+2}) \geq 2n+4$. Hence $\gamma_{sp}(M(C_{3n+2})) = 2n+4$.

Theorem 3.3: Let $G = K_{1,n} n \geq 1$,then $\gamma_{sp}(M(K_{1,n})) = n\, \gamma_{sp}(K_{1,n})$.

Proof: Let $V(G) = \{v, v_1, v_2, \ldots, v_n\}$ deg v = n and deg $v_i = 1, 1 \leq i \leq n$, $E(G) = \{ e_i / e_i = vv_i, 1 \leq i \leq n\}$. $V(M(K_{1,n})) = \{v, v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$ v and v_i are adjacent with $e_i, 1 \leq i \leq n$ and each e_i is adjacent with all other e_j's. Therefore the sub graph induced by $\{v, e_1, e_2, \ldots, e_n\}$is a complete graph$K_{n+1}$,deg v = n, deg$e_i = n+1 = \Delta(G), 1 \leq i \leq n$ and deg $v_i = 1$, $1 \leq i \leq n$. Each e_i strongly dominates v ,v $_i$and e $_j$, $1 \leq j \leq n$, for $1 \leq i \leq n$, $i \neq j$. $S_i = \{ e_i\} \cup \{ v_j / 1 \leq j \leq n, j \neq i\}$ is a γ_{sp} – set of $M(K_{1,n})$ and $|S_i| = n$. Therefore$\gamma_{sp}(M(K_{1,n})) = n\, \gamma_{sp}(K_{1,n})$.

Theorem 3.4: $\gamma_{sp}(M(D_{r,s})) = r+s+1$, $r,s \geq 1$

Proof: Let $V(D_{r,s}) = \{u, v, u_1, u_2, \ldots, u_r, v_1, v_2, \ldots, v_s\}$. Let e = uv, $e_i = u_i u$, $1 \leq i \leq r$, $f_j = v_j v$, $1 \leq j \leq s$. Let $G = M(D_{r,s})$. $V(G) = V(D_{r,s}) \cup \{e, e_i, f_j, 1 \leq i \leq r, 1 \leq j \leq s\}$, deg e = r+s+2 = $\Delta(G)$, deg u = r+1, deg v = s+1 degu_i = degv_j = 1, $1 \leq i \leq r$, $1 \leq j \leq s$, dege_i= r+1, $1 \leq i \leq r$, deg f_j= s+1, $1 \leq j \leq s$.$\{e, u_1, u_2, \ldots, u_r, v_1, v_2, \ldots, v_s\}$ is the unique strong perfect dominating set of G. Hence $\gamma_{sp}(G) = r+s+1$.

IV. CONCLUSION

In this paper strong perfect domination number of standard graphs and their middle graphs are determined.

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