Behaviour of Soret and Dufour Effects on Unsteady Micropolar Fluid Considering Viscosity and Thermal Conductivity as Variable Quantities under Mixed Convection and Mass Transfer

[1]Indira Baruah, [2]G.C. Hazarika
Department of Mathematics, Tinsukia college, Assam, India
Department of Mathematics, Dibrugarh University , Assam, India

Abstract

In the paper, an analysis is performed to study the unsteady behaviour of micropolar fluid under mixed convection and mass transfer. In the process of study, similarity transformational approach has been considered under variable viscosity and thermal conductivity. Governing equations for the problem concerned are developed under prescribed constraints and latter on numerically solved by using Runge-Kutta method. The residual graphs represent various important parameters of the problem, viz., velocity, temperature, micro-rotation, concentration of the fluid and also considered the effects of Soret and Dufour within the boundary layers and analyzed.

Keywords - *Micro-polar fluid, unsteady flow, Soret and Dufour effects.*

I. INTRODUCTION

The micropolar fluids are non-Newtonian fluids firstly introduced by Eringen[1, 2], which cannot be described normally by classical Navier-Stokes theory. Micropolar fluid is that fluid which contains micro-structures, can undergo rotation, the presence of which can affect the hydrodynamics of the flow. Eringen [1, 2] was the first to introduce the theory of micropolar fluid. The extension on this account towards the theory and applications can be found in the literature of Ariman et al.[16], Lukaszewicz[8] and Eringen[1, 2]. Micropolar fluids practically appear in the field of navigation, submarine, aeronautics, solidification of liquid crystals, prediction of environment pollution in the ocean, atmosphere and in many other industrial fields. Different problems of heat and mass transfer had been undertaken by the researchers considering the importance of micropolar fluid. Peddison and Mc. Nitt[12] derived the boundary layer theory which was important in the light of a number of technical processes. Oianrewaju[15], Hazarika and Phukan[10] and also many researchers studied the behaviour of micropolar fluids in steady state on different environments. Hayat[18], El Aziz[14],], Thakur et al.[16] , Baruah and Hazarika[11] studied Micropolar fluids in the unsteady state. The study of unsteady flow over stretching surface has its own importance as it is not possible to maintain steady-state conditions in the velocity and thermal fields due to impulsive change in the surface velocity or temperature (heat flux).

The energy flux caused by composition gradient is called the Dufour or Diffusion thermo effect, on the other hand mass fluxes which can be created by temperature gradient is the Soret effect. As for instance, Soret effect has been utilized for isotopic separation and in the mixture between gases with very light molecular weight (H_2, He). For medium molecular weight (N_2, air), the Dufour effect was found to be significant in magnitude and as such, it can't be neglected. The results in this direction was proved by the works of Eckert and Drake [6]. Succinctly, Soret and Dufour effects seemed to be very much apposite when temperature and concentration gradients are high.

Taking into account of the importance of the aforesaid effects many researchers such as D. Srinivasacharya et al.[5], MBK Moorty et al.[13], Aurangzaib et al.[] discussed micropolar type of problems in various different environments. In all the above cases, viscosity and thermal conductivity consider as constant quantities. The aim of the present paper is to study Soret and Dufour effects heat transfer on unsteady boundary layer electrically conducting micropolar fluid in porous medium, considering viscosity and thermal conductivity as variable, where there has been a very less work available in this direction .

II. FORMULATION OF THE PROBLEM

Unsteady micropolar fluid flow of an incompressible fluid is being considered in a porous medium over a stretching sheet with Soret and Dufour effects. A transverse magnetic field B_0 is imposed along y axis. Magnetic Reynold number is taken as very small so that induced magnetic field can be neglected. The surface stretched with linear velocity considering as $U_w = \frac{ax}{1-\beta t}$ ($a \geq 0\, , \beta \geq 0$ and $\beta t < 1$). The basic equations of the boundary value problem of micropolar fluid are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{……………………………(1)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial N}{\partial y}\right) + \beta_T g(T - T_\infty) + \beta_c g(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u - \frac{\nu u}{k^*}$$

……. (2)

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \qquad \text{…………………………..(3)}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{D_m \lambda}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + \left(\frac{\mu + \kappa}{\rho c_p}\right)\left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{……………………..(4)}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{D_m \lambda}{\lambda_m}\frac{\partial^2 T}{\partial y^2} \qquad \text{……………………………………………(5)}$$

With boundary conditions:

$$\left.\begin{array}{l} u = u_w \ \ , v = 0 \ , N = -n\frac{\partial u}{\partial y} \ , \ T = T_w \ , C = C_w \ as \ \ y = 0 \\ u \to 0 \ , v \to 0 \ \ , T \to T_\infty \ \ , C \to \infty \ \ as \ \ y \to \infty \end{array}\right\} \qquad \text{…………………………..(6)}$$

u, v are velocity in x and y directions, β_T and β_c are coefficients of thermal and concentration expansion. Density of the micropolar fluid represented by ‘ρ’, vortex viscosity by κ, spin gradient viscosity by ‘γ’, σ is the electrical conductivity, permeability of porous medium given by ‘k^*’, micro inertia per unit mass by ‘j’, g acceleration due to gravity, c_p is the specific heat at constant pressure, c_s concentration susceptibility, D_m mass diffusivity, ‘λ’ is thermal conductivity, λ_m mean fluid temperature. The surface temperature of the stretching sheet T_w (x, t) and concentration field are considered as:

$$T_w = T_\infty + \frac{bx}{(1-\beta t)^2} \quad ; \quad C_w = C_\infty + \frac{cx}{(1-\beta t)^2} \qquad \text{…………………..(7)}$$

Where b and c are positive constants, but b > 0 corresponding for assisting, and b< 0 for opposing flows, also, in case of force convection i.e., in absence of buoyancy force b=0. In the above equations considering a linear relation for spin gradient and micro-inertial per unit mass as:

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j \ = \mu\left(1 + \frac{1}{2}\Delta\right)j \ where \ \Delta = \frac{\kappa}{\mu} \qquad \text{………………………(8)}$$

Which by Ahmadi [9], gives the correct behavior of the fluid when microstructure effects became negligible in limiting case and the microrotation reduces to angular velocity. The equations (1)-(5) are changed into ordinary differential equations by using similarity variable as:

$$\left.\begin{array}{l}\psi=\left(\upsilon x U_w\right)^{1/2} f(\eta) \quad , \quad \eta=\left(\dfrac{U_w}{\upsilon x}\right)^{1/2} y \ , N=U_w\left(\dfrac{U_w}{\upsilon x}\right)^{1/2} g(\eta) \ , B(t)=\dfrac{B_0}{\sqrt{1-\beta t}} \\ \theta(\eta)=\dfrac{T-T_\infty}{T_w-T_\infty} \quad , \quad \phi(\eta)=\dfrac{C-C_\infty}{C_w-C_\infty}\end{array}\right\} \quad \text{.............(9)}$$

Where $f(\eta)$, $g(\eta)$, $\theta(\eta)$, $\phi(\eta)$ are dimensionless stream function, micro-rotation function, temperature distribution , concentration function respectively. Assuming fluid viscosity and thermal conductivity as linear function of temperature (Lai and Kulaki[7])

$$\left.\begin{array}{l}\dfrac{1}{\mu}=\dfrac{1}{\mu_\infty}\left[1+\delta(T-T_\infty)\right]=b^*(T-T_r) \\ where \quad b^*=\dfrac{\delta}{\mu_\infty} \quad , T_r=T_\infty-\dfrac{1}{\delta}\end{array}\right\} \text{ and } \left.\begin{array}{l}\dfrac{1}{\lambda}=\dfrac{1}{\lambda_\infty}\left[1+\delta(T-T_\infty)\right]=c^*(T-T_k) \\ where \quad c^*=\dfrac{\xi}{\lambda_\infty} \quad , T_k=T_\infty-\dfrac{1}{\delta}\end{array}\right\} \quad \text{.....(10)}$$

In this relation b^* , T_r and c^*, T_k are constants , their values depend on reference state and the thermal property of the fluid. Using the above relations in equations (1) to (5), the equations take the form as below:

$$(1+\Delta) f^{///}-\left(\frac{1}{2}A\eta-f-\frac{\theta^{/}}{\theta_c-\theta}\right)f^{//}-f^{/2}-\left(A+M^2+\frac{1}{K^p}\right)f^{/}+G_r\theta+G_m\phi+\Delta g^{/}=0 \text{.........(11)}$$

$$\left(1+\frac{1}{2}\Delta\right)g^{//}-gf^{/}+(f-\frac{1}{2}A\eta)g^{/}+\frac{3}{2}Ag-\Delta B(2g+f^{//})=0 \quad \text{.....................(12)}$$

$$\theta^{//}-\frac{\theta^{/}}{\theta-\theta_r}-P_r(\theta f^{/}-\theta^{/}f)-\frac{1}{2}AP_r(4\theta+\eta\theta^{/})+D_fP_r\phi^{//}+(1+\Delta)P_rE_c f^{//2}=0 \quad \text{..................(13)}$$

$$\phi^{//}-S_c\left\{\frac{1}{2}A(4\phi+\eta)+\phi f^{/}-f\phi^{/}-S_r\theta^{//}\right\}=0 \quad \text{......................(14)}$$

With boundary conditions:

$$\left.\begin{array}{l}f^{/}(\eta)=1 \ , f(\eta)=0 \quad , g(\eta)=-nf^{//}(\eta) \ , \theta(\eta)=1, \ \phi(\eta)=1 \ as \ \eta=0 \\ , \ f^{/}(\infty)=0 \ , \quad g(\infty)=0 \ , \quad \theta(\infty)=0, \quad \phi(\infty)=0 \quad as \ \eta\to\infty\end{array}\right\} \quad \text{..................(15)}$$

Where A=β/a , unsteady parameter.

$M=B_0\sqrt{\sigma/\rho a}$ magnetic parameter , $B=\dfrac{\upsilon(1-\beta t)}{ja}=\dfrac{\upsilon x}{jU_w}$ dimensional parameter.

$\mathrm{Re}=\dfrac{U_w x}{\upsilon}$ Reynold number , $E_c=\dfrac{U_w^2}{C_p(T_w-T_\infty)}$ Eckert number

$G_r=\dfrac{g\beta_T b}{a^2}=\dfrac{G_{rx}}{\mathrm{Re}_x^2}$, $G_{rx}=\dfrac{g\beta_T(T_w-T_\infty)x^3}{\upsilon^2}$ Local Grashof number

$G_c=\dfrac{g\beta_c c}{a^2}=\dfrac{G_{cx}}{\mathrm{Re}_x^2}$, $G_c=\dfrac{g\beta_c(C_w-C_\infty)x^3}{\upsilon^2}$ Local mass Grashof number

$\Delta=\dfrac{\kappa}{\mu}$ micropolar parameter, $\dfrac{1}{K^p}=\dfrac{\upsilon(1-\beta t)}{ak^*}$ permeability parameter, $P_r=\dfrac{\lambda}{\mu c_p}$ Prandtl number.

$D_f=\dfrac{D_m\lambda(C_w-C_\infty)}{C_pC_s\upsilon(T_w-T_\infty)}$ Dufour number , $S_r=\dfrac{D_m\lambda(T_w-T_\infty)}{T_m\upsilon(C_w-C_\infty)}$ Soret numbe , $S_c=\dfrac{\upsilon}{D_m}$ Schmidt number .

Two important physical quantities the skin friction C_f and Nusselt number N_u are defined by

$$C_f = \frac{\left[(\mu+\kappa)\frac{\partial u}{\partial y} + \kappa N \right]_{y=0}}{\rho U_w^2}$$ and

$$Nu = \frac{-x\left(\lambda \frac{\partial T}{\partial y} \right)_{y=0}}{\lambda_\infty (T_w - T_\infty)}$$ **…………………(16)**

After simplification it becomes:

$$C_f R_e^{1/2} = \left[\left(\frac{\theta_c}{\theta_c - 1} \right) + \Delta(1-n) \right] f^{//}(0)$$ and $$N_u R_e^{-1/2} = \left(\frac{\theta_r}{\theta - \theta_r} \right) \theta^{/}(0)$$

………………………..(17)

III. RESULTS AND DISCUSSIONS

Final boundary value equations (11) to (14) are solved by Runge- Kutta fourth order equation with shooting technique by developing a computer oriented program. The computed results of the equations for various values of parameters such as unsteady parameter A, thermal conductivity parameter θr, viscosity parameter θc, magnetic parameter M, Grasof number G_r, Dufour number D_f Soret number S_r are expressed as velocity , temperature, micro-rotation and concentration profile etc. and presented graphically in fig:1 to 18.

The effects of various parameters on velocity profile are presented in fig:1-4. Velocity increases with the increasing value of Grashof number and micropolar parameter. On the other hand velocity decreases for various values of viscosity parameter θ_c and magnetic parameter M. Lorentz force exists due to the presence of applied magnetic field, this force acts opposite to the motion of the fluid for which, physically velocity decreases.

The temperature profile for various values of important parameters presented by fig: 5-8. Temperature increases for magnetic parameter, Grashof and Eckert number but decreases with the increasing value of unsteady parameter and Prandtls number. Magnetic parameter increases the viscous and Lorentz force both of them resists the fluid flow. Fluid has to work against these forces as a result temperature increases on increasing value of M. Because of the increasing value of Prandtl number, thermal conductivity decreases for which temperature also decreases.

The micro-rotation effect and concentration of the fluid for various parameters are shown in fig:9-12. Micro-rotation decreases as the increasing value of micropolar parameter and Grashof number and increases as for the increasing value of unsteady and magnetic parameter. Due to increasing value of parameter D_f, S_c, G_m and S_r, micro-rotation effect increases but for E_c it is decreases . All the numerical results are calculated by taking initial values of the parameter as M=1, S_r =0.2, E_c =0.5 ,θ_c =2, ,θ_r =3, B=-1, A=0.25, P_r=.72, S_c=1, D_f =0.15 , G_r=0.1, G_m=0.1, N=0.5 ,K_p=0.5 etc.

The values of Skin friction and Nusselt number are given in Table: 1 to 3 for combined values of different parameters. The missing values of the functions $f^{//}(0)$, $\theta^{/}(0)$, $\phi^{/}(0)$, $g^{/}(0)$ also given in the same tables. The Table:I (one) reflects that increasing value of thermal conductivity and Prandtls number give increase of both the coefficient of Skin friction and the Nusselt number. In Table:II (two) it has been observed that increasing value of magnetic parameter give decrease of both of them. For increasing value of viscosity parameter, skin friction increases but Nusselts number decreases. Comparing values of Skin friction and Nusselt number are presented in Table:IV (four), where it is observed that due to consideration of variable viscosity and thermal conductivity, the results are appeared to be with some differences.

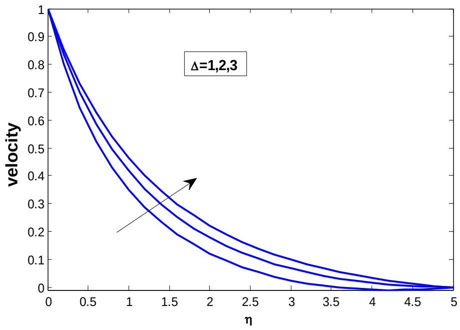

Fig: 1

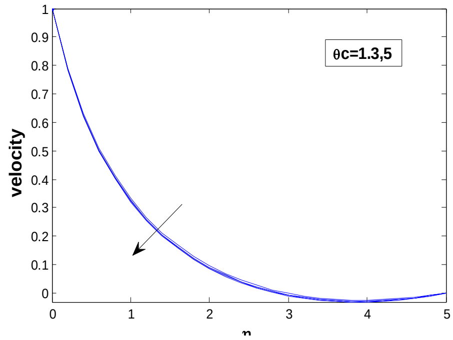

Fig:2

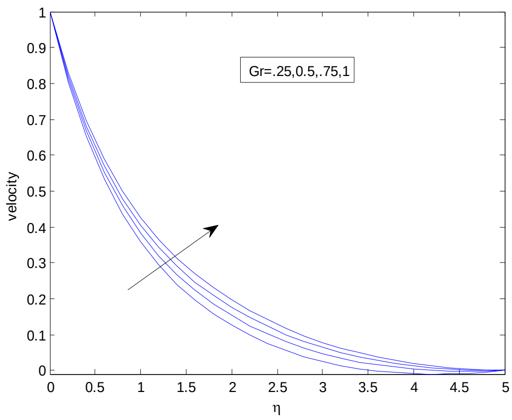

Fig:3

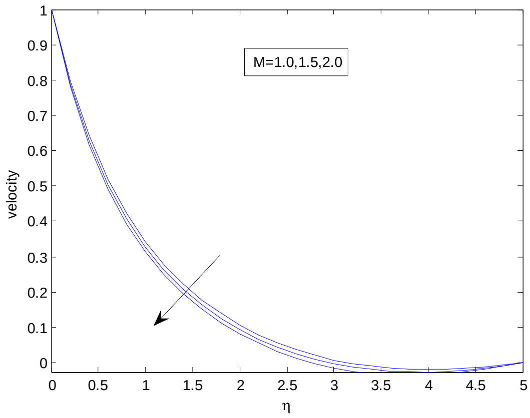

Fig:4

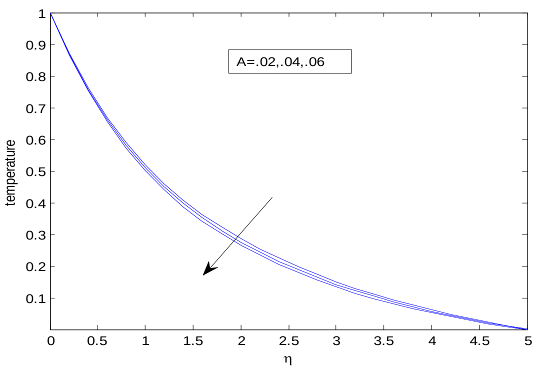

Fig:5

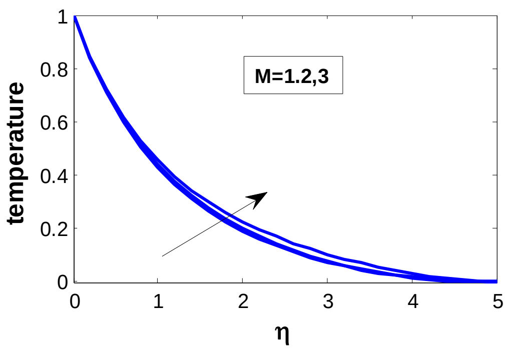

Fig:6

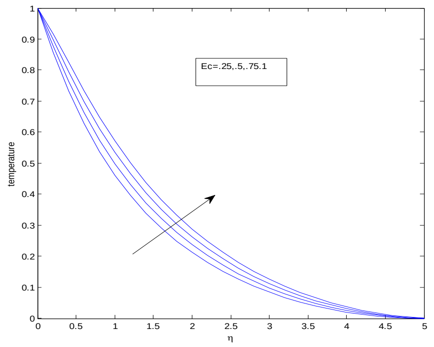

fig:7

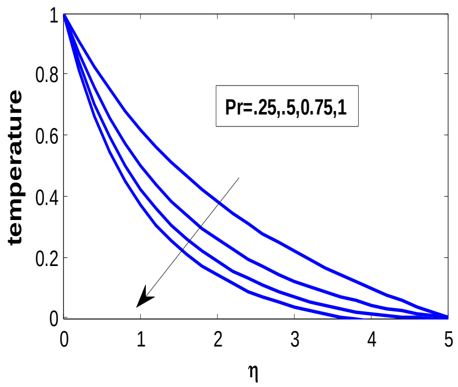

fig:8

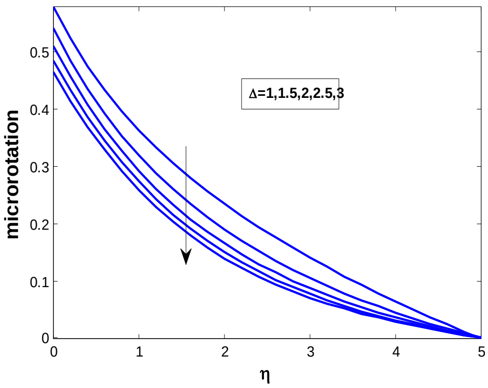

fig: 9

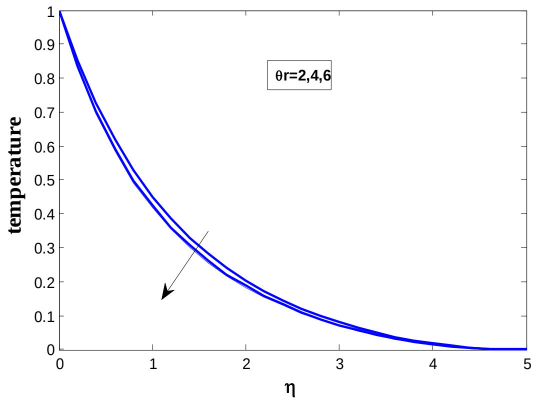

fig:10

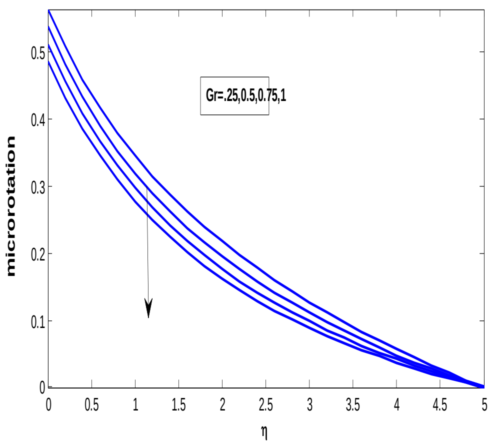

fig :11

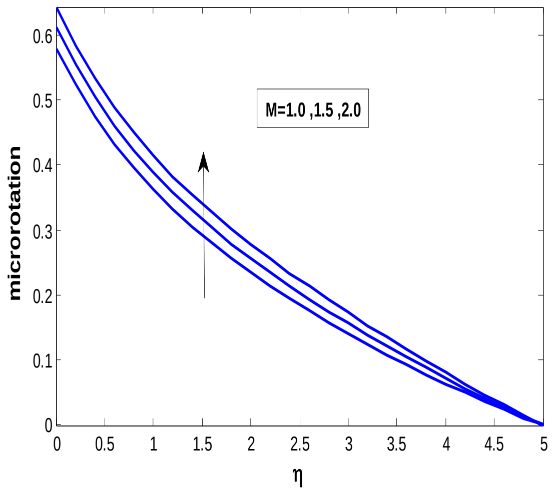

fig:12

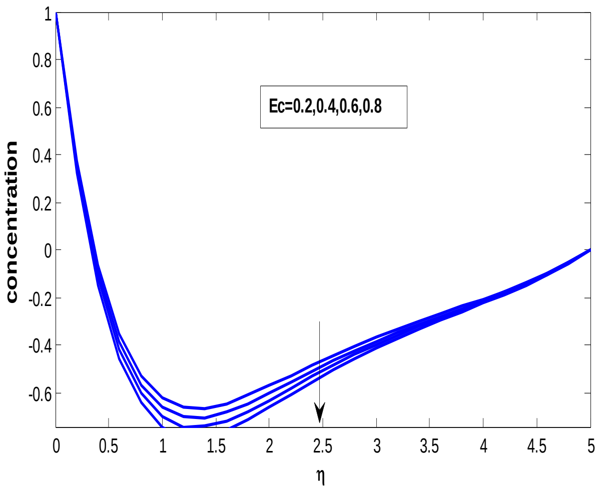

Fig: 13

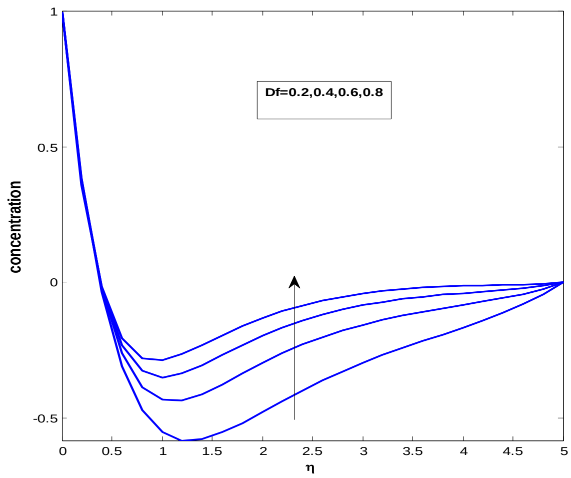

Fig:14

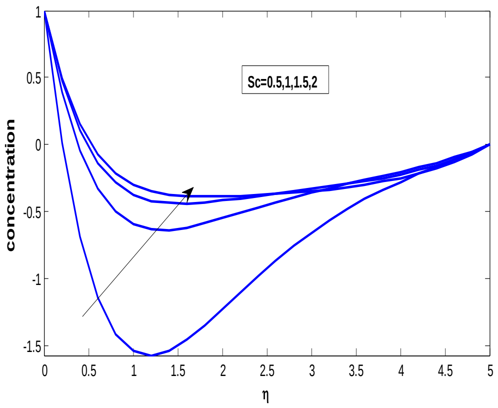

fig:15

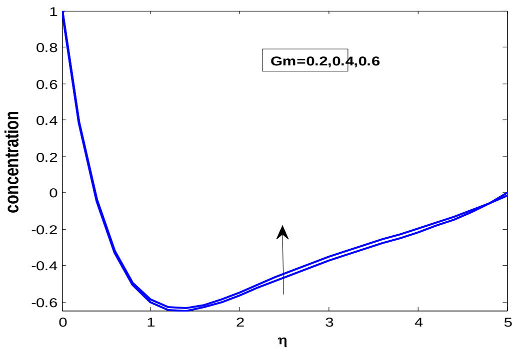

fig:16

Table1

θr	Pr	f''(o)	θ'(o)	ϕ'(0)	g'(o)	cf	Nu
2	0.5	-1.07821	-0.72476	0.068673	-2.85615	-2.69553	1.449528
2	1	-1.06283	-1.0167	0.068884	-4.09018	-2.65709	2.033405
3	0.5	-1.07567	-0.79117	0.068763	-3.15986	-2.68918	1.186751
3	1	-1.06061	-1.10092	0.06895	-4.56296	-2.65152	1.651384
4	0.5	-1.07463	-0.81874	0.0688	-3.28601	-2.68658	1.091648
4	1	-1.0597	-1.13533	0.068978	-4.75576	-2.64926	1.513778

Table2

θc	M	f''(o)	θ'(o)	ϕ'(0)	g'(o)	cf	Nu
2	0.25	-0.96254	-0.95563	0.066737	-3.81427	-2.40634	1.433448
2	0.5	-0.99853	-0.95246	0.067492	-3.80672	-2.49632	1.428692
2	0.75	-1.03349	-0.94941	0.068204	-3.79939	-2.58371	1.424108
2	1	-1.06749	-0.94646	0.068876	-3.79226	-2.66872	1.419682
3	0.25	-1.01019	-0.95245	0.067375	-3.80549	-2.02039	1.428669
3	0.5	-1.04675	-0.94926	0.068118	-3.79785	-2.09351	1.423885
3	0.75	-1.08224	-0.94618	0.068819	-3.79044	-2.16448	1.419276
3	1	-1.11674	-0.94322	0.06948	-3.78324	-2.23347	1.414827

4	0.25	-1.02841	-0.95114	0.067646	-3.80202	-1.88542	1.42671
4	0.5	-1.06514	-0.94795	0.068384	-3.79435	-1.95275	1.42192
4	0.75	-1.10078	-0.94487	0.069079	-3.78692	-2.01809	1.417306

Table3

θr	M	f"(o)	θ'(o)	ϕ'(0)	g'(o)	cf	Nu
2	0.25	-0.96455	-0.8786	0.066658	-3.42402	-2.41138	1.757191
2	0.5	-1.00069	-0.87569	0.067414	-3.4178	-2.50172	1.751386
2	0.75	-1.03578	-0.87289	0.068127	-3.41173	-2.58946	1.745787
2	1	-1.06992	-0.87019	0.0688	-3.40582	-2.6748	1.740377
3	0.25	-0.96254	-0.95563	0.066737	-3.81427	-2.40634	1.433448
3	0.5	-0.99853	-0.95246	0.067492	-3.80672	-2.49632	1.428692
3	0.75	-1.03349	-0.94941	0.068204	-3.79939	-2.58371	1.424108
3	1	-1.06749	-0.94646	0.068876	-3.79226	-2.66872	1.419682
4	0.25	-0.96172	-0.98736	0.06677	-3.9749	-2.40429	1.316486
4	0.5	-0.99765	-0.98409	0.067525	-3.96681	-2.49412	1.312116
4	0.75	-1.03255	-0.98093	0.068236	-3.95897	-2.58137	1.307905
4	1	-1.0665	-0.97788	0.068908	-3.95135	-2.66625	1.303841

Table4

Gr	Δ	C_f		N_u	
		Present result	Reference[3]	Present result	Reference[3]
0.2	0	-2.81015	-1.4221	1.39649	1.1276
	1.5	-2.60341	-2.4815	1.43040	1.1284
0.6	0	-2.57224	-1.3087	1.41435	1.132
	1.5	-2.4247	-2.2899	1.44146	1.1327
1	0	-2.34314	-1.1916	1.430406	1.4992
	1.5	-2.24997	-2.1190	1.451736	1.4997

IV. CONCLUSIONS

Unsteady micropolar fluid flow on streatching surface with heat and mass transfer in presence of magnetic field with Soret and Dufour effects has been considered taking viscosity and thermal conductivity as time dependent variable. In the light of above numerical results, an analysis of the graph is being conducted and conluded as follows :

1.The momentum boundary layer field increases as the increasing values of Δ , G_r but decreaes as as the increasing values of θ_c , M .

2.Thermal boundary layer thickness increases due to increasing value of magnetic parameter and G_r , E_c but reverse effect has been seen for unsteady paramter and P_r .

3.Thickness of concentration boundary layer increases due to the increase of physical parameter S_r, G_m , D_f and decreses for E_c.

4.Micro-rotation effect increases due to increase of unsteady and magnetic parameters while it decreses for G_r and Δ .

5. Increasing value of thermal conductivity both Skin friction coefficient and Nusselts number decreases while increases for P_r.

6. Increasing value of thermal conductivity and decreas value of magnetic parameter together increas the value of Skin friction but decreas the Nusselts number.

7. Changing both the parameter thermal conductivity (increasing) and magnetic parameter (decreasing) distintctively effect the Skin friction.

REFERENCES

[1] A.C Eringen " Theory of simple microfluids", Intl, J. Sci., pp.205-217(1964).
[2] A.C. Eringen "Theory of micropolar fluids", J. Math. Mech. 16, pp.1-18(1966)
[3] A.R.Aurangzaib, M Kasim , N. F.Mohammad and S. Sharidan " Soret and Dufour on Unsteady MHD flow of micropolar fluid in the presence of thermophoresis decomposition particle ", World Applied Science Journal, 21(5), Pp. 766-773, (2013)
[4] B.C.Sakiadis " Boundary-layer behaviour on continuous solid surface" J AIChe, Vol-7, pp. 26-28 (1961)orous plate" J.Appl.Sci.&

Engg.Vol.16, N0.2, pp-141-150(2013)
[5] D.Srinivasacharya ,Ch.Ram Reddy 'Soret and Dufour effect of mixed convection in mcropolar fluid' Int.J.Non linear Sci.Vol-II Pp:246-255.(2011)
[6] ERG Eckere , R.M.Drake.Analysis of heat and mass transfer. Mc.Graw Hill ,Newyark (1972)
[7] F.C.Lai. & F.A.Kulacki, "The effect of variable viscosity and mass transfer along a vertical surface in a saturated porous medium".Int.J.Heat and Mass transfer."Vol.33,pp.1028-1031(1990)
[8] G.Lukaszewicz 'Micropolar fluids,theory and application'(1990)
[9] G.Ahmadi "Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate." Int.J.Eng.Sci.Vol.14 .pp.639-646(1976)
[10] G.C.Hazarika and B. Phukan , "Effect of variable viscosity and thermal conductivity on MHD flow of micropolar fluid in a continuous moving flat plate." IJCA Vol-122, pp.29-37(2015).
[11] I.Baruah & G.C.Hazarika, "Effect of variable viscosity and thermal conductivity of unsteady micropolar fluid flow under mixed convection in presence of uniform magnetic field on stretching surface." IJCA vol-166 pp.17-24(2017).
[12] J.Peddison and R.P. McNitt., "Boundary layer theory for micropolar fluid" Recent Adv.Eng.Sci.,Vol.5,pp.405-426(1970).
[13] MBK Moorthy ,K Senthilvadivu ' Soret and Dufour effect on natural convection flow past a vertical surface in porus medium with variable viscosity' J.App.Math (2012)
[14] M.A. El-Aziz (1995). "Unsteady fluid and heat flow induced by a stretching sheet with mass transfer and chemical reaction". Chemical Engineering Communications, Vol. 197, pp. 1261- 1272, (2010)
[15] P.O Olanrewaju., G.T Okedayo,., J.A Gbadeyan ,"Effect of thermal radiation on magnetohydrodynamics flow of micropolar fluid towards a stagnation point on a vertical plate", Int. J. Appl. Sc. & Tech, Vol 1 , No pp. 219-230.(2011)
[16] P.Thakur & G.C.Hazarika, "Effect of variable viscosity and thermal conductivity on unsteady free convction heat and mass transfer MHD flow of micropolar fluid with constant heat flux through porous medium" Int.J.C.Appl.vol.-110,No-8 pp.22-32 (2015)
[17] T.Ariman , M. A.Turk and N.D. Sylvester 'Application of microcontinum fluid mechanics'.Int. J.Eng.Sci 12 ,pp- 273-293 (1974)
[18] T.Hayat, Z. Abbas, T. Javed, " Mixed convection flow of a micropolar fluid over a non linearly stretching sheet" Physics Letter A, 372, Issue 5 pp:637-647(2008).