2-ABSORBING AND WEAKLY 2-ABSORBING SUBSEMIMODULES

N. RAVI BABU[1*], DR. T.V. PRADEEP KUMAR[2], DR. P.V. SRINIVASA RAO[3]

Abstract. A partial semiring is a structure possessing an infinitary partial addition and a binary multiplication, subject to a set of axioms. The partial functions under disjoint-domain sums and functional composition is a partial semiring. In this paper we obtain equivalent conditions and some characteristics of 2-absorbing subsemimodules and weakly 2-absorbing subsemimodules in partial semirings.

Index Terms: Semimodule, 2-absorbing subsemimodule, weakly 2-absorbing subsemimodule, commutative partial semiring.

Introduction

Partially defined infinitary operations occur in the contexts ranging from integration theory to programming language semantics. The general cardinal algebras studied by Tarski in 1949, Housdorff topological commutative groups studied by Bourbaki in 1966, Σ-structures studied by Higgs in 1980, sum ordered partial monoids and sum ordered partial semirings (so-rings) studied by Arbib, Manes and Benson[2], [3], and Streenstrup[13] are some of the algebraic structures of the above type.

In 2014, M. S. Reddy[12] introduced the notion of 2-absorbing subsemimodules in partial semirings which is the generelisation of subsemimodules in partial semirings. In this paper, we consider the 2-absorbing subsemimodules of partial semirings and obtain various equivalent conditions of it. Also we obtain the characterizations of weakly 2-absorbing subsemimodules interms of weakly 2-absorbing partial ideals of partial semirings.

2010 *Mathematics Subject Classification.* 16Y60.

[1*]Department of Basic Science & Humanities, Narasaraopet Engineering College, Narasaraopet, Andhra Pradesh, INDIA

[2]Department of Science and Humanities, ANU College of Engineering, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur - 522510, Andhra Pradesh, INDIA

[3]Department of Basic Engineering, DVR & Dr.HS MIC College of Technology, Kanchikacherla - 521180, Andhra Pradesh, INDIA

1. Preliminaries

In this section we collect some definitions and results for our use in this paper.

Definition 1.1. [3] *A partial monoid is a pair* (M, Σ) *where* M *is a nonempty set and* Σ *is a partial addition defined on some, but not necessarily all families* $(x_i : i \in I)$ *in* M *subject to the following axioms:*

(1) Unary Sum Axiom. If $(x_i : i \in I)$ *is a one element family in* M *and* $I = \{j\}$*, then* $\Sigma(x_i : i \in I)$ *is defined and equals* x_j*.*

(2) Partition-Associativity Axiom. If $(x_i : i \in I)$ *is a family in* M *and* $(I_j : j \in J)$ *is a partition of* I*, then* $(x_i : i \in I)$ *is summable if and only if* $(x_i : i \in I_j)$ *is summable for every* j *in* J *and* $(\Sigma(x_i : i \in I_j) : j \in J)$ *is summable. we write* $\Sigma(x_i : i \in I) = \Sigma(\Sigma(x_i : i \in I_j) : j \in J)$*.*

Definition 1.2. [13] *A partial semiring is a quadruple* $(R, \Sigma, \cdot, 1)$*, where* (R, Σ) *is a partial monoid,* $(R, \cdot, 1)$ *is a monoid with multiplicative operation '$\cdot$' and unit* 1*, and the additive and multiplicative structures obey the following distributive laws: If* $\Sigma(x_i : i \in I)$ *is defined in* R*, then for all* y *in* R*,* $\Sigma(y \cdot x_i : i \in I)$ *and* $\Sigma(x_i \cdot y : i \in I)$ *are defined and* $y \cdot \Sigma(x_i : i \in I) = \Sigma(y \cdot x_i : i \in I)$*,* $\Sigma(x_i : i \in I) \cdot y = \Sigma(x_i \cdot y : i \in I)$*.*

Definition 1.3. [3] *A partial semiring* $(R, \Sigma, \cdot, 1)$ *is said to be commutative, if* $xy = yx$ $\forall\ x,\ y \in R$*.*

Definition 1.4. [10] *Let* $(R, \Sigma, \cdot, 1)$ *be a partial semiring and* $(M, \overline{\Sigma})$ *be a partial monoid. Then* M *is said to be a left partial semimodule over* R *if there exists a function* $*: R \times M \longrightarrow M : (r, x) \longmapsto r * x$ *which satisfies the following axioms for* x*,* $(x_i : i \in I)$ *in* M *and* r_1, r_2*,* $(r_j : j \in J)$ *in* R

(i). if $\overline{\Sigma_i} x_i$ *exists then* $r * (\overline{\Sigma_i} x_i) = \overline{\Sigma_i}(r * x_i)$*,*

(ii). if $\Sigma_j r_j$ *exists* $(\Sigma_j r_j) * x = \overline{\Sigma_j}(r_j * x)$*,*

(iii). $r_1 * (r_2 * x) = (r_1 \cdot r_2) * x$ *and*

(iv). $1_R * x = x$*.*

Definition 1.5. [10] *Let* $(M, \overline{\Sigma})$ *be a left partial semimodule over a partial semiring* R*. Then a nonempty subset* N *of* M *is said to be a subsemimodule of* M *if* N *is closed under* $\overline{\Sigma}$ *and*

Definition 1.6. [10] *If N is a proper subsemimodule of a partial semimodule M over R then* $(N : M) = \{r \in R \mid rM \subseteq N\}$.

Definition 1.7. [10] *Let M be a partial semimodule over R. Then M is said to be multiplication partial semimodule if for all subsemimodules N of M there exists a partial ideal I of R such that $N = IM$.*

Theorem 1.8. [10] *A partial semimodule M over R is a multiplication partial semimodule if and only if there exists a partial ideal I of R such that $Rm = IM$ for each $m \in M$.*

Definition 1.9. [10] *Let M be a multiplication partial semimodule over R and N, K be subsemimodules of M such that $N = IM$ and $K = JM$ for some partial ideals I, J of R. Then the multiplication of N and K is defined as $NK = (IM)(JM) = (IJ)M$.*

Definition 1.10. [10] *Let M be a multiplication partial semimodule over R and $m_1, m_2 \in M$ such that $Rm_1 = IM$ and $Rm_2 = JM$ for some partial ideals I, J of R. Then the multiplication of m_1 and m_2 is defined as $m_1m_2 = (IM)(JM) = (IJ)M$.*

Definition 1.11. [12] *Let M be a partial semimodule over R and N be a proper subsemimodule of M. Then N is said to be a 2-absorbing subsemimodule of M if for any $a, b \in R$ and $m \in M$, $ab * m \in N$ implies $ab \in (N : M)$ or $a * m \in N$ or $b * m \in N$.*

Remark 1.12. [8] *Let R be a so-ring and I be a proper ideal of R. If I is a 2-absorbing ideal then I is a semi-2-absorbing ideal of R.*

Theorem 1.13. [12] *Let M be a multiplication partial semimodule over R and N be a subsemimodule of M. Then the following conditions are equivalent:*

(i). N is a 2-absorbing subsemimodule of M

(ii). for any subsemimodules U, V of W of M, $UVW \subseteq N$ implies $UV \subseteq N$ or $VW \subseteq N$ or $UW \subseteq N$

(iii). for any $m_1, m_2, m_3 \in M$, $m_1m_2m_3 \subseteq N$ implies $m_1m_2 \subseteq N$ or $m_2m_3 \subseteq N$ or $m_1m_3 \subseteq N$.

Definition 1.14. [11] *Let M be a multiplication partial semimodule over R. A subset S of M is said to be multiplication closed subset (in short closed subset) if for any $m, n \in S$, $mn \bigcap S \neq \emptyset$.*

Definition 1.15. [3] *A left partial semimodule M over R is said to be entire if and only if $r * m \neq 0_M$ whenever $0 \neq r \in R$ and $0_M \neq m \in M$.*

Throughout this paper, R denotes a commutative semiring.

2. 2-Absorbing Subsemimodules

Following the notion of [11], we proved the following results:

Theorem 2.1. *Let K be a multiplication partial semimodule over R. Let B be a subsemimodule of K and T be a closed subset of K such that $B \bigcap T = \emptyset$. Then there is a subsemimodule P of K which is maximal with respect to the property that $B \subseteq P$ and $P \bigcap T = \emptyset$. Furthermore, P is a 2-absorbing subsemimodule of K.*

Proof. Take $\mathcal{A} = \{J \mid J\ is\ a\ subsemimodule\ of\ K,\ B \subseteq J\ and\ J \bigcap T = \emptyset\}$. Clearly $B \in \mathcal{A}$. Moreover $(\mathcal{A}, \subseteq)$ is a partially ordered set in which every simply ordered family has an upper bound. By Zorn's lemma, $\mathcal{A}$ has a maximal element. Let it be P. i.e., P is a subsemimodule of M which is maximal with respect to the property that $B \subseteq P$ and $P \bigcap T = \emptyset$. Now we have to prove P is a 2-absorbing subsemimodule of M. Let $a,\ b \in R$ and $k \in K$ such that $(ab)*k \in P$. Suppose $ab \notin (P : K)$, $a*k \notin P$ and $b*k \notin P$. Then $(ab)K \nsubseteq P$, $a*k \notin P$ and $b*k \notin P$. $\Rightarrow P \subset P+R(ab)K$, $P \subset P+R(a*k)$ and $P \subset P+R(b*k)$. By the maximality of P, $(P+R(ab)K) \bigcap T \neq \emptyset$, $(P+R(a*k)) \bigcap T \neq \emptyset$ and $(P+R(b*k)) \bigcap T \neq \emptyset$. $\Rightarrow \exists\ r,\ s,\ t \in T$ such that $r \in P + R(ab)K$, $s \in P + R(a*k)$ and $t \in P + R(b*k)$. Since T is a closed subset of K, $rst \bigcap T \neq \emptyset$. Moreover $rst = [P + R(ab)K][P + R(a*k)][P + R(b*k)] \subseteq P$. Thus $P \bigcap T \neq \emptyset$, a contradiction. Therefore P is a 2-absorbing subsemimodule of K. Hence the theorem. □

Theorem 2.2. *Every 2-absorbing subsemimodule T of a multiplication partial semimodule K over R contains a minimal 2-absorbing subsemimodule.*

Proof. Take $\mathcal{A} = \{L \mid L\ is\ a\ 2-absorbing\ subsemimodule\ of\ K\ such\ that\ L \subseteq T\}$. Since $T \in \mathcal{A}$, $(\mathcal{A}, \subseteq)$ is a non-empty partially ordered set. Let $\{L_i \mid i \in I\}$ be a decreasing chain of subsemimodules of K such that $L_i \subseteq T\ \forall i \in I$ and let $L' = \bigcap_{i \in I} L_i$. Then L' is a subsemimodule of K and $L' \subseteq T$. Now we prove that L' is 2-absorbing. Let $k_1,\ k_2,\ k_3 \in K$ such that $k_1k_2k_3 \subseteq L'$, $k_1k_2 \nsubseteq L'$ and $k_2k_3 \nsubseteq L'$. Then $k_1k_2 \nsubseteq L_k$, $k_2k_3 \nsubseteq L_k$ for some $k \in I$ (since $\{L_i \mid i \in I\}$ is a decreasing chain). $\Rightarrow k_1k_3 \subseteq L_k$. Now for any $i \leq k$, $L_i \supseteq L_k$ and hence $k_1k_3 \subseteq L_i$. For any $i > k$, $L_i \subseteq L_k$. $\Rightarrow k_1k_2 \nsubseteq L_i$, $k_2k_3 \nsubseteq L_i$ and hence $k_1k_3 \subseteq L_i$ $\forall i > k$. Hence $k_1k_3 \subseteq L'$. By theorem 1.13., L' is a 2-absorbing subsemimodule of K and $L' \in \mathcal{A}$. Then by Zorn's lemma, $\mathcal{A}$ has a minimal element. Hence the theorem. □

Definition 2.3. *Let K be a partial semimodule over R and T be a subsemimodule of K. Then T is said to be semi-2-absorbing if and only if its associated partial ideal $(T : K)$ is semi-2-absorbing.*

Remark 2.4. *Every 2-absorbing subsemimodule of a partial semimodule over R is semi-2-absorbing subsemimodule.*

Proof. Let K be a partial semimodule over R. Let T be a subsemimodule of K. Suppose T is a 2-absorbing subsemimodule of K. Then its associated partial ideal $(T : K)$ is a 2-absorbing partial ideal of R. By remark 1.12., $(T : K)$ is a semi-2-absorbing partial ideal of R. Hence T is a semi-2-absorbing subsemimodule of K. □

The following is an example of a partial semimodule over R in which a semi-2-absorbing subsemimodule is not 2-absorbing.

Example 2.5. *Let R be a partial semiring $\mathbb{N}$ with finite support addition and usual multiplication. Then $K = \mathbb{N} \times \mathbb{N}$ is a left partial semimodule over R by the scalar multiplication $* : (x, (a, b)) \longrightarrow (xa, xb)$. Take $T' = 0 \times 4\mathbb{N}$. Then $(T' : K) = \{0\}$ is a semi-2-absorbing partial ideal of R and hence T' is semi-2-absorbing subsemimodule of K. For $2, 2 \in R$, $(0, 1) \in K$, $(2, 2) * (0, 1) = (0, 4) \in T'$.But $2 \cdot 2 = 4 \notin (T' : K) = \{0\}$ and $2 * (0, 1) = (0, 2) \notin T'$. Hence T' is not a 2-absorbing subsemimodule of K.*

Theorem 2.6. *Let K be a multiplication partial semimodule over R and S be a subsemimodule of K. Then the following conditions are equivalent:*
(i). S is a semi-2-absorbing subsemimodule
(ii). for any subsemimodule H of K, $H^3 \subseteq S$ implies $H^2 \subseteq S$
(iii). for any $k \in K$, $k^3 \subseteq S$ implies $k^2 \subseteq S$.

Proof. (i) $\Rightarrow$ (ii): Suppose S is a semi-2-absorbing subsemimodule. Then $(S : K)$ is a semi-2-absorbing partial ideal of R. Let H be a subsemimodule of K such that $H^3 \subseteq S$. Since K is a multiplication partial semimodule, $\exists$ a partial ideal J of R such that $H = JK$. $\Rightarrow$ $H^3 = (JK)(JK)(JK) = J^3K \subseteq S$. $\Rightarrow J^3 \subseteq (S : K)$. $\Rightarrow J^2 \subseteq (S : K)$ (since $(S : K)$ is a

semi-2-absorbing partial ideal). $\Rightarrow J^2K \subseteq S$. $\Rightarrow (JK)(JK) \subseteq S$. $\Rightarrow H^2 \subseteq S$.
(ii) $\Rightarrow$ (i): Suppose for any subsemimodule H of K, $H^3 \subseteq S$ implies $H^2 \subseteq S$. Let J be a partial ideal of R such that $J^3 \subseteq (S : K)$. $\Rightarrow J^3K \subseteq S$. $\Rightarrow (JK)^3 \subseteq S$. By assumption, $(JK)^2 \subseteq S$. $\Rightarrow J^2K \subseteq S$. $\Rightarrow J^2 \subseteq (S : K)$. Therefore $(S : K)$ is a semi-2-absorbing partial ideal of R. Hence S is a semi-2-absorbing subsemimodule of K.
(ii) $\Rightarrow$ (iii): Suppose for any subsemimodule H of K, $H^3 \subseteq S$ implies $H^2 \subseteq S$. Let $k \in K$ such that $k^3 \subseteq S$. Since K is a multiplication partial semimodule, $\exists$ a partial ideal J of R such that $Rk = JK$. $\Rightarrow k^3 \subseteq (Rk)^3 = (JK)^3 \subseteq S$. Since JK is a subsemimodule of K, by assumption $(JK)^2 \subseteq S$. $\Rightarrow (Rk)^2 \subseteq S$. Since $k \in Rk$, $k^2 \subseteq (Rk)^2$. Hence $k^2 \subseteq S$.
(iii) $\Rightarrow$ (ii): Suppose for any $k \in K$, $k^3 \subseteq S$ implies $k^2 \subseteq S$. Let H be a subsemimodule of K such that $H^3 \subseteq S$. Suppose $H^2 \nsubseteq S$. $\Rightarrow \exists\ h \in H$ such that $h^2 \subseteq H^2$ and $h^2 \nsubseteq S$. $\Rightarrow h^3 \subseteq H^3 \subseteq S$. $\Rightarrow h^3 \subseteq S$. By assumption, $h^2 \subseteq S$, a contradiction. Therefore $H^2 \subseteq S$. □

3. Weakly 2-Absorbing Subsemimodules

Following the notion of weakly 2-absorbing ideals of so-rings in [5], we define weakly 2-absorbing subsemimodules in partial semirings as follows.

Definition 3.1. *Let L be a subsemimodule of a partial semimodule K over R. Then L is said to be weakly 2-absorbing if $0 \neq (xy) * l \in L$, $x, y \in R$, $l \in K$ then $xy \in (L : K)$ or $x * l \in L$ or $y * l \in L$.*

Remark 3.2. *Every 2-absorbing subsemimodule of a partial semimodule K is a weakly 2-absorbing subsemimodule of K.*

Proof. Let K be a partial semimodule over R. Let L be a 2-absorbing subsemimodule of K. Let $x, y \in R$, $l \in K$ such that $0 \neq (xy) * l \in L$. Since L is 2-absorbing, $xy \in (L : K)$ or $x * l \in L$ or $y * l \in L$. Hence L is a weakly 2-absorbing subsemimodule of K. □

The following is an example of a partial semiring R in which the converse need not be true.

Example 3.3. *Consider the partial semiring Z_8. Take $R: = Z_8 \times Z_8$. Clearly R is a commutative partial semiring w.r.t. cartesian product operations. Consider $L = \{(0, 0), (0, 4)\}$ be a subsemimodule of R. Clearly L is a weakly 2-absorbing subsemimodule of R. Now $(2, 0)(2, 0)(0, 2) = (0, 0) \in L$ but $(2, 0)(2, 0) = (4, 0) \notin L$. Hence L is not a 2-absorbing subsemimodule of R.*

Theorem 3.4. *Let L be a weakly 2-absorbing subsemimodule of an entire partial semimodule K over R. Then the associated partial ideal $(L : K)$ is a weakly 2-absorbing partial ideal of R.*

Proof. Let $x, y, z \in R$ such that $0 \neq xyz \in (L : K)$, $xy \notin (L : K)$ and $yz \notin (L : K)$. Then $0 \neq (xyz)K \subseteq L$ and $\exists\ 0 \neq a, b \in K$ such that $(xy) * a \notin L$, $(yz) * b \notin L$. $\Rightarrow (xy) * a \neq 0$, $(yz) * b \neq 0$. Since K is entire, $0 \neq (xyz) * a \in (xyz)K \subseteq L$ and $0 \neq (xyz) * b \in (xyz)K \subseteq L$. $\Rightarrow (xz)(y*a) \in L$, $(xz)(y*b) \in L$. Since L is weakly 2-absorbing, $(xy)*a \notin L$ and $(yz)*b \notin L$ then $xz \in (L : K)$. Hence $(L : K)$ is a weakly 2-absorbing partial ideal of R. □

The following example illustrates if $(L : K)$ is a weakly 2-absorbing partial ideal of R then L need not be a weakly 2-absorbing subsemimodule of an entire partial semimodule K over R.

Example 3.5. *Let R be the partial semiring $\mathbb{N}$ with finite support addition and usual multiplication. Define the scalar multiplication as $* : (a, (x, y)) \longrightarrow (ax, ay)$. Clearly $K = \mathbb{N} \times \mathbb{N}$ be an entire partial semimodule over R by $*$. Let $L = 0 \times 4\mathbb{N}$ be a subsemimodule of K. Now $(L : K) = \{0\}$ is a weakly 2-absorbing partial ideal of R. Since $(0, 0) \neq (2, 2) * (0, 1) \in L$, $2.2 \notin (L : K)$ and $2 * (0, 1) \notin L$, L is not a weakly 2-absorbing subsemimodule of K.*

The following is an example in which the above theorem is not true if K is not an entire partial semimodule over R.

Example 3.6. *Let $R = \mathbb{N}$ be the partial semiring and $K = (Z_6, +_6)$ be the partial semimodule over R. Clearly K is not entire and $\{0\}$ is a weakly 2-absorbing subsemimodule of K. Also, $(\{0\} : K) = 6\mathbb{N}$. Since $0 \neq 2.2.2 \in 6\mathbb{N}$, $2.2 \notin 6\mathbb{N}$. Hence $(\{0\} : K) = 6\mathbb{N}$ is not a weakly 2-absorbing partial ideal of R.*

Theorem 3.7. *Let L be a subsemimodule of an entire multiplication partial semimodule K over R. Then L is weakly 2-absorbing subsemimodule of K if and only if the associated partial ideal $(L : K)$ is a weakly 2-absorbing partial ideal of R.*

Proof. Suppose L is weakly 2-absorbing subsemimodule of an entire multiplication partial semimodule K over R. Then by theorem 3.4., the associated partial ideal $(L : K)$ is a weakly 2-absorbing partial ideal of R. Conversely suppose that $(L : K)$ is a weakly 2-absorbing partial ideal of R. Let $x, y \in R$, $l \in K$ such that $0 \neq (xy) * l \in L$, $x * l \notin L$ and $y * l \notin L$. Since

K is a multiplication partial semimodule then $\exists$ partial ideal A of R such that $Rl = AK$. Now $(xyA)K = xy(AK) = xy(Rl) = R[(xy) * l] \subseteq L$. Also $(xA)K = x(AK) = x(Rl) = R(x * l) \nsubseteq L$ and $(yA)K = y(AK) = y(Rl) = R(y * l) \nsubseteq L$. $\Rightarrow xA \nsubseteq (L : K)$ and $yA \nsubseteq (L : K)$. If $(xy)A = 0$. Then $(xyA)K = 0$. $\Rightarrow R[(xy) * l] = 0$. $\Rightarrow (xy) * l = 0$, a contradiction. So assume that $(xy)A \neq 0$. Now $0 \neq (xy)A \subseteq (L : K)$, $xA \nsubseteq (L : K)$, $yA \nsubseteq (L : K)$ and $(L : K)$ is a weakly 2-absorbing partial ideal of R, We have $xy \in (L : K)$. Hence L is a weakly 2-absorbing subsemimodule of K. □

Theorem 3.8. *Let L be a subsemimodule of an entire partial semimodule K over R. Then the following conditions are equivalent:*

(i). L is a weakly 2-absorbing subsemimodule of K

(ii). for any subsemimodules A, B, C of K, $0 \neq ABC \subseteq L$ implies $AB \subseteq L$ or $BC \subseteq L$ or $AC \subseteq L$

(iii). for any $k_1, k_2, k_3 \in K$, $0 \neq k_1k_2k_3 \subseteq L$ implies $k_1k_2 \subseteq L$ or $k_2k_3 \subseteq L$ or $k_1k_3 \subseteq L$.

Proof. (i) $\Rightarrow$ (ii): Suppose L is a weakly 2-absorbing subsemimodule of K. Let A, B and C be the subsemimodules of K such that $0 \neq ABC \subseteq L$. Since K is a multiplication partial semimodule, $\exists$ partial ideals P, S, T such that $A = PK$, $B = SK$ and $C = TK$. Now $0 \neq ABC = (PK)(SK)(TK) = (PST)K \subseteq L$. $\Rightarrow 0 \neq PST \subseteq (L : K)$. Since $(L : K)$ is a weakly 2-absorbing partial ideal of R, $PS \subseteq (L : K)$ or $ST \subseteq (L : K)$ or $PT \subseteq (L : K)$. $\Rightarrow (PS)K \subseteq L$ or $(ST)K \subseteq L$ or $(PT)K \subseteq L$. $\Rightarrow (PK)(SK) \subseteq L$ or $(SK)(TK) \subseteq L$ or $(PK)(TK) \subseteq L$. $\Rightarrow AB \subseteq L$ or $BC \subseteq L$ or $AC \subseteq L$.

(ii) $\Rightarrow$ (iii): Suppose for any subsemimodules A, B, C of K, $0 \neq ABC \subseteq L$ implies $AB \subseteq L$ or $BC \subseteq L$ or $AC \subseteq L$. Let $k_1, k_2, k_3 \in K$, $0 \neq k_1k_2k_3 \subseteq L$. Since K is a multiplication partial semimodule, $\exists$ partial ideals P, S, T of R such that $Rk_1 = PK$, $Rk_2 = SK$ and $Rk_3 = TK$. Now $0 \neq k_1k_2k_3 = (Rk_1)(Rk_2)(Rk_3) = (PK)(SK)(TK) = (PST)K \subseteq L$. $\Rightarrow$ $0 \neq (Rk_1)(Rk_2)(Rk_3) = (PST)K \subseteq L$. By assumption, $(Rk_1)(Rk_2) \subseteq L$ or $(Rk_2)(Rk_3) \subseteq L$ or $(Rk_1)(Rk_3) \subseteq L$. $\Rightarrow k_1k_2 \in L$ or $k_2k_3 \in L$ or $k_1k_3 \in L$.

(iii) $\Rightarrow$ (i): Suppose for any $k_1, k_2, k_3 \in K$, $0 \neq k_1k_2k_3 \subseteq L$ implies $k_1k_2 \in L$ or $k_2k_3 \in L$ or $k_1k_3 \in L$. Let $a, b, c \in R$ such that $0 \neq abc \subseteq (L : K)$. $\Rightarrow 0 \neq (abc)K \subseteq L$. $\Rightarrow$ $0 \neq (aK)(bK)(cK) \subseteq L$ (since K is a multiplication semimodule). $\Rightarrow 0 \neq (a * k)(b * k)(c * k) \subseteq L \ \forall \ k \in K$. By assumption, $(a * k)(b * k)]subseteqL$ or $(b * k)(c * k)]subseteqL$ or $(a * k)(c * k)]subseteqL$. $\Rightarrow (aK)(bK) \subseteq L$ or $(bK)(cK) \subseteq L$ or $(aK)(cK) \subseteq L$. $\Rightarrow$

$(ab)K \subseteq L$ or $(bc)K \subseteq L$ or $(av)K \subseteq L$. $\Rightarrow ab \in (L : K)$ or $bc \in (L : K)$ or $ac \in (L : K)$. Therefore $(L : K)$ is a weakly 2-absorbing partial ideal of R. Hence by theorem 3.7., L is a weakly 2-absorbing subsemimodule of K. □

CONCLUSION: In this paper, we introduced the notions of semi-2-absorbing and weakly 2-absorbing subsemimodules as a generalization of 2-absorbing subsemimodules in partial semimodules over partial semirings. Further, we obtained the equivalent conditions of these subsemimodules for a special class of partial semimodules and expressed these subsemimodules in terms of their associated partial ideals of partial semirings.

References

[1] Acharyulu, G.V.S.: *A Study of Sum-Ordered Partial Semirings*, Doctoral thesis, Andhra University, 1992.

[2] Arbib, M.A., Manes, E.G.: *Partially Additive Categories and Flow-diagram Semantics*, Journal of Algebra, Vol. 62, pp. 203-227, 1980.

[3] Manes, E.G., and Benson, D.B.: *The Inverse Semigroup of a Sum-Ordered Partial Semirings*, Semigroup Forum, Vol. 31, pp. 129-152, 1985.

[4] Ravi Babu, N., Pradeep Kumar, T.V., Srinivasa Rao, P.V.: *2-absorbing ideals in so-rings*, Accepted by International Journal of Pure and Applied Mathematics.

[5] Ravi Babu, N., Pradeep Kumar, T.V., Srinivasa Rao, P.V.: *weakly 2-absorbing ideals of so-rings*, International Journal of Scientific and Innovative Mathematical Research (IJSIMR),Volume 5, Issue 7, 2017, PP 29-35, ISSN:2347-3142, DOI: http://dx.doi.org/10.20431/2347-3142.0507003.

[6] Ravi Babu, N., Pradeep Kumar, T.V., Srinivasa Rao, P.V.: *2-absorbing primary ideals in so-rings*, Accepted by Jordan Journal of Mathematics and Statistics.

[7] Ravi Babu, N., Pradeep Kumar, T.V., Srinivasa Rao, P.V.: *Weakly 2-absorbing primary ideals in so-rings*, Accepted by International Journal of Pure and Applied Mathematics.

[8] Ravi Babu, N., Pradeep Kumar, T.V., Srinivasa Rao, P.V.: *Semi-2-absorbing ideals in so-rings*, International Journal for Research in Engineering Application & Management (IJREAM),ISSN : 2454-9150, 2018.

[9] Srinivasa Rao, P.V.: *Ideals Of Sum-ordered Semirings*, International Journal of Computational Cognition(IJCC), Vol. 7(2), pp. 59-64. June 2009.

[10] Srinivasa Rao, P.V., Siva Mala, M.: *A Class of Primary subsemimodules*, Jordan Journal of Mathematics and Statistics, Vol. 6(4), pp. 251-264. 2013.

[11] Srinivasa Rao, P.V.: *Ideal Theory of Sum-ordered Partial Semirings*, Doctoral thesis, Acharya Nagarjuna University, 2011.

[12] Srinivasa Reddy, M., Amarendra Babu, V., Srinivasa Rao, P.V.: *2-absorbing Subsemimodules of Partial Semimodules*, Gen.Math.Notes, Vol. 23(2), pp. 43-50, 2014.

[13] Streenstrup, M.E.: *Sum-ordered Partial Semirings*, Doctoral thesis, Graduate school of the University of Massachusetts, Feb 1985 (Department of computer and Information Science).