LRS Bianchi Type-I Cosmological Model with Variable Deceleration Parameter

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Abstract:-
Locally rotationally symmetric(LRS) Bianchi type-Icosmological model for perfect fluid distribution is obtained by considering variable deceleration parameter.Some physical and geometrical properties of the model arealsodiscussed.

Keywords:- *LRS Bianchi type-I model, variable deceleration parameter, perfect fluid.*

I. INTRODUCTION

Einstein's field equations are a coupled system of highly non-liner differential equations and we seek physical solutions of the field equations for application in cosmology and astrophysics. In order to solve Einstein's field equations, we normally assume a form of the matter content or suppose thatthe space time admits killing vector symmetries.Kramer et al. [1] have pointed out that most authors solve the Einstein's field equations with a stress energy tensor of perfect fluid type by assuming an equation of state linking the pressure p and energy density ρ in order to build an analytical methodsnear the singularity. Davidson [2] and later many others (Coley and Tupper [3]) have considered models with variable equation of state, which essentially deals with the Friedman RoberstonWaker (FRW) metric. Solutionsof field equationscan be generated by applying the law of variation of Hubble's parameter proposed by Berman [4] which yields a constant value of deceleration parameter.It is worth observing that most of the well-known models of Einstein's theory and Brans-Dicke theory with curvatureparameter k=0 including inflationary models are models with constant deceleration parameter.The cosmological models with a constant deceleration parameter have been studied by several researchers such as Kramer et al. [1], Berman and Gomide [5],Maharaj and Naidoo[6], Reddy et al.[7]. In 1998, published observations of type Ia supernovae by the High-Z Supernovasearch team (Riess et al. [8]) followed by 1999 Supernova cosmology project (Perlmutteret al.[9]) suggested that the expansionof the universe is accelerating.Recent observations of SNe Ia of high confidence level (A.G.Riess et al.[10]) have further confirm this. Now for a Universe which was decelerating in past and accelerating at the present time, the deceleration parameter must show signature flipping(Padmanabham and Roychowdhury [11],Amendola[12]).So, there is no scope for constant declaration parameter at present epoch. Thus, in general, the deceleration parameter is not a constant but time variable.

Motivated by the recent results, in this paper, we have investigated LRS Bianchi type -I cosmological model by taking the deceleration parameter to be variable.The outline of the paper is as follows:In Section 2, the metric and field equations are described.Section 3 deals with the solution of the field equations. In Section 4, we have discussed the physical and geometrical behaviour of the model.Finally conclusions are summarised in the last Section 5.

II. THE METRIC AND FIELDS EQUATIONS

We consider LRS Bianchi type- I space- time

$$ds^2 = -dt^2 + A^2dx^2 + B^2\left(dy^2 + dz^2\right) \quad (1)$$

Where the metric potentials A and B are functions of cosmic time t alone.
We define the following parameters to be used in solving Einstein's field equations for the matric (1).
The average scale factor a of Bianchi type-I model (1) is defined as

$$a = \left(AB^2\right)^{\frac{1}{3}} \quad (2)$$

A volume scale factor V is given by

$$V = a^3 = AB^2 \tag{3}$$

In analogy with FRW universe, we also define the generalized Hubble parameter H and decelaration parameter q as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \tag{4}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} \tag{5}$$

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} \tag{6}$$

The energy- momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} \tag{7}$$

with equation of state

$$p = \gamma\rho, 0 \le \gamma \le 1 \tag{8}$$

Where p and p are pressure and energy density respectively and u_i is the 4 velocity vector satisfying $u_i u^i = -1.$

In a co-moving coordinate system we have $u_i = (0,0,0,1)$.Forthe energy momentum tensor (7) and LRS Bianchi type-I space- time (1), Einstein's field equations (6) yield the following independent equations

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi G p \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p \tag{10}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi G\rho \tag{11}$$

Where over dots on A and B denotes the ordinary differentiation with respect to t and G is the gravitational constant.

The law of energy- conservation equation $T;_j^{ij} = 0$ gives

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \tag{12}$$

Using equations (4) and (8) in theequation (12), we obtain

$$\dot{\rho} + \rho(1+\gamma)\frac{3\dot{a}}{a} = 0$$

This on integration leads to

$$\rho = \frac{K}{a^{3(1+\gamma)}} \tag{13}$$

Where k is the constant of integration

From the field equations (9) - (11), the pressure p and the energy density ρ, in terms of physical parameters, can be written as follows

$$H^2(2q-1) - \sigma^2 = 8\pi G p \tag{14}$$

$$3H^2 - \sigma^2 = 8\pi G\rho \tag{15}$$

where σ is shear scalar given by

$$\sigma^2=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+2\left(\frac{\dot{B}}{B}\right)^2\right]-\frac{1}{6}\theta^2 \qquad (16)$$

and the expansion scalar θ is defined as

$$\theta=u^i_{;i}=\frac{3\dot{a}}{a}=3H. \qquad (17)$$

Subtracting (9) from (10), one finds the relation between A and B as

$$\frac{A}{B}=d_1 exp\left(K_1\int\frac{dt}{a^3}\right) \qquad (18)$$

where d_1 and k_1 are constants of integration.

By using equation (17) in equation (15), we get

$$\frac{\theta^2}{3}-\sigma^2=8\pi G\rho \qquad (19)$$

For $8\pi G=1, G>0,$ from eqution (19), we get

$$\frac{3\sigma^2}{\theta^2}=1-\frac{3\rho}{\theta^2}$$

which implies that

$$0<\frac{\sigma^2}{\theta^2}<\frac{1}{3} \text{ and } 0<\frac{\rho}{\theta^2}<\frac{1}{3}(\rho>0)$$

Therefore the presence of positive G lowers the upper limit of anisotropy and presence of negative G is to halt this decrease.

Re- writting equation (5) as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-\left(\frac{\dot{H}+H^2}{H^2}\right)$$

or

$$q=-\frac{3\dot{\theta}}{\theta^2}-1, \qquad (20)$$

By the use of Eqs. (8),(17) and (20) in Eqs. (14)and (15), we obtain

$$\dot{\theta}=-3\sigma^2-12\pi G(1+\gamma)\rho \qquad (21)$$

It follows from (21) that the rate of volume expansion decreases during the time evolution and the presence of negative G is to halt this decreases where as the positive G promotes it.

III. SOLUTION OF THE FIELD EQUATIONS

We assume the deceleration parameter to be variable and set

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=b(Variable)$$

(22)

The above equation may be re- written as

$$\frac{\ddot{a}}{a}+b\frac{\dot{a}^2}{a^2}=0 \qquad (23)$$

The general solution of equation(23) is given by

$$\int e^{\int\frac{b}{a}da}da=t+d \qquad (24)$$

wheredisan integrating constant. Inorder to solve the problem completely, we have to choose $\int\frac{b}{a}da$ in such a manner that equation (24) be integrable.

Without loss of generality, we consider

$$\int \frac{b}{a} da = \ln L(a) \tag{25}$$

Which does not effect the nature of generality of solution.
Hence, from equastions (24) and (25), we get

$$\int L(a) da = t + d \tag{26}$$

Of course the choice of $L(a)$,in equation (26), is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observation.

So,weconsider $L(a) = \frac{1}{2k_2\sqrt{a+k_3}}$,

where k_2 and k_3 are constants.
In this case, on integrating, equation (26), we obtain

$$a(t) = \alpha_1 t^2 + \alpha_2 t + \alpha_3, \tag{27}$$

where α_1, α_2, and α_3 are arbitrary constants.

If we take $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 0$ in equation(27), we get

$$a(t) = t^2 + t \tag{28}$$

By using equation (28) in equation (18), we get

$$\frac{A}{B} = d_1 \exp\left(k_1 f(t)\right), \tag{29}$$

where $f(t) = \int \frac{dt}{a^3} = \int \frac{dt}{\left(t^2+t\right)^3}$

By using equations (28) and (29) in the relation

$a^3 = AB^2$, we get

$$A = n_1\left(t^2+t\right)\exp\left(\frac{2k_1 f(t)}{3}\right) \tag{30}$$

And

$$B = n_2\left(t^2+t\right)\exp\left(\frac{-k_1 f(t)}{3}\right), \tag{31}$$

Where $n_1 = d_1^{\frac{2}{3}}$ and $n_2 = d_1^{-\frac{1}{3}}$ are the new constants related by the relation $n_1 n_2^2 = 1$.
For this solution, The metric (1) reduces to the form

$$ds^2 = -dt^2 + \left(t^2+t\right)^2\left[n_1^2 \exp\left(\frac{4k_1 f(t)}{3}\right)dx^2 + n_2^2 \exp\left(\frac{-2k_1 f(t)}{3}\right)\left(dy^2+dz^2\right)\right] \tag{32}$$

The expressions for the Hubble parameter H, scalar of expansion θ , magnitude of shear σ^2, the spatial volume V, cosmological energy density ρ , pressure p and deceleration parameter q for the model (32) are given by

$$H = \frac{\dot{a}}{a} = \frac{2}{(t+1)} + \frac{1}{t(t+1)} \tag{33}$$

$$\theta = 3H = \frac{6}{(t+1)} + \frac{3}{t(t+1)} \tag{34}$$

$$\sigma^2 = \frac{k_1^2}{3\left(t^2+t\right)^6} \tag{35}$$

$$V = \left(t^2+t\right)^3 \tag{36}$$

$$\rho = \frac{k}{\left(t^2+t\right)^{3(1+\gamma)}} \tag{37}$$

$$p = \gamma\rho = \frac{\gamma k}{\left(t^2+t\right)^{3(1+\gamma)}} \tag{38}$$

$$q = \frac{-2\left(t^2+t\right)}{\left(2t+1\right)^2} \tag{39}$$

IV. DISCUSSION

From the above results, we observe that the spatial volume Vis zero at t=0. This shows that the universe starts evolving with zero volume at the initial epoch and expands with cosmic time t.The expansion scalar $\theta \to \infty$ as $t \to 0$ which shows that Universe starts evolving with infinite rate of expansion. The energy density ρ , pressure p and shear scalar σ tend to infinity as $t \to 0$ .The scale factor a also vanishes at t=0 and hence the model has a point type singularity at the initial epoch.As $t \to \infty$, the scale factor a and volume V tend to infinitywhere as the energy density ρ , the pressure p, the enpansion scalar θ and shear scalar σ tend to zero. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion andshear scalar decreases to zero and tend to isotropic. At the initial stage of expansion,when p and ρ are large, the Hubble parameter H is also large and with the expansion of universe H, θ decrease as ρ does.Since $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$ the model approachesisotropy for large value t. Our model satisfies all conditions of homogeneity and isotropization according to the formal definition given by collins and Hawking [13].

V. CONCLUSION

In this paper we have studied a spatially homogeneous and isotropic LRS Bianchi Type-I cosmological model with the variable deceleration parameter.We have obtained some importantcosmological parameters and physical behaviour of the model are also discussed in detail. The model shows shearing, non-rotating and expanding model with a big-bang start.The model has a point type singularity at the initial epoch and approach isotropy at late time.Finally the solutions presented in this paper are new and may be useful for better understanding of the characteristics of LRS Bianchi type-I cosmological models with variable deceleration parameter.

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