

Nature of Mathematical Content as a Contributing Factor for Students' Mathematical Errors

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Abstract

This research study has explored, analyzed and theorized the opinions of mathematics teachers in regard to their students' mathematical errors which were evolved and developed due to the nature of the content of mathematics. There were five main areas of investigation to explore that how the typical and hierarchical nature of mathematics contributes in formation of students' mathematical errors. The study has basically followed a qualitative research design though it has processed some simple factual data. Thus, the perceptual information was supplemented and strengthened through the data obtained from the survey task. As a data collection procedure, it has adapted interview, focus group discussion (FGD) and survey. For this, the required research tools developed were interview and FGD guidelines along with a set of surveying questionnaire. The qualitative information was coded as per the main themes of investigation of five areas and then they were critically judged, analyzed and interpreted. At the end, the findings of the study were triangulated with reference to previous research studies and relevant theories.

Keywords – *Mathematical content, nature, misconception, errors, teachers' perceptions, constructivism*

I. INTRODUCTION

Mathematics serves as a foundation course mainly for studying all the science subjects so, it is considered as a queen of sciences. It is not only the language of science, but the essential nutrient for thought, logic, reasoning and progress. Further, mathematics has been proven as a founding stone of technology and bedrock of modern development. In this way, mathematics is the base for science, technology and modern development. For any nation to survive and develop, it has to improve its technology which can only be achieved through effective teaching and learning of mathematics (Sirajo, 2015). Therefore, mathematics is taken as a compulsory subject in our schools though not all the students are expected to become mathematicians but because of its applications in everyday life.

By nature, mathematics is subjected to reasoning. The reasoning in mathematics means thinking in a logical manner, formulating and testing conjecture, making sense of things, forming and justifying judgments, showing inferences and drawing conclusions. The students demonstrate mathematical behavior when they recognize and describe patterns, construct physical and conceptual models of phenomena, create symbol systems to help us represent, manipulate, and reflect on ideas, and invent procedures to solve problems (Hill, Rowan & Bass, 2005). While doing so, the students may commit various errors. Their learning achievement is inversely proportional to their errors. So, there is a vital role of remediating errors to ensure the quality of learning. It indicates that the good quality of learning depends upon teachers' constructive perception towards typical and hierarchical nature of mathematics which is quite different than that of other subjects.

The students who learned by committing errors achieve stronger conceptual foundation in the comparison of those who did not commit errors in the first attempt Matz (1980). He also argued that normal errors should not be a hindrance to the learning process. More than that, they serve as a purpose of constructive and adaptive tools for promoting conceptual understanding. Thus, in the process of correcting or searching for the origins of errors, students reach a better understanding of their own mathematical reasoning. In fact, making errors while learning mathematics is a worldwide phenomenon. They teach students to make them perfect. This indicates the students' life having experience of dealing with numerous errors that can have positive effects. The theory of critical knowledge is

also necessary to identify the boundaries of correct facts and processes. Thus, the students should also be taught about incorrect facts or processes (Oser & Spsychiger, 2005, cited in Rach, Ufer & Heinze, 2013).

Teachers' right teaching methods along with their right perceptions and knowledge about error analysis make the learning mathematics meaningful. They should entertain students' errors and orient them in student-friendly environment. Teachers need to understand that the roots of errors are reached to the nature of mathematical content. The content of mathematics is of hierarchical nature so that the earlier concepts influence the later ones. The teachers need orientation about the nature of math-content that how it contributes to form errors. The orientation of errors includes handling errors with positive attitude, proper communication among teachers and students, no risk for students to make errors, content wise identification of errors, anticipation of errors and learning from them. But problem is that we don't have such a formal system of error analysis and make a specific plan and strategies to address them. Even being an important pedagogical part of learning, neither it can be seen in any course of pedagogy nor teacher training programs. In this context, this study was envisioned to explore teachers' perceptions towards the relationship between the nature of mathematical content and development of students' errors.

II. STATEMENT OF THE PROBLEM

When we turns towards T/L process of mathematics in schools, it has been a common practice that most of the mathematics teachers let their students sit by themselves with papers, workbooks and pencils to struggle independently to understand the concepts and solve the problems. Such a learning process can be boring, lonely and frustrating. Therefore, it is not surprising that most of the students, trapped in errors, and lost their interest in learning mathematics. Even a single error may prevent the progress and accomplishment, so we cannot say what happens if a child has a variety of errors. Obviously, the more errors a child has the more likely s/he is to experience failure in examinations which the teachers should take seriously.

We see that most of the students fail to properly develop particular mathematical concepts, knowledge and skills even if they struggle hard in learning mathematics. The main issue is students' learning difficulties because of rampant errors as they are neither taken seriously by teachers nor properly treated in early stages. In this context, Li (2006) added that committing errors are neither inborn nor instantaneous; actually they are acquired in T/L process. Many students may have bitter experience with errors, which probably arise from being criteria to assess their performance level. Teachers are furious when they see students' errors and deal the errors with superficial correction. Because of which our T/L practice has hampered students' creativity, progress and learning mathematics. In this situation, how can students share their feelings, learning experiences and perceptions towards errors in order to learn and be benefitted from their errors. Actually, learning from errors depends on the perceptions of teachers.

The efficient teachers put students' current knowledge and interests at the centre of their instructional decision making. The teachers adjust their instruction to meet the learning needs of their students being informed about students' competencies including their learning ability to cope with errors. Thus, to help students to learn from their errors, teachers should be informed about the factors that cause students commit errors. As a major factor; the nature of mathematical content should be analyzed properly. So, the teachers should have knowledge of identifying and addressing students' errors in order to draw the attention of the students and guide them in the known difficulties. Further they need to have a positive perception and behavior so that students' errors could be practiced in classroom T/L approach for their systematic remediation. But, it is obvious that our school level T/L practices have not been following this mechanism. Thus, this study has aimed to uncover teachers' real perceptions towards students' errors which are formed due to nature of the content of mathematics.

III. OBJECTIVE OF THE STUDY

This study had set forth a main objective as to explore mathematics teachers' perspectives towards the contributing role of nature of mathematical content for formation and extension of students' mathematical errors.

IV. RESEARCH QUESTIONS

In order to explore the perceptions of mathematics teachers towards students' errors based on the nature of mathematical content, there were prepared five research questions as follows.

1. What kinds of mathematical content and problems are subjected to students' errors?
2. How do mathematical models of solving problems contribute in making students' errors?
3. How is the role of mathematical practice and application in creating students' errors?
4. How is the role of prior knowledge and skills in contributing students' errors?
5. What do teachers think about students' mathematical errors?

V. REVIEW OF LITERATURE

The review of literature gives overall ideas to the research work without any duplication. Actually, it provides the idea to develop each section of the research including the research design and tools, and their implementation. Thus, the researcher went through theoretical and empirical both of the types of researches in this research study.

A. Review of Theoretical Literature

In order to explore and analyze teachers' perceptions subjected to contributing factors for students' errors, the study has mainly been guided by the philosophy of constructivism. Before the advent of constructivism, errors were negatively viewed as digressions, a result of student's confusions and as unfortunate events that had to be eliminated and avoided (Gagatsis & Kyriakides, 2000). If there are any gaps in comprehending and understanding the mathematical concepts correctly, they threaten students' construction of knowledge and the coherent structure of mathematics (Li, 2006). The errors are caused by applying previously acquired and correct knowledge to mathematical situations where the knowledge is inapplicable (Gagatsis & Kyriakides, 2000). Some of the errors are germinated while using prior conceptions or misconceptions to interpret phenomena, events and situations in their construction of knowledge in the classroom (Chauraya & Mashingaidze, 2017). These constructivist explanations of errors highlight the centrality of students' mis-conceptual structures and how these structures are reused in developing further mathematical concepts and ideas.

The students drag their prior knowledge or experiences to make sense of the new situations where they need to recall and apply them accordingly. According to Brodie (2014, cited in Chauraya and Mashingaidze, 2017), when students attempt to use previously acquired knowledge in novel situations, their prior knowledge becomes inadequate for explaining new concepts and solving new problems as a result errors occur. Thus, errors are seen as reasonable and sensible for students that they think what they are doing is correct. This view of errors suggests the need for teachers to engage with students' errors in such a way that teachers enable them to identify the students' thinking or conceptions behind any observed errors. Such knowledge will enable teachers to deal with students' errors in appropriate ways that support students to reach to the correct mathematical concepts. But for this, teachers need to deal students' errors with positive attitude.

Regarding students' errors, teachers' perspectives can be categorized into three classes like; explaining the reasons for students' errors, investigating teachers' interpretations of common students' errors and understanding of students' errors (Gagatsis and Kyriakides, 2000). This study targeted to investigate teachers' perceptions in the periphery of these phenomena. Further, care has been given on the teachers who may describe the errors without explaining the students' reasoning behind the errors, that is, they might have explained students' errors by focusing procedural rather than conceptual.

The ingredients extracted from these literatures have been encoded and cited in different places as the guiding insights to understand teachers' perceptions in regard to students' varieties of mathematical errors caused by typical nature of the content of mathematics.

B. Review of Empirical Literature

In order to confine the area of study on specific issues and locate them in new area, it was essential to study empirical literatures which were provided as given below.

Gagatsis and Kyriakides (2000) carried out a research where they have asked teachers who participated in an in-service course in mathematics about their understanding of causes of students' errors and their explanations of particular errors. In the study, they found that the teachers no longer attributed errors to students' attitudes. They saw and explained errors as a result of the nature of mathematical knowledge and the rules in mathematics, for example viewing errors as a result of previous correct knowledge which is not applicable in a new situation. The teachers also attributed students' errors to the 'didactic contract' in which students don't see the practical part instead they deal mathematical problems in non-applicable or impossible situation. Here, their study helped to investigate the teachers' understanding of the nature of errors and their attribution of the sources of errors in mathematics. The study has mainly used their questionnaire with some modifications.

Chauraya and Mashingaidze (2017) studied about in-service teachers' perceptions and interpretations of students' errors in mathematics. They used a survey research design which incorporated questionnaire having two parts. In first part, they found out teachers' perceptions towards nature of errors whereas in second part, teachers were asked to describe five common algebraic errors. It helped to design a research method. Further, this study helped to differentiate mathematics errors are different from slips. They have argued that in teaching/learning situations, students make slips these are often easily identified and corrected either by the student or the teacher. Slips usually do not recur once they are corrected. Errors are mistakes that tend to recur. Teachers need to have this

understanding of errors if they are to engage productively with errors in their teaching for the benefit of students’ understanding of mathematical concepts. Students’ errors were regarded as evidence of learner thinking on which teachers could draw to help learners understand mathematical concepts. It helped to view errors as connected to learning mathematics and examine how mathematics teachers view and explain students’ errors.

Upadhyay (2001) has carried out a research on the effectiveness of constructivism in students’ mathematics achievements. His study has given the philosophical, psychological and anthropological bases of constructivism which supported to capture teachers’ perceptions towards students’ construction and misconstruction of mathematical knowledge and skills, and understand the T/L situation of Nepalese mathematics classrooms as well.

Cathy (1987) found that the students’ academic performance in mathematics was weak and they were showing low interest and confident in learning mathematics. The social interaction in the class during the teaching and learning of mathematics in the class was low as well. Moreover, most of the children often made some errors that caused them to lose marks in the mathematics tests. But, they showed a high interest in learning when they were treated on the basis of their errors. It gave researcher confidence to be confined in this area of research.

Hudson and Miller’s (2006) argued that students’ lack of knowledge could be a major reason for making error so that they cannot solve certain problems consistently. They found that there were three types of errors: procedural (not following correct steps), factual (not recalling facts or not mastered basic facts), and conceptual (unknown to specific concepts). Procedural and factual errors (also known as ‘slips’) are generally not due to inherent misunderstandings. They found that the slips are due to memory deficits, impulsivity, or visual-motor integration problems and are easier to identify than conceptual errors. They argued that the conceptual errors (or ‘bugs’) which are more serious may look like procedural errors, but they occur because the student does not fully understand a specific mathematics concept. This study helped researcher to identify the type of students’ errors.

C. Conceptual Framework of the Study

On the basis of reviewed literatures (theoretical and empirical), the ideas developed thereafter helped to conceptualize a road-map (for exploration of mathematics teachers’ perceptions towards the nature of math-content which is responsible for evolving and developing students’ mathematical errors) has been sketched below.

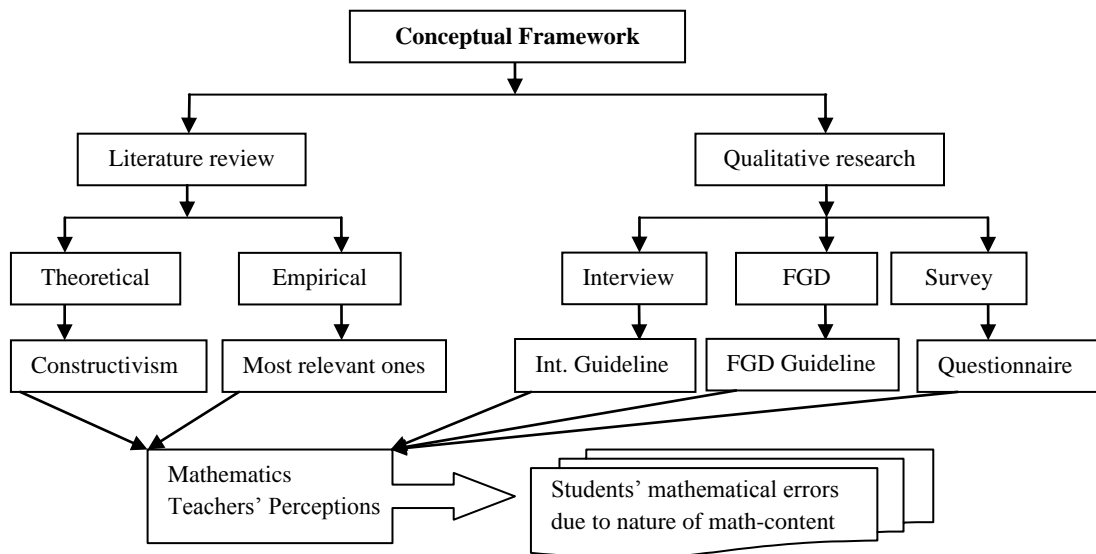


Diagram 1: Conceptual framework of the study

VI. RESEARCH METHODOLOGY

As per the nature of the study, the research method followed was mainly a qualitative though there were some quantitative data collected through survey questionnaire. The quantitative data were processed and dealt through the simple calculation and diagrammatical presentation. The researcher collected and used all the primary data. The sources of data were opinions and responses of mathematics teachers. The main purpose of the study was to explore teachers’ perceptions towards students’ mathematical errors regarding role of nature of mathematical content. The

qualitative data were collected through interviews and focus group discussion. In addition, the study conducted a survey among the mathematics teachers of Kathmandu Valley.

A survey- questionnaire was adapted from the study carried out by Gagatsis and Kyriakides (2000) with required modifications. They had studied teachers' understanding of students' errors by focusing some mathematical concepts and problems. But, this study focused on teachers' perceptions towards students' mathematical errors with reference to role of the content of mathematics. For this, an interview guideline, FGD guideline and survey questionnaire were developed. The role of math-content and its nature for contributing students' error consisted five themes. The statements of the questionnaire for surveying purpose were prepared by following the Likert scale design. The research tools were piloted among the four teachers of other two schools. They were also consulted with senior teachers, experts and researchers before finalizing them.

Then, the researcher took interview of ten mathematics teachers one-by-one who were from four public schools. Other nine mathematics teachers of two public schools (each has four and five teachers) were engaged in FGD separately. In order to see teachers' opinions in broader field one hundred sixteen sets of questionnaires were administered in forty public schools of Kathmandu Valley. Out of which one hundred three were duly filled up and received back. In order to make a task of converting teachers' responses into percentage easier, those three sets were removed randomly and made a total respondents' number one hundred.

The results obtained from the qualitative information were verified with the help of the surveyed data. Further the information was triangulated with the findings of the previous studies and related theories. In this way, trustworthiness of the findings of the study has been established with the help of interviews, FGD, surveyed questionnaire, pre-existed findings and theoretical closure.

VII. ANALYSIS AND INTERPRETATION

The data on teachers' perceptions towards students' mathematical errors formed due to nature of math-content were collected by interviewing ten mathematics teachers, conducting two FGDs and administering survey questionnaire among other mathematics teachers. The qualitative information collected through interviewing the teachers were verified, refined and strengthened by conducting focus group discussions among other mathematics teachers. Further, the data were broadly scrutinized by collecting quantitative data through survey- questionnaire administered among larger group of mathematics teachers. The data thus obtained were organized and presented in five thematic areas for their systematic analysis process. In this study, the critical judgment process was adapted to analyze and interpret all the qualitative information under the notion of constructivism. The data were minutely triangulated in order to authenticate, strengthen and generalize the findings of this study. The study has also calculated and simply analyzed quantitative data statistically and presented them diagrammatically. However, the qualitative information of five themes of the study have been analyzed and interpreted, turn by turn, as follows.

A. *The mathematical content and problems*

In this regard, the mathematics teachers argued that the school level mathematical concepts should be applied in daily life circle, instead, students are just taught through lecture method and they are made to memorize not only the formulae rather they also rote problem solving steps. One of the teachers further said:

The students also want to learn mathematics through listening and reading like other subjects instead of practicing its problems. Further, he said, the students read mathematics more at the time of examination. So, there is no way other than falling in confusion and making errors as they have to memorize a big content at a time. It produces mismatch concepts with numerous confusions which are the basis for errors.

Some teachers claimed that the content of mathematics demands some imagination, estimation and mind mapping. It is perfect when they get opportunities to be indulged in practical works. Otherwise, while doing so, students commit errors. As argued by Egodawatte (2011), the causes of errors related to nature of mathematics found in this study were as students' wrong intuitive assumptions, pragmatic reasoning habits, and immature ways of transitioning learning process from one concept to another like arithmetical numerals to algebraic symbols.

They also claimed that the steps of solving mathematical problems are different than they are calculated by using calculator. There is a hierarchical nature of mathematics so the students who have no clear concepts from early grades they make more errors. A group of the teachers said:

There are eight major units of mathematics which teachers teach separately as if they are completely different whereas some of them should have taught by linking their similar concepts. If it were done so, they would have effective learning which helped them to avoid errors.

Other teachers also shared their experience that the students make more errors in abstract part of mathematics like algebraic content (surds and indices, equations, polynomials, similar types of formulae etc.). It means, the students are with lack of fundamental meanings and skills to deal with those abstract concepts of algebra, so they are encountered with errors.

B. Mathematical models for solving problems

The teaching activities are based on definitions, axioms and postulates rather than following any particular model for solving mathematics problems. Further, there are no concrete and visible models of teaching/learning in mathematics. So, it depends upon teachers' iterated language until and unless they get fed up. A teacher argued:

It took me two days to teach concept of limit in grade IX. It was difficult for me too as there was no particular example to be drawn from real life situation. Similarly, teacher teach all the content of three dimensions by drawing figures in two dimensional board and copies which creates more confusion and students make many errors.

It showed that there are neither concrete problem solving models nor teaching/learning materials used by the teachers. According to Campbell (2009), students wanted to have a mechanical rule for finding the answers rather than making the learning creative, meaningful and useful. So, in the lack of meaningful learning, the students fell in complicated misconceptions.

Actually, it needs to have a provision of some marks for practical works as in other subjects like computer, science etc. and a mathematics laboratory in each school as the teachers suggested.

C. The role of mathematical practices and applications

The students forget soon if they have no practical knowledge of mathematics. In this regard, one of the experienced teachers claimed:

If mathematics is not taken in use or practice regularly, it will be new in nine days (nau dinma nau), it will be forgotten in twenty days (bish dinma birsane) and it will scare in thirty days (tish dinma tarsine).

Another teacher added that it is easy to teach students about land slide as they can see it but it is difficult to teach about discount in mathematics as they cannot see it like land slide. They also shared their experience that as long as they teach students by making teaching and learning strategies both compatible, the learning becomes effective. Otherwise, all efforts contribute to generate errors in either way as claimed by Campbell (2009).

D. Use of prior knowledge and skills in new situation

The teachers agreed that students could not recall well the previously learnt mathematical concepts, as and when needed, to apply in new situation in the lack of having permanent learning. In order to have a permanent learning, it needs to have a practical knowledge and skills. They said that students have a rote learning which does not work longer. One of the teachers argued:

Learning mathematics for the students has been like watching video games by sitting where they just look at white board as a screen as if teacher is an artist who solves problems fantastically. In the meantime, students have no role and they need not to do anything over there. So, once the class is over, they get up from the seats and then there is no more mathematics for them.

In the same vein, another teacher added that learning mathematics is like learning typing skill, if there is no regular practice then it will be forgotten. Yet another teacher added that if cycling is learnt theoretically, it cannot be handled properly. All they mean that if there is no practice, it becomes difficult to recall and use previously learnt mathematical concepts in new situation which could be learnt even in earlier grades as argued by Matz (1980).

E. Teachers' perspectives towards errors (as a part of learning)

A teacher shared her experience, "I allow students to do make mistakes because if it is not a case like this, they won't start doing mathematics". She further added her experience:

I used a method of Union and Interaction while teaching HCF and LCM. Students made many mistakes in the beginning but once they learnt it they felt so easy and I think they would remember for a long time. Thus, we should use alternative methods by considering errors are part of learning which are obvious to be appeared but removed if they are tackled with their causes.

It means until and unless the teachers consider errors are part of learning, take them as a normal phenomenon of learning and deal them positively they cannot be treated and minimized properly as claimed by Matlin (2005). One of the teachers took it so positively and claimed that the mathematics teachers also make errors.

So, errors are not big things and learning does not take place without trying and making some errors. Thus, teachers should not be afraid of students' errors. In spite, they should develop a positive attitude towards students' errors.

F. Analytical Presentation of Factual Data

In order to extend and take the research into a broader domain of investigation the survey research was also carried out. It helped to strengthen and authenticate the qualitative information. For, as mentioned above, the statements in the questionnaire were related with five themes of mathematical content and nature which contribute to generate students' errors. In order to take the responses of the teachers, there were five levels of choices. They were as 'agree (A)' and 'strongly agree (SA)' for representing their agreement in the statements whereas 'disagree (DA)' and 'strongly disagree (SDA)' for representing disagreement upon the given statements. Further, there was a provision of 'undecided (UD)' if they are not that much sure about their opinions. But, finally these five options were categorized into three choices as per their responses which were squeezed in agreed and very few in disagreed and undecided. There were one hundred three questionnaires which were duly filled up and received back in hands. The three questionnaires were randomly removed in order to express easily the frequencies of each statement into percentage as given below in the table. However, the collective responses of the teachers in each statement have been presented categorically in below given diagram which is followed by their analysis and interpretation.

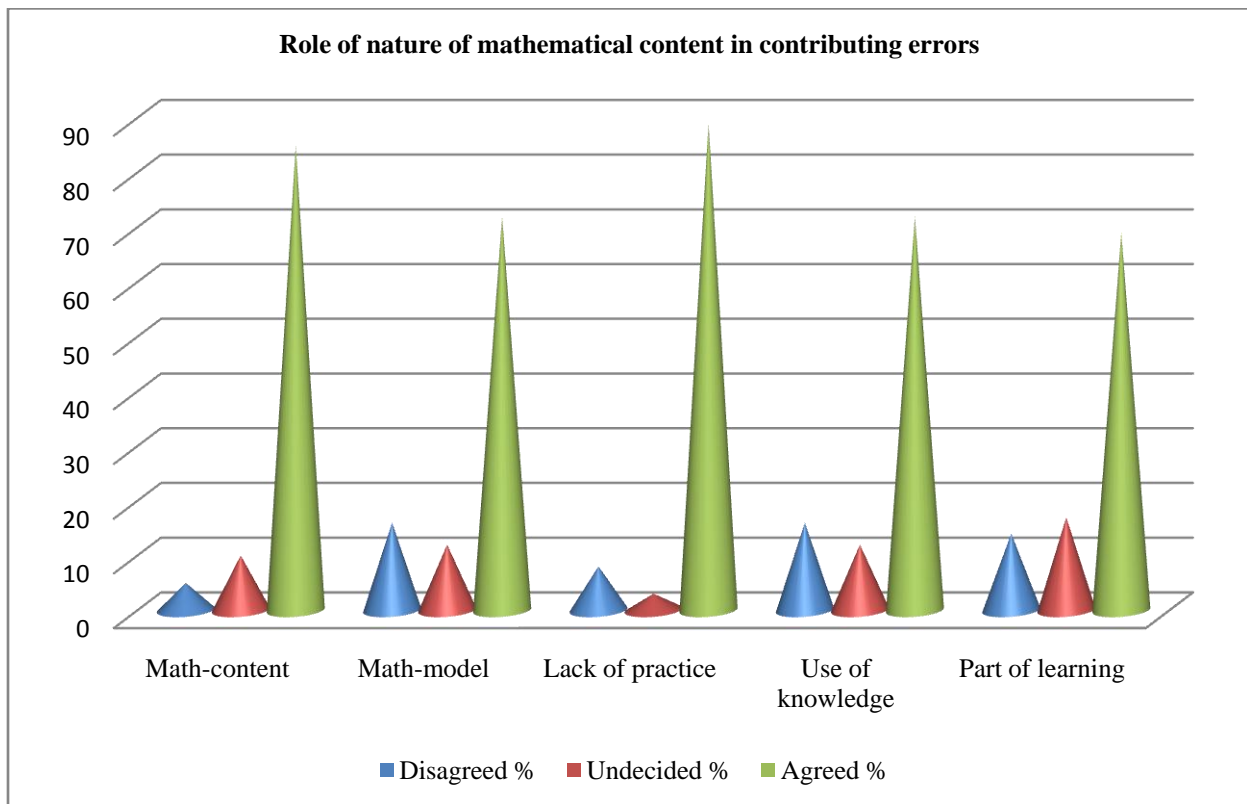


Diagram 2: Role of nature of math-content in contributing errors

The diagram showed the teachers' responses have highly been converged in the favor of all the given statements. The responses in disagreement and undecided have been found to be very less in the comparison of agreement. The agreed responses have highly been ranging from 69 to 89 percentages. Among them, the statement related with the factor of less practicing and not using the mathematics in daily life causes high percentage (89%) of errors that students commit. It could be thought that this kind of understanding about nature of mathematical content is useful and progressive. It means the errors are rampant, persistent and unavoidable in teaching/learning situation as claimed by Matlin (2005). Further, the teachers who have such an understanding that the nature and content of the mathematics can play a vital role to form errors they will not neglect errors instead they will engage students to learn even from errors.

VIII. FINDINGS OF THE STUDY

The teachers were found to be with no unique way of perceiving and interpreting students' mathematical errors though they agreed that errors are formed not only because of students' role. There is a role of nature of mathematical content in evolving and developing students' errors. Regarding the nature of mathematical content, the teachers came in consensus that the related major factors were as confused mathematical models for solving problems, lack of practice and use of mathematics in daily life circle, difficult to use previous knowledge in new situation, and the errors were considered as a part of learning mathematics so that they could not be avoided completely. Further, mathematics is a rule dominant subject. In this situation, the errors could be the results of confusing concepts and rules overlapped or contradicted to right rules. Teachers realized that they taught to memorize rules or steps of solving problems without having clear concepts of their meanings. Such a process leads students to forget or mix up the processes and commit errors.

As teachers shared, the students have taken mathematics as a foreign subject and it would be used in area of science and technology of well developed countries. The students do not see any application of other parts of mathematics except arithmetic as they were unable to connect it with their real life situation. These kinds of psychological learning, thoughts and behaviors were also found responsible to detach the students from the attraction of learning mathematics and further laid foundation to breed errors.

There was no system of diagnosing students' errors and treating them. The teachers also agreed that errors could be useful sources of inquiring students' learning process logically and consistently. It indicated that errors are part and parcel of mathematical content and learning process. In this way, erring is not a matter of blaming students. However, there were few teachers who thought that they knew where students make errors, and errors are avoidable too. Further, they were likely to blame students for their errors. These kinds of teachers either correct students' errors superficially or show them correct solution of problems without engaging them in errors. In this situation, students cannot have a meaningful learning.

IX. CONCLUSION

Learning mathematics by erring is a natural phenomenon but overlook of errors has got continuity since a long time and still there is no error analysis and treatment system. Actually, neither such a system and plan is taught in any level of course nor it has been a content of teacher training program. In this context, this research has unveiled the perceptions of teachers towards students' errors though the teachers had no idea and skills for their extraction and remediation. Even there is a gap between how students commit errors and teachers understand them. Until and unless they get break through the gap, the teachers cannot be engaged productively to address them. Thus, teachers should learn about origin, nature, causes and adverse impact of errors. Teachers should be aware on how the nature of mathematical content contributes in cultivating students' errors. If so, it will help teachers to be more effective and professional instead of blaming students for their errors.

The teachers' beliefs in mathematics as absolute knowledge needed to be changed to mathematical knowledge as fallible and human activity so that teachers could give importance to every reasoning of the students. The conceptual learning demands construction of own knowledge in their own way being based on own experience, context, and culture. Teachers should not be anxious of students' errors. Even the students' wrong answers can guide to reach to the origin of errors that they may be the best tools for crafting their learning experiences. Thus, for teaching towards understanding of mathematical concepts, content, nature and removing errors, it is first necessary to understand students' prior knowledge, examine it, and then provide opportunities to learn mathematics in fearless environment where students can take a risk of erring and learning meaningfully.

X. IMPLICATIONS OF THE STUDY

The study has drawn many prominent implications which can be used by students and teachers in daily T/L practices of mathematics. This study has provided an opportunity for students to have a better understanding towards specific problems caused by special nature of mathematical content. It would make easy to address students' erroneous problems so that their study would be efficient.

This study provides a window for teacher to observe and determine that where a student lacks basic conceptual understanding. So by identifying and locating contributing factors of students' errors, the teacher can provide required instructions targeted to their area of need. For this, there is a crucial role of teachers so that their real status of perceptions towards students' errors should be known for their further professionalism development.

Moreover, the findings of this study could be useful for subject experts, trainers, book writers, educationists, curriculum and policy makers. Thus, it has drawn valuable results from the finding so as to bring a positive change in teaching/learning mathematics.

XI. DELIMITATIONS OF THE STUDY

Due to the constraints of time, cost, resources, and specific purpose of the study, the research was confined among the public schools and mathematics teachers of Kathmandu valley. However, the in-depth interviews and focus group discussions have been carried out in addition to conducted a surveying through questionnaires, to make it a landmark in the area of learning mathematics successfully and draw some important conclusions along with the educational implications based on the findings.

The study was concentrated on investigation of teachers' perception towards students' errors caused by the nature of math-content. On the way, the researcher was interested in a number of issues that were surfaced in this study; and these issues have further broadened the scope of research work to be carried out in the future. For example, this study has not included classroom observation for the reflected behaviors of teachers and the opinions of students. Also, the study has not collected the data from researchers, educationists, and policy makers. Thus, the study is delimited to have further researches by incorporating these issues.

It is also delimited to conduct other researches being concentrated on any other micro-level practices like individual case study type so that it may diagnose the errors intensively and provide feedbacks for the treatment of individual problems.

The study is delimited to conduct further researches of this kind in diverse geographical locations and different types of schools (private and community). Similar types of researches will be a goodness of fit if they are conducted in multi-lingual and multi-cultural classroom setting as well.

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