Fixed Coefficients for A Subclass of Spirallike Functions

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Abstract:

The objective of this paper is to give some properties of a new subclass with negative coefficients and with fixed second coefficients

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I. INTRODUCTION AND DEFINITIONS

Let*S* denote the class of functions of the form $f(z)=z+\sum_{n=2}^{\infty}a_nz^n$ which are analytic and univalent in the open unit disc *U* = {*z*∈C:|*z*|≤1}. Also let S^* and C denote the subclasses of *S* that are respectively, starlike and convex.

Motivated by certain geometric conditions, Goodman [1,2] introduced an interesting subclass of starlike functions called uniformly starlike functions denoted by UST and an analogous subclass of convex functions called uniformly convex functions, denoted by UCV. From [5,7] we have

$$f\in UCV \Leftrightarrow \operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}\geq\left|\frac{zf''(z)}{f'(z)}\right|,\quad z\in U.$$

In [7], Ronning introduced a new class S_p of starlike functions which has more manageable properties. The classes *UCV* and S_p were further extended by Kanas and Wisniowska in [3,4] as *k*–*UCV*(α) and *k*–*ST*(α). The classes of uniformly spirallike and uniformly convex spirallike were introduced by Ravichandran et al [6]. This was further generalized in [9] as *UCSP*(α,β). In [10], Herb Silverman introduced the subclass *T* of functions of the form

$$f(z)=z-\sum_{n=2}^{\infty}a_nz^n, \tag{1.1}$$

which are analytic and univalent in the unit disc *U*. Motivated by [11], new subclasses with negative coefficients *UCSPT*(α,β) and $SP_pT(\alpha,\beta)$ were introduced and studied in [8].

A function *f*(*z*) defined by (1.1) is in *UCSPT*(α,β) if

$$\operatorname{Re}\left\{e^{-i\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)\right\}\geq\left|\frac{zf''(z)}{f'(z)}\right|+\beta, \tag{1.2}$$

$|\alpha|<\frac{\pi}{2}, 0\leq\beta<1.$

A function *f*(*z*) defined by (1.1) is in $SP_pT(\alpha,\beta)$ if

$$\operatorname{Re}\left\{e^{-i\alpha}\left(\frac{zf'(z)}{f(z)}\right)\right\}\geq\left|\frac{zf'(z)}{f(z)}-1\right|+\beta, \tag{1.3}$$

$|\alpha|<\frac{\pi}{2}, 0\leq\beta<1.$

For the classes *UCSPT*(α,β) and $SP_pT(\alpha,\beta)$ [8] proved the following lemmas.

Lemma 1.1.

A function $f(z)=z-\sum_{n=2}^{\infty}a_nz^n$ is in *UCSPT*(α,β) if and only if

$$\sum_{n=2}^{\infty}(2n-\cos\alpha-\beta)n\,a_n \le \cos\alpha-\beta. \tag{1.4}$$

Corollary 1.1.1.

Let the function $f(z)=z-\sum_{n=2}^{\infty}a_nz^n$, $a_n\ge 0$ be in the class $UCSPT(\alpha,\beta)$, $|\alpha|<\frac{\pi}{2}, 0\le\beta<1$, then

$$a_n \le \frac{\cos\alpha-\beta}{n(2n-\cos\alpha-\beta)}, \quad n\ge 2. \tag{1.5}$$

Lemma 1.2.

A function $f(z)=z-\sum_{n=2}^{\infty}a_nz^n$ is in $SP_pT(\alpha,\beta)$ if and only if

$$\sum_{n=2}^{\infty}(2n-\cos\alpha-\beta)\,a_n \le \cos\alpha-\beta. \tag{1.6}$$

Corollary 1.2.1.

Let the function $f(z)=z-\sum_{n=2}^{\infty}a_nz^n$, $a_n\ge 0$ be in the class $SP_pT(\alpha,)$, $|\alpha|<\frac{\pi}{2}, 0\le\beta<1$, then

$$a_n \le \frac{\cos\alpha-\beta}{(2n-\cos\alpha-\beta)}, \quad n\ge 2. \tag{1.7}$$

Using (1.7), the functions f(z) $\in SP_pT(\alpha,\beta)$ will satisfy

$$a_2 \le \frac{(\cos\alpha-\beta)}{(4-\cos\alpha-\beta)}. \tag{1.8}$$

Let $SP_pT_c(\alpha,\beta)$ be the subclass of functions in $SP_pT(\alpha,\beta)$ of the form

$$f(z)=z-\frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)}-\sum_{n=3}^{\infty}a_nz^n, \tag{1.9}$$

($a_n\ge 0$), where $0\le c\le 1$. When c = 1 we get

$$SP_pT_1(\alpha,\beta)=SP_pT(\alpha,\beta).$$

II. COEFFICIENT ESTIMATE

Theorem 2.1.
The function $f(z)$ defined by (1.5) belongs to $SP_pT_c(\alpha,\beta)$ if and only if

$$\sum_{n=3}^{\infty}(2n-\cos\alpha-\beta)\,a_n \le (1-c)(\cos\alpha-\beta). \tag{2.1}$$

The result is sharp.

Proof.
Taking

$$a_2=\frac{c(\cos\alpha-\beta)}{4-\cos\alpha-\beta}, 0\le c\le 1, \tag{2.2}$$

in (1.6) we get the required result. Also the result is sharp for the function

$$f(z)=z-\frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)}-\frac{(1-c)(\cos\alpha-\beta)z^n}{(2n-\cos\alpha-\beta)}, (n\ge 3). \tag{2.3}$$

□

Corollary 2.1.1.
If $f(z)$ defined by (1.9) is in the class $SP_pT_c(\alpha,\beta)$ then,

$$a_n \le \frac{(1-c)(\cos\alpha-\beta)}{(2n-\cos\alpha-\beta)},\ (n\ge 3). \tag{2.4}$$

The result is sharp for the function $f(z)$ given in (2.3).

III. CLOSURE THEOREMS

Theorem 3.1.
The class $SP_pT_c(\alpha,\beta)$ is closed under convex linear combination.

Proof.
Let $f(z)$ defined by (1.9) be in $SP_pT_c(\alpha,\beta)$. Let $g(z)$ be defined by

$$g(z) = z - \frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)} - \sum_{n=3}^{\infty} b_n z^n, \quad (b_n \ge 0). \tag{3.1}$$

If $f(z)$ and $g(z)$ belong to $SP_pT_c(\alpha,\beta)$ then it is enough to prove that the function $H(z)$ defined by

$$H(z) = \lambda f(z) + (1-\lambda) g(z), \quad (0 \le \lambda \le 1) \tag{3.2}$$

is also in $SP_pT_c(\alpha,\beta)$.

$$H(z) = z - \frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)} - \sum_{n=3}^{\infty} (\lambda a_n + (1-\lambda) b_n) z^n. \tag{3.3}$$

Using theorem (2.1) we get

$$\sum_{n=3}^{\infty} (2n-\cos\alpha-\beta)(\lambda a_n + (1-\lambda) b_n) \le (1-c)(\cos\alpha-\beta). \tag{3.4}$$

Hence $H(z)$ is in $SP_pT_c(\alpha,\beta)$. Thus $SP_pT_c(\alpha,\beta)$ is closed under convex linear combination. □

Theorem 3.2.
Let the functions

$$f_j(z) = z - \frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)} - \sum_{n=3}^{\infty} a_{n,j} z^n, \quad (a_{n,j} \ge 0), \tag{3.5}$$

be in the class $SP_pT_c(\alpha,\beta)$ for every j = 1,2,...m. Then the function $F(z)$ defined by

$$F(z) = \sum_{j=1}^{m} d_j f_j(z), \quad (d_j \ge 0), \tag{3.6}$$

is also in the same class $SP_pT_c(\alpha,\beta)$ where

$$\sum_{j=1}^{m} d_j = 1. \tag{3.7}$$

Proof.
Using (3.5) and (3.7) in (3.6) we have

$$F(z) = z - \frac{c(\cos\alpha-\beta)z^2}{4-\cos\alpha-\beta} - \sum_{n=3}^{\infty} \left[\sum_{j=1}^{m} d_j a_{n,j} \right] z^n. \tag{3.8}$$

Each $f_j(z) \in SP_pT_c(\alpha,\beta)$ for j = 1,2,...m, theorem (2.1) gives

$$\sum_{n=3}^{\infty} (2n-\cos\alpha-\beta) a_{n,j} \le (1-c)(\cos\alpha-\beta), \tag{3.9}$$

for j =1,2, ...m.Hence we get

$$\sum_{n=3}^{\infty} (2n-\cos\alpha-\beta) \left[\sum_{j=1}^{m} d_j a_{n,j} \right] = \sum_{j=1}^{m} d_j \left[\sum_{n=3}^{\infty} (2n-\cos\alpha-\beta) a_{n,j} \right] \le (1-c)(\cos\alpha-\beta).$$

This implies $F(z) \in SP_pT_c(\alpha,\beta)$, by theorem(2.1). □

Theorem 3.3.
Let

$$f_2(x) = z - \frac{c(\cos\alpha-\beta)z^2}{4-\cos\alpha-\beta} \tag{3.10}$$

and

$$f_n(x) = z - \frac{c(\cos\alpha-\beta)z^2}{4-\cos\alpha-\beta} - \frac{(1-c)(\cos\alpha-\beta)z^n}{2n-\cos\alpha-\beta}, \tag{3.11}$$

for $n = 3,4,...$. Then $f(z)$ is in $SP_pT_c(\alpha,\beta)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=2}^{\infty} \lambda_n f_n(z) \tag{3.12}$$

where $\lambda_n \geq 0$ and $\sum_{n=2}^{\infty} \lambda_n = 1$.

Proof.
Let us assume that $f(z)$ can be expressed in the form(3.12). Then we have

$$f(z) = z - \frac{c(\cos\alpha-\beta)z^2}{4-\cos\alpha-\beta} - \sum_{n=3}^{\infty} \frac{(1-c)(\cos\alpha-\beta)}{2n-\cos\alpha-\beta}\lambda_n z^n. \tag{3.13}$$

But

$$\begin{aligned}\sum_{n=3}^{\infty} \frac{(1-c)(\cos\alpha-\beta)}{2n-\cos\alpha-\beta}\lambda_n(2n-\cos\alpha-\beta) &= (1-c)(\cos\alpha-\beta)(1-\lambda_2) \\ &\leq (1-c)(\cos\alpha-\beta).\end{aligned} \tag{3.14}$$

Hence from (2.1) it follows that $f(z) \in SP_pT_c(\alpha,\beta)$.Conversely, we assume that $f(z)$ defined by (1.6) is in the class $SP_pT_c(\alpha,\beta)$. Then by using (2.4), we get

$$a_n \leq \frac{(1-c)(\cos\alpha-\beta)}{(2n-\cos\alpha-\beta)}, \quad (n = 3,4,\ldots).$$

Taking $\lambda_n = \dfrac{(2n-\cos\alpha-\beta)a_n}{(1-c)(\cos\alpha-\beta)}, \quad (n = 3,4,\ldots)$and $\lambda_2 = 1 - \sum_{n=3}^{\infty}\lambda_n$, we have (3.12).

Hence the proof is complete. □

Corollary 3.3.1.
The extreme points of the class $SP_pT_c(\alpha,\beta)$ are the functions $f_n(z),(n\geq 2)$ given by theorem (3.3) .

IV.DISTORTION THEOREMS

For finding the distortion bounds of $f(z) \in SP_pT_c(\alpha,\beta)$, we need the following lemmas.

Lemma 4.1.
Let the function $f_3(z)$ be defined by

$$f_3(z) = z - \frac{c(\cos\alpha-\beta)z^2}{4-\cos\alpha-\beta} - \frac{(1-c)(\cos\alpha-\beta)z^3}{6-\cos\alpha-\beta}. \tag{4.1}$$

Then, for $0 \leq r < 1$ and $0 \leq c \leq 1$,

$$|f_3(re^{i\theta})| \geq r - \frac{c(\cos\alpha-\beta)r^2}{4-\cos\alpha-\beta} - \frac{(1-c)(\cos\alpha-\beta)r^3}{6-\cos\alpha-\beta}, \tag{4.2}$$

with equality for $\theta = 0$. For either $0 \le c < c_0$ and $0 \le r \le r_0$ or $c_0 \le c \le 1$,

$$\left|f_3(re^{i\theta})\right| \le r + \frac{c(\cos\alpha - \beta)r^2}{4 - \cos\alpha - \beta} - \frac{(1-c)(\cos\alpha - \beta)r^3}{6 - \cos\alpha - \beta}, \tag{4.3}$$

with equality for $\theta = \pi$. Further, for $0 \le c < c_0$ and $r_0 \le r < 1$,

$$\left|f_3(re^{i\theta})\right| \le r\left[\left[1 + \frac{c^2(\cos\alpha-\beta)(6-\cos\alpha-\beta)}{4(1-c)(4-\cos\alpha-\beta)^2}\right]\right.$$
$$+ r^2(\cos\alpha-\beta)\left[\frac{2(1-c)}{6-\cos\alpha-\beta} - \frac{c^2(\cos\alpha-\beta)}{2(4-\cos\alpha-\beta)^2}\right]$$
$$\left.+\frac{r^4(1-c)(\cos\alpha-\beta)^2}{(6-\cos\alpha-\beta)}\left[\frac{(1-c)}{(6-\cos\alpha-\beta)} + \frac{c^2(\cos\alpha-\beta)}{4(4-\cos\alpha-\beta)^2}\right]\right]^{1/2},$$

with equality for $\theta = \cos^{-1}\left[\frac{c(\cos\alpha-\beta)(1-c)r^2 - c(6-\cos\alpha-\beta)}{4(1-c)(4-\cos\alpha-\beta)r}\right]$, where

$$c_0 = \frac{1}{2(\cos\alpha-\beta)}\left[(6\cos\alpha + 4\beta - 22)\right)$$
$$\left. + \sqrt{(22 - 6\cos\alpha - 4\beta)^2 + 16(\cos\alpha-\beta)(4-\cos\alpha-\beta)}\right] \tag{4.4}$$

and

$$r_0 = \frac{1}{c(1-c)(\cos\alpha-\beta)}\left[-2(1-c)(4-\cos\alpha-\beta)\right.$$
$$\left.+\sqrt{4(1-c)^2(4-\cos\alpha-\beta)^2 + c^2(1-c)(6-\cos\alpha-\beta)(\cos\alpha-\beta)}\right]. \tag{4.5}$$

Proof. The techniques used by Silverman and Silvia [11] give,

$$\frac{\partial\left|f_3(re^{i\theta})\right|^2}{\partial\theta} = \frac{2(\cos\alpha-\beta)r^3\sin\theta}{(4-\cos\alpha-\beta)}\left[c + \frac{4(1-c)(4-\cos\alpha-\beta)r\cos\theta}{(6-\cos\alpha-\beta)}\right.$$
$$\left.- \frac{c(1-c)r^2(\cos\alpha-\beta)}{(6-\cos\alpha-\beta)}\right]. \tag{4.6}$$

Also $\frac{\partial\left|f_3(re^{i\theta})\right|^2}{\partial\theta} = 0$, for $\theta_1 = 0$, $\theta_2 = \pi$ and

$$\theta_3 = \cos^{-1}\left[\frac{(\cos\alpha-\beta)c(1-c)r^2 - c(6-\cos\alpha-\beta)}{4(1-c)(4-\cos\alpha-\beta)r}\right], \tag{4.7}$$

since θ_3 is a valid root only when $-1 \le \cos\theta_3 \le 1$. Hence there is a third root if and only if $r_0 \le r < 1$ and $0 \le c \le c_0$. The result follows by comparing the extremal values $\left|f_3(re^{i\theta k})\right|$, ($k$=1,2,3) on the appropriate intervals. □

Lemma 4.2.
Let the function $f_n(z)$ be defined by (3.11) and $n \ge 4$. Then

$$\left|f_n(re^{i\theta})\right| \le \left|f_n(-r)\right|. \tag{4.8}$$

Proof.

Since $f_n(z) = z - \dfrac{c(\cos\alpha-\beta)\,z^2}{(4-\cos\alpha-\beta)} - \dfrac{(1-c)(\cos\alpha-\beta)\,z^n}{(2n-\cos\alpha-\beta)}$ and r^n is a decreasing function of n, we have

$$\left|f_n(re^{i\theta})\right| \le r + \frac{c(\cos\alpha-\beta)r^2}{(4-\cos\alpha-\beta)} + \frac{(1-c)(\cos\alpha-\beta)r^n}{(2n-\cos\alpha-\beta)}$$

$$\le r + \frac{c(\cos\alpha-\beta)r^2}{(4-\cos\alpha-\beta)} + \frac{(1-c)(\cos\alpha-\beta)r^4}{(2n-\cos\alpha-\beta)} = -f_4(-r),$$

which gives (4.8) . □

Theorem 4.3.
Let the function $f(z)$ defined by (1.6) belong to the class $SP_pT_c(\alpha,\beta)$. Then for$0 \le r<1$,

$$\left|f(re^{i\theta})\right| \ge r - \frac{c(\cos\alpha-\beta)r^2}{(4-\cos\alpha-\beta)} - \frac{(1-c)(\cos\alpha-\beta)r^3}{(6-\cos\alpha-\beta)},$$

with equality for $f_3(z)$ at $z=r$ and

$$\left|f(re^{i\theta})\right| \le \max\{\max_\theta\left|f_3(re^{i\theta})\right|, -f_4(-r)\},$$

where $\max_\theta\left|f_3(re^{i\theta})\right|$ is given by lemma 4.1.

The proof is obtained by comparing the bounds of Lemma 4.1 and Lemma 4.2.

Corollary 4.3.1.
Let the function $f(z)$ be defined by (1.1) be in the class $SP_pT(\alpha,\beta)$. Then for $|z| =r<1$, we have

$$r - \frac{(\cos\alpha-\beta)r^2}{(4-\cos\alpha-\beta)} \le \left|f(z)\right| \le r + \frac{(\cos\alpha-\beta)r^2}{(4-\cos\alpha-\beta)}.$$

The result is sharp.

Corollary 4.3.2.
Let the function $f(z)$ be defined by (1.5) be in the class $SP_pT_c(\theta,\beta)$. Then the disk $|z|<1$ is mapped onto a domain that contains the disk

$$|w| < \frac{(6-\cos\alpha-\beta)(4-\cos\alpha-\beta)-(\cos\alpha-\beta)(4+2c-\cos\alpha-\beta)}{(6-\cos\alpha-\beta)(4-\cos\alpha-\beta)}.$$

The result is sharp with the extremal function

$$f_3(z) = z - \frac{c(\cos\alpha-\beta)z^2}{(4-\cos\alpha-\beta)} - \frac{(1-c)(\cos\alpha-\beta)z^3}{(6-\cos\alpha-\beta)}.$$

Proof.
The result follows by letting $r\to 1$ in theorem 4.3.

Lemma 4.4.
Let the function $f_3(z)$ be defined by (4.1) . Then for $0\le r <1$ and $0 \le c \le 1$,

$$\left|f_3'(re^{i\theta})\right| \ge 1 - \frac{2c(\cos\alpha-\beta)r}{(4-\cos\alpha-\beta)} - \frac{3(1-c)(\cos\alpha-\beta)r^2}{(6-\cos\alpha-\beta)},$$

with equality for $\theta=0$. For either $0 \le c<c_1$ and $0\le r\le r_1$ or $c_1\le c\le 1$,

$$\left|f_3'(re^{i\theta})\right| \le 1 + \frac{2c(\cos\alpha-\beta)r}{(4-\cos\alpha-\beta)} - \frac{3(1-c)(\cos\alpha-\beta)r^2}{(6-\cos\alpha-\beta)},$$

with equality for $\theta=\pi$. Further, $0\le c<c_1$ and $r_1\le r<1$,

$$\left|f_3'(re^{i\theta})\right| \le \left\{\left[1+\frac{c^2(\cos\alpha-\beta)(6-\cos\alpha-\beta)}{3(1-c)(4-\cos\alpha-\beta)^2}\right]\right.$$
$$+(\cos\alpha-\beta)\left[\frac{6(1-c)}{(6-\cos\alpha-\beta)}+\frac{2c^2(\cos\alpha-\beta)}{(4-\cos\alpha-\beta)^2}\right]r^2$$
$$\left.+\frac{3(1-c)(\cos\alpha-\beta)^2}{6-\cos\alpha-\beta}\left[\frac{3(1-c)}{(6-\cos\alpha-\beta)}+\frac{c^2(\cos\alpha-\beta)}{(4-\cos\alpha-\beta)^2}\right]r^4\right\}^{1/2},$$

with equality for

$$\theta=\cos^{-1}\left[\frac{c(1-c)(\cos\alpha-\beta)3r^2-c(6-\cos\alpha-\beta)}{6(1-c)r(4-\cos\alpha-\beta)}\right],$$

where

$$c_1=\frac{-(30-10\cos\alpha-4\beta)+\sqrt{(30-10\cos\alpha-4\beta)^2+72(4-\cos\alpha-\beta)(\cos\alpha-\beta)}}{6(\cos\alpha-\beta)}$$

and

$$r_1=\frac{1}{3c(1-c)(\cos\alpha-\beta)}\left\{-3(1-c)(4-\cos\alpha-\beta)\right.$$
$$\left.+\sqrt{9(1-c)^2(4-\cos\alpha-\beta)^2+3c^2(1-c)(\cos\alpha-\beta)(6-\cos\alpha-\beta)}\right\}.$$

The proof of lemma(4.4) is given in the same way as lemma(4.1).

Theorem 4.5.
Let the function $f(z)$ defined by (1.6) be in the class $SP_pT_c(\alpha,\beta)$. Then for $0\le r<1$,

$$\left|f'(re^{i\theta})\right| \ge 1 - \frac{2c(\cos\alpha-\beta)r}{(4-\cos\alpha-\beta)} - \frac{3(1-c)(\cos\alpha-\beta)r^2}{(6-\cos\alpha-\beta)},$$

withequality for $f_3'(z)$ at $z=r$ and

$$\left|f'(re^{i\theta})\right| \le \max\{\max_\theta\left|f_3'(re^{i\theta})\right|, -f_4'(-r)\},$$

where $\max_\theta\left|f_3'(re^{i\theta})\right|$ is given by lemma (4.4).

When c=1 in theorem 4.5 we get the following corollary.

Corollary 4.5.1.
Let the function $f(z)$ defined by (1.1) be in the class $SP_pT_c(\alpha,\beta)$. Then for $|z| = r<1$, we have

$$1-\frac{2(\cos\alpha-\beta)r}{(4-\cos\alpha-\beta)} \le \left|f'(z)\right| \le 1+\frac{2(\cos\alpha-\beta)r}{(4-\cos\alpha-\beta)},$$

the result is sharp.

V. CONCLUSION

The class $SP_pT_c(\alpha,\beta)$can be further studied for various other properties.

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