A Fuzzy Dynamical Model Developed by the Concept of Vedic Mathematics

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Abstract

Vedic Mathematics is the name given to the ancient system of Indian Mathematics which was rediscovered from the Vedas. Kosko Bart (1988) described the notion of fuzzy cognitive maps with the concept of Vedic Mathematics.

The paper is therefore addressed to the introduction of a fuzzy dynamical model by the help of fuzzy relational maps as an extension of fuzzy cognitive maps.

Keywords: *Vedic Mathematics, Fuzzy Cognitive maps, Fuzzy relational maps, Dynamical model.*

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I. INTRODUCTION

Vedic Mathematics was discovered from the Vedas between 1911 and 1918 by Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja of Govardhana Maṭha, Puri, India (1884-1960). According to his research, all of Mathematics is based on sixteen sutras or word formulae. For example vertically and crosswise is one of such formulae. These formulae describe the way the mind naturally works.

The most striking feature of the Vedic system is its co-herence. Instead of a hotch – potch of unrelated technique, the whole system is beautifully inter related and unified. In general multiplication method for example is easily reversed to allow one line division and simple squaring method can be reversed to give one line square root. This unifying quality is very satisfying. It makes mathematics easy and enjoyable.

In the Vedic system, difficult problem can often be solved immediately by Vedic method. The simplicity of Vedic Mathematics means that calculation can be carried out mentally, though the method can also be written down. Pupils can invent their own method. They are not limited to one correct method.

The sixteen sutras of Vedic method as discovered by Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja along with their corollaries have their importance [1].

The first relevant sutra 'Ekadhikena Purvena' which rendered into English says 'By one more than the previous one'. In this way all the sixteen sutras have their word wise meaning Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja in his famous book Vedic Mathematics has mentioned a point in chapter forty 'There are also various subjects of a miscellaneous character which are of great practical interest. Vedic sutras may be very useful to them. Dynamics is one of them.

It is perhaps for this reason that adopting the co-herence of a hotch – potch of unrelated technique Kosko Bart [4] in his paper introduced fuzzy cognitive maps as a directed graph with concepts like events so as to obtain hidden pattern of a dynamical system.

In this paper, we have introduced fuzzy relational maps as an extension of fuzzy cognitive maps and constructed a fuzzy dynamical model.

II. FUZZY SETS AND FUZZY GRAPH

A fuzzy set Ã on a universal set X of discourse is a set of ordered pairs

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \in X \}$$

Where, $\mu_{\tilde{A}} : x \rightarrow [0,1]$ is membership function and $\mu_{\tilde{A}}(x)$ is designated as the degree of membership of x.

Let X be a crisp set of nodes, the extreme point of any graph, then a fuzzy graph is defined by

$$\tilde{G}(x_i,y_j) =\{ ((x_i , y_j), \mu_{\tilde{G}}(x_i , y_j)) : (x_i , y_j)\in XxX\}$$

which are obtained by fuzzy binary relation $\tilde{R}$ (x). The elements of XxX with non zero membership graph of $\tilde{R}$ (x) are replaced by the lines connecting the nodes.

Such lines are labelled with the values of the membership grade $\mu_{\tilde{R}}(x, y)$ $\forall$ x, y$\in$ X.

A path of a fuzzy graph $\tilde{G}$ (x_i ,y_j) is a sequence { x_1, x_2, ……………..,x_n } of distinct nodes such that

$\mu_{\tilde{G}}(x_i, x_{i+1})>0$ whose minimum value is called strength. Total number of nodes is called length and $\mu_{\tilde{G}}(x_i, x_{i+1})$ is called edge.

A path is called a cycle if $x_0 = x_n \forall\ n\geq 3$. Degree of node x_i is the number of arc weights by means of which it is supported [5].

III. FUZZY COGNITIVE MAPS

A fuzzy cognitive map [4] is a conceptual directed graph with events as nodes and causalities as edge weights. It has a major role to play mainly when the data concerned is an unsupervised one.

As for example the increase of graduates these days is disproportionate with the need of their jobs. This has resulted in thousands of unemployed and under employed graduates.

Suppose an expert spells out five major concepts relating to the unemployed graduates

A_1 = Frustration
A_2 = Unemployment
A_3 = Increase of educated criminals
A_4 = Under employment
A_5 = Taking up drugs
Let A_1, A_2, ………… A_5 be nodes and causalitiesbe edges one cannot actually get data, But can use experts opinion for this unsupervised data to obtain idea about real situation.

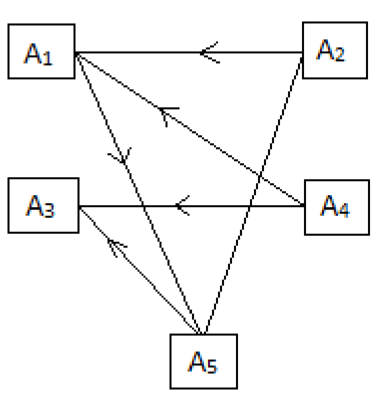

Fig.-1

If increase or decrease in one concept leads to the increase or decrease in another we give the value 1. If there is no relation between two concepts, value 0 is given and if increase or decrease of one concept leads to the decrease or increase of another, the value -1 is given.

Let the edges weight belongs to the set {-1, 0, 1}, Then supposing $A_1, A_2, \ldots\ldots A_n$ as nodes and $e_{ij} \in \{-1, 0, 1\}$ as weights of directed edge (A_i, A_j), let $E= [e_{ij}]$ be the square matrix with diagonals zero elements known as connective matrix.

If $V = \{ v_1, v_2, \ldots\ldots v_n\}$ where $v_i \in \{0, 1\}$, Then V is called instantaneous state vector which denotes the on – off position of the node.

i.e., $v_i = \left.\begin{matrix} 0 & if & v_i & is & off \\ 1 & if & v_i & is & on \end{matrix}\right\}$ $i = 1, 2, \ldots\ldots n.$

Let $(A_1, A_2), (A_2, A_3), \ldots\ldots$be the edges when $i \neq j$ ($1 \leq i \leq n$; $1 \leq j \leq n$). Then the edges form a directed cycle.

A fuzzy cognitive map with cycle is called a feedback. A fuzzy cognitive map with cycle is called a dynamical system.

Let $(A_1, A_2), (A_2, A_3), \ldots\ldots$ be a cycle when A_i is switched on and casualty flows through the edges of a cycle and causes A_i again, we say that the dynamical system goes round and round. The equilibrium state for this dynamical system is known as hidden pattern.

IV. FUZZY RELATIONAL MAPS (FRM)

Here we divide the very casual association into two disjoint units, for example the relation between a teacher and a students, a doctor and a patient and so on.

To define a fuzzy relational maps, we need a domain space D and a range space R such that $D \cap R = \phi$ in the sense of concepts.

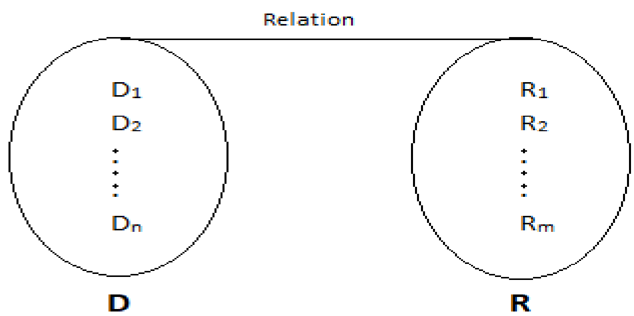

Fig.-2

We assume that no intermediate relation exists within the domain elements and range space elements.

Let the elements of domain are from real vectors space of dimension n and that of range space are real vectors from the vector space of dimension m ($n \neq m$).

We denote by R the set of nodes $R_1, R_2, \ldots\ldots R_m$ of the range space,

Where $R = \{ (x_1, x_2, \ldots\ldots X_m) / x_i = 0 \text{ or } 1 \}$

If $x_i = 1$, the node R_i is on and if $x_i = 0$, the node R_i is off.

Similarly D denotes the nodes $D_1, D_2, \ldots\ldots D_n$

where D = { (x_1, x_2, ……. X_n) / x_j = 0 or 1, j = 1, 2, ………. n }. If x_j = 1, node D_j is on and if x_j = 0, it is off.

Thus we have the following definitions.

Definition (4.1)

A fuzzy relational map (FRM) is a directed graph from D to R with concepts like policies as nodes and causalities as edge which represents causal relationship between spaces D and R.

The directed edge (D_j, D_i) denotes the causality of D_j on R_i called relations. Each edge is weighted with a number $e_{ji} \in$ {-1, 0, 1}. If increase and decrease in D_j implies the increase and decrease in R_i.

Then, e_{ji} = 1. If D_j does not have any effect on R_i, e_{ji} = 0.

We shall not discuss the cases when increase and decrease in D_j implies the decrease and increase in R_i.

If the nodes of fuzzy relational maps are fuzzy sets, we shall call them fuzzy nodes and the matrix E = [e_{li}] will be called the relational matrix of fuzzy relational maps.

If A = (a_1, a_2, ………… a_n), $a_j \in \{0, \pm 1\}$

B= (b_1, b_2, ……….. b_m), $b_i \in \{0, \pm 1\}$

Then, A and B are called the instantaneous state vectors of domain and range space respectively

$$\therefore\ a_j = \left.\begin{matrix} 0 & \textit{means} & a_j & \textit{is} & \textit{off} \\ 1 & \textit{means} & a_j & \textit{is} & \textit{on} \end{matrix}\right\} ,j = 1, 2, \ldots\ldots\ldots\ldots n.$$

$$\text{Similarly } b_i = \left.\begin{matrix} 0 & \textit{means} & b_i & \textit{is} & \textit{off} \\ 1 & \textit{means} & b_i & \textit{is} & \textit{on} \end{matrix}\right\} ,\ i = 1, 2, \ldots\ldots\ldots\ldots n.$$

If the edges form a directed cycle, fuzzy relational maps is said to be a cycle with feedback otherwise acycle. When there is a feedback in fuzzy relational maps, it is called a dynamical system.

In $D_jR_i(1 \le j \le n\ ;\ 1 \le i \le m\ )$when D_j and R_i is switched on and casually flows through edges of cycle and again causes D_j or R_i , the dynamical system goes round and round. The equilibrium state of this system is called hidden pattern.

V. FUZZY DYNAMICAL MODEL

Fuzzy dynamical model is constructed when we have at hand the opinion of several experts. It functions more like fuzzy relational maps and in the operations, max – min principle is used.

Let us consider n experts who gave their opinions about the problem using p nodes along the columns and m nodes along the rows.

We define a new fuzzy system $\tilde{F}$ = [a_{ij}] to be $m \times p$ matrix

$$\begin{bmatrix} a_{11} & a_{12} & \ldots\ldots\ldots & a_{1p} \\ a_{21} & a_{22} & \ldots\ldots\ldots & a_{2p} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} & \ldots\ldots\ldots & a_{mp} \end{bmatrix}$$

giving equal importance to the views of experts where $a_{ij} \in [0, 1], 1 \leq i \leq m ; 1 \leq j \leq p$.

Let all the experts choose to work with the same p sets of nodes along the columns and m sets of nodes along the rows.

Let $P_1, P_2, \ldots\ldots P_p$ be nodes related to columns and $C_1, C_2, \ldots\ldots C_m$ be nodes related to rows. Then a_{ij} indicates as to which degree C_i influences P_j which is a membership degree in [0, 1]

$\therefore\ a_{ij} \in [0, 1], 1 \leq i \leq m ; 1 \leq j \leq p$ by any K th expert.

Now $\tilde{F}_k = [\, a_{ij}(k) \,]$ is a fuzzy $m \times p$ matrix defined as the new fuzzy vector matrix.

Taking the views of n experts and denoting $\tilde{F}_1, \tilde{F}_2, \ldots\ldots \tilde{F}_n$ as n numberof $m \times p$ matrices.

$$\text{Let } \tilde{F} = \frac{\tilde{F}_1 + \tilde{F}_2 + \ldots\ldots\ldots. + F_n}{n}$$

$$= \frac{[\, a_{ij}(1) \,] + [\, a_{ij}(2) \,] + \text{-----------} + [\, a_{ij}(n) \,]}{n}$$

$$= [a_{ij}], 1 \leq i \leq m ; 1 \leq j \leq p$$

$$\Rightarrow a_{11} = \frac{[\, a_{11}(1) \,] + [\, a_{11}(2) \,] + \text{-----------} + [\, a_{11}(n)}{n}$$

$$\Rightarrow a_{12} = \frac{[\, a_{12}(1) \,] + [\, a_{12}(2) \,] + \text{-----------} + [\, a_{12}(n)}{n}$$

and so on

$$\text{Thus } a_{ij} = \frac{[\, a_{ij}(1) \,] + [\, a_{ij}(2) \,] + \text{-----------} + [\, a_{ij}(n)}{n}$$

The matrix $\tilde{F} = [\, a_{ij} \,]$ is defined as the new fuzzy dynamical model.

VI. CONCLUSION

We have used cognitive maps theory Kosko Bart in this paper and have extended his theory to fuzzy relational maps into two disjoint units and have developed a new fuzzy dynamical model.

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