Relation between Zeroes and Coefficients of a Polynomial

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Abstract

Understanding the relationship between zeroes and coefficients of a given polynomial of any order. Establishing the relationship between zeroes and coefficients of a given polynomial.

Keywords - *Zeroes of a Polynomial*

I. INTRODUCTION

Polynomial:

A function $p(x)$ of the form $p(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where $a_0, a_1, a_2, \ldots\ldots a_n$ are real numbers and n is a non-negative integer is called a 'polynomial in x over reals'.
The highest exponent of the variable involved in a polynomial is called its Degree.
A polynomial of first degree is called a linear polynomial. Ex: $ax + b$, $a \neq 0$.
A polynomial of second degree is called a Quadratic polynomial. Ex: $ax^2 + bx + c$, $a \neq 0$.
A polynomial of third degree is called a Cubic polynomial. Ex: $ax^3 + bx^2 + cx + d$, $a \neq 0$.
The value obtained by substituting $x = \alpha$ in a polynomial is called the value of the polynomial at $x = \alpha$.
The value obtained by substituting $x = \alpha$ in a polynomial is zero then $x = \alpha$ is called the zero of the polynomial.
Every linear polynomial has at the most one zero.
Every quadratic polynomial can have at the most two zeroes.
Similarly a cubic polynomial has three zeroes, a biquadratic (4^{th} degree) polynomial has 4 zeroes, a 5^{th} degree polynomial has 5 zeroes and so on.

II. ZEROES OF A POLYNOMIAL

Zero of a linear polynomial $ax + b$ is $x = \frac{-b}{a}$.

Zeroes of a quadratic polynomial $ax^2 + bx + c$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Let the two zeroes of a quadratic polynomial are α and β. Now

Sum of the zeroes $= \alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-b}{a} = \frac{-\,coefficient\ of\ x}{coefficient\ of\ x^2}$.

Product of zeroes $= \alpha \times \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{c}{a} = \frac{constant\ term}{coefficient\ of\ x^2}$.

The general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$, $a \neq 0$. It has at most three zeroes. Let the three zeroes be α, β and γ, then

Sum of the zeroes $= \alpha + \beta + \gamma = \frac{-\,coefficient\ of\ x^2}{coefficient\ of\ x^3}$

Sum of the product of zeroes taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$.

Product of zeroes $= \alpha\beta\gamma = \frac{-\,constant\ term}{coefficient\ of\ x^3}$.

The general form of a biquadratic polynomial is $ax^4 + bx^3 + cx^2 + dx + c$, $a \neq 0$. It has at most four zeroes. Let the four zeroes are α, β, γ and δ then

Sum of the zeroes $= \alpha + \beta + \gamma + \delta = \frac{-\,coefficient\ of\ x^3}{coefficient\ of\ x^4}$

Sum of the product of zeroes taken three at a time* $= \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{coefficient\ of\ x^2}{coefficient\ of\ x^4}$.

(*This can be done in $4\,c_3 = \frac{4\times3\times2}{3\times2\times1} = 4$ ways)

Sum of the product of zeroes taken two at a time* $= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = -\frac{coefficient\ of\ x}{coefficient\ of\ x^4}$.

(*This can be done in 4 $c_2 = \frac{4\times3}{2\times1}$ = 6 ways)

Product of zeroes = $\alpha\beta\gamma\delta$ $= \frac{constant\ term}{coefficient\ of\ x^4}$.

The general form of a fifth degree polynomial is $ax^5 + bx^4 + cx^3 + dx^2 + cx + e$, $a \neq 0$. It has at most five zeroes. Let the five zeroes are $\alpha, \beta, \gamma, \delta$ **and** τ then

Sum of the zeroes = $\alpha + \beta + \gamma + \delta + \tau = -\frac{coefficient\ of\ x^4}{coefficient\ of\ x^5}$

Sum of the product of zeroes taken four at a time* $= \alpha\beta\gamma\delta + \alpha\beta\gamma\tau + \alpha\beta\delta\tau + \alpha\gamma\delta\tau + \beta\gamma\delta\tau = \frac{coefficient\ of\ x^3}{coefficient\ of\ x^5}$.

(*This can be done in 5 $c_4 = \frac{5\times4\times3\times2}{4\times3\times2\times1}$= 5 ways)

Sum of the product of zeroes taken three at a time* =

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\beta\tau + \alpha\gamma\delta + \alpha\gamma\tau + \alpha\delta\tau + \beta\gamma\delta + \beta\gamma\tau + \beta\delta\tau + \gamma\delta\tau = -\frac{coefficient\ of\ x^2}{coefficient\ of\ x^5}.$$

(*This can be done in 5 c_3 =10 ways)

Sum of the product of zeroes taken two at a time* =

$$\alpha\beta + \alpha\gamma + \alpha\delta + \alpha\tau + \beta\gamma + \beta\delta + \beta\tau + \gamma\delta + \gamma\tau + \delta\tau = \frac{coefficient\ of\ x}{coefficient\ of\ x^5}.$$

(*This can be done in 5 c_2 =10 ways)

Product of zeroes = $\alpha\beta\gamma\delta\tau = -\frac{constant\ term}{coefficient\ of\ x^5}$.

Similarly in this way we can find out the relationship between zeroes of a polynomial and coefficients of the polynomial of any non-negative integral degree polynomial.

Eg: Consider the polynomial $x^6 - 21x^5 + 175x^4 - 735x^3 + 1624x^2 - 1764x + 720$.

The degree of the polynomial is 6. So it has 6 zeroes. Let the six zeroes be α β γ δ τ and σ.

Its zeroes are 1, 2, 3, 4, 5 and 6.

Coefficient of x^6 (The coefficient of highest exponent of variable) = 1,

Coefficient of x^5 = –21,

Coefficient of x^4 = 175,

Coefficient of x^3 = – 735,

Coefficient of x^2 = 1624,

Coefficient of x = –1764,

Constant = 720.

Relation between zeroes and coefficients:

Sum of zeroes = 1+2+3+4+5+6 = 21

$-\frac{coefficient\ of\ x^5}{coefficient\ of\ x^6} = \frac{-(-21)}{1} = 21.$

Product of zeroes = $1\times2\times3\times4\times5\times6 = 720$

$\frac{constant\ term}{coefficient\ of\ x^6} = \frac{720}{1}$.

III. CONCLUSION

A polynomial of n^{th} degree has ‘n’ zeroes, then

sum of the zeroes = $-\frac{coefficient\ of\ (n-1)th\ degree\ term}{coefficient\ of\ n\ th\ degree\ term}$

Sum of the product of zeroes taken (n-1) at a time = $\frac{coefficient\ of\ (n-2)\ th\ degree\ term}{coefficient\ of\ n\ th\ degree\ term}$

Sum of the product of zeroes taken (n-2) at a time = $-\frac{coefficient\ of\ (n-3)rd\ degree\ term}{coefficient\ of\ n\ th\ degree\ term}$

Sum of the product of zeroes taken (n-3) at a time = $\frac{coefficient\ of\ (n-4)th\ degree\ term}{coefficient\ of\ n\ th\ degree\ term}$ and so on.

REFERENCES

[1] A text book of Mathematics for class 10, NCERT.