Optimal Prediction of Queue Lengths in the Stationary Multiserver Queueing System

Sadhna Golash[#1], G.C. Sharma[*2]
Department of Mathematics, IET, Khandari, Agra (India), 2820025

Abstract

This paper present a stationary multiserver queueing system with first in first out discipline with poisson arrival and general service system we make the queue lengths predictions and conditional expectation predictors for the M/G/C queue. Specially the optimal mean square predictors for several variables have been derived for this system.

Keywords - *Stochastic processes, transition probabilities, stationary multiserver queue.*

I. INTRODUCTION

This paper concern the optimal mean-square prediction of future queue lengths based on measured queue length data in the stationary multiserver queue.

Several authors have studied optimal prediction of queue lengths in the different models Stanford (1983) found optimal prediction of times and queue lengths in the GI/M/1 queue.

These authors also presented optimal prediction of queue length and delays in GI/M/m multiserver queues. Pag Vrek et al. (1988) studied optimal prediction of times and queue lengths -in the M/G/1 queue. Alouf, Nain and Towsley found inferring network characteristics via moment based estimators. In INFOCOM (2001). Aus, Wiper and Lillo. Bayesian (2004) estimation for the M/G/l queue using a phase type approximation. Journal of statistical planning and inference. Aus, Lillo, Wiper and Bayesian (2007) control of the number of servers in a G1/M/C queueing system. In (2008) They predicted the transient behavior and busy period in short and long tailed GI/G/1 queueing systems. Acharya, Rodriquez, Sanchez and Villarreal (2013) analysed maximum likelihood estimates in an MM/c queue with heterogeneous servers, Anusha, Sharma, Vanajakshi, Subramanian and Rilett (2016) described modelbased approach for queue and delay estimation at. signalized intersections with erroneous automated data.

Multiserver predictors have application in telephony, multiprocessing computers and industrial job shop systems containing service centres with parallel servers or channels. These algorithms have been used to study optimal mean square predictors for queue lengths. Prediction and estimation are practical powerful tools in many areas of stochastic processes, and hence one expects similar value in their application to queues.

We have organized this paper as follows:-

In Section 1 we consider necessary definitions and preliminaries. In section 2 a recursive from for queue length predictors and in section 3 we derive efficient computation for the queue length predictions. Section 4 contains our conclusions and references.

II. NOTATION AND GENERAL RESULTS

In a mulltiserver M/G/C queue with Poisson arrivals at rate λ. Service time distribution function $F_y\left(t\right)$, traffic intensity $\rho = \lambda / c\mu < 1$, the n^{th} customer is denoted by B_n and leaves behind X_n customers in the customers in the queue when he departs.

The expectation and variance of random variable R will be denoted by R or $N_R = E\{R\}$ and $\sigma_R^2 = Var\left\{R\right\}$

We define the usual transition probabilities,

$$P_{ij} = \Pr\left\{x_{n+1} = j \mid x_n = i\right\} = \begin{bmatrix} k_j & i = 0 \\ k_j - i + 1 & i \le j \le c \\ 0 & else\ where \end{bmatrix}$$

Where

$$K = \int_0^\infty e^{-\lambda t}\left(\lambda t\right)^1 / 1! \mid dF_y\left(t\right) \qquad 1 \ge 0$$

Moreover, we define)

$$\pi_i = \Pr\left\{x_n = i\right\}, i \ge 0$$

the; stationary distribution for the Chain $\{X_n\}$ n = 0,1,2,........

In this case Chapman Kolmogorov equation becomes

$$P_{ij}^n = \sum_{l=i-1}^{\infty} P_{ij} P^{n-1} ij \qquad ...(1)$$

For the n-step transition Probabilities

$$P_{ij}^n = \Pr\left\{X_{K+n} = j \mid X_k = i\right\}$$

Let Q be distributed as the number of arrivals that occur during an arbitrary service time.

Then

$$\Pr\left\{Q = 1\right\} = K_1 \qquad 1 \ge 0$$

and

$$Q = \lambda / c\mu, \sigma_Q^2 = \lambda / c\mu + \lambda^2 \sigma_y^2 \qquad ...(2)$$

III.QUEUE -LENGTH- PREDICTIONS

The queue length - prediction is $E = \{X_n \mid x_0 = \mathrm{i}\}$ which is well known to minimize the mean square prediction error. Its conditional variance and over all mean squared error are given below :

$$Var\left\{X_n \mid x_0 = i\right\} = E\left\{X_n^2 \mid x_n = i\right\} - E\left\{X_n \mid x_0 = i\right\}^2 \qquad ...(3)$$

and

$$MSE\left(X_n\right) = E\left[\left\{X_n - \left\{X_n \mid x_0\right\}^2\right]\right. = E\left\{Var\left(X_n \mid x_0\right)\right\}$$

$$= \sum_{i=0}^{\infty} \pi_i\, Var\left\{X_n / x_0 = i\right\} \qquad ...(4)$$

[(Pepul is (1986) and Sage and Melsa 1971)]

In equation (1) on applying the Chapman -Kolmogrov equation we find that

$$E\left\{X_n \mid x_0 = i\right\} = \sum_{j=0}^{\infty} j\, P_{ij}^n = \sum_{1=\max(0,i-1)}^{\infty} P_{ij} E\left\{X_{n-1} \mid x_0 = 1\right\} \qquad ...(5)$$

And

$$E\left\{X_n^2 \mid x_0 = i\right\} = \sum_{j-0}^{\infty} j^2 P_{ij}^n = \sum_{1=\max(0,i-1)}^{\infty} P_{i1} E\left\{X_{n-1}^2 \mid x_0 = 1\right\} \qquad ...(6)$$

Using equation (3) and (4) we can obtain the conditional variance $Var\left\{X_n \mid x_0 = i\right\}$ and $MSE\left(X_n\right)$

Since $P_{0j} = P_{ij}, \quad \forall j$ and by equation (5) and (6)

$$E\left\{X_n^k \mid x_0 = 0\right\} = E\left\{X_n^k \mid x_0 = 1\right\} \forall k = 1, 2$$

$$Var\left\{X_n \mid x_0 = 0\right\} = Var\left\{X_n \mid x_0 = 1\right\}$$

IV. EFFICIENT COMPUTATION OF THE QUEUE -LENGTH PREDICTIONS

We now derive a computationally convenient form of equation (4), (5) and (6), in which the infinite sums are replaced by finite sums.

First we consider the case $i \ge n$, The expected queue size left behind by *Bn* will merely be $(i-n)$, plus the expected number that arrive during the *n* service times. Since the numbers of arrivals during the service times are independent.

$$E\{X_n \mid x_0 = i\} = i - n + \frac{n\lambda}{c\mu}, i \ge n \qquad ...(7)$$

And

$$Var\{X_n \mid x_0 = i\} = n\,\sigma_Q^2 = n\left(\frac{\lambda}{c\mu} + \lambda^2\sigma_y^2\right) i \ge n \qquad ...(8)$$

Now, we take the case $1 \ge n-1$, we get

$$E\{X_{n-1} \mid x_0 = 1\} = (1-n+1) + (n-1)\frac{\lambda}{c\mu}, \quad 0 < i < n$$

Hence

$$E\{X_n \mid x_0 = i\} = \sum_{1=i-1}^{n-2} P_{i1}\, E\{X_{n-1} \mid x_0 = 1\} + \sum_{1=n-1}^{\infty} P_{i1}\left[1-(n-1)\left(1-\frac{\lambda}{c\mu}\right)\right] \qquad ...(9)$$

and

$$E\{X_n \mid x_0 = i\} = (i-n) + \frac{n\lambda}{c\mu} + \sum_{1=i-1}^{n-2} P_{i1}$$
$$\left[E\{X_{n-1} \mid x_0 = 1\} - 1 \mid + (n-1)\left(1-\frac{\lambda}{c\mu}\right)\right] 0 \le i < n \qquad ...(10)$$

We have replaced the infinite summation of equation (5) by the finite sum of equations (7) and (10), By the same technique we replace (6) for the case ($i < n$) by

$$E\{X_n^2 \mid x_0 = i\} = \sum_{1=i-1}^{n-2} P_{i1}\, E\{X_{n-1}^2 \mid x_0 = 1\} - \left\{(1-n+1)(n-1)\left(\frac{\lambda}{c\mu}\right)\right\}^2$$
$$+n\left(\frac{\lambda}{c\mu} + \lambda^2\sigma_y^2\right) + \left(1-n-\frac{n\lambda}{c\mu}\right)^2, 0 < i < n \qquad ...(11)$$

Equation (10) and (11) are also valid for $i \ge n$,

The mean - squared error (4) can also be expressed by a finite sum -namely,

$$MSE(X_n) = \sum_{i=0}^{n-1} \pi_i\, Var\{X_n \mid x_0 = i\} + \sum_{i=n}^{\infty} \pi_i\, n\left(\frac{\lambda}{c\mu} + \lambda^2\sigma_y^2\right)$$
$$= \sum_{i=0}^{n-1} \pi_i\, Var\{X_n \mid x_0 = i\} + n\left(\frac{\lambda}{c\mu} + \lambda^2\sigma_y^2\right)\left(1 - \sum_{i=0}^{n-1} \pi_i\right)$$

V. NUMERICAL EXAMPLE

A call lasts 162 sec on the average, whereas the standard deviation *a* of the holding time is 169, *i.e.* σ = 169, in order to visualize the distribution of calls, a histogram as; shown in figure 1.0 has been created. The shape of this fig. is of an exponential distribution mean and standard deviation of the exponential distribution are both equal to $1/\lambda$, it can be seen, that measured data, exhibit a similar value for sample mean and standard deviation. As a grouping of data with interval length 15 sec. has been introduced, the average holding time will be scaled as well, *i.e.* $1/\lambda$ = 162/15 = 10.8.

Plotting the formula for the exponential probability density function (PDF)

VI. CONCLUSIONS

In this investigation all predictions are generated from queue lengths which are convenient to measure and the Predictors are not restricted to being Linear. Computable Predictors for queue lengths based on queue lengths at departure instants have been derived.

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