Truncated Life Test Sampling Plan Under Odd-Weibull Distribution

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Abstract

Skip lot acceptance sampling plan is proposed for the truncated life test based on product quality following odd Weibull distribution. For the proposed plan the minimum sample size necessary to ensure the specified median life are obtained at the given consumer's confidence level. The operating characteristic values are analyzed with various ratios of the true median lifetime to the specified lifetime of the product. The minimum ratio of true population median life to the specified median life is also obtained at the specified producer's risk. Selection and application of sampling plan is illustrated with a numerical example.

Keywords - *Skip-lot acceptance sampling plan, consumer's confidence level, producer's risk, operating characteristic function, binomial model.*

I. INTRODUCTION

Life test sampling plan is a technique, which consist of decision making based on sampling inspection of batch of product by experiments for examining the continuous utility of the products for the specified function. Acceptance sampling plans in statistical quality control are concerned with accepting or rejecting a submitted lot on the basis of the quality of the products inspected in a sample taken from the lot. If the quality of the product is the life time of the product then acceptance sampling plan becomes a life test plan. Products or items have variations in their lifetimes even though they are produced by the same producer, same machine and under the identical manufacturing condition. Producer and Consumer are under risky condition due to this variation. Increasing the sample size may minimize both risks to certain level but this will obviously increase the cost. To reduce these risks and cost an efficient acceptance sampling scheme with truncated life test is proposed.

Several studies have been done for designing single acceptance sampling plans based on truncated life tests under various statistical distributions. With the introduction of modern quality management system such as ISO 9000, the manufacturing processes produce units of homogenous quality and provide lots of superior quality. Under this situation it is feasible and desirable to use a skip lot procedure, where by every lot of product need not be sample inspected, and inspection of certain lots may be skipped. Under this scenario Dodge and Perry (1973) introduced skip-lot sampling plan to achieve sampling economy. Goode and Kao (1961) developed an acceptance sampling plan using Weibull distribution as a life time distribution. Gomathi and Muthulakshmi (2014) proposed Truncated life test sampling plan under Log-logistic Model. Gomathi and Muthulakshmi (2014) developed Life test skip-lot sampling plan based on Generalized Log-logistic Model. Malathi and Muthulakshmi (2015) developed Acceptance single sampling plan and double sampling plan for truncated life test under odd Weibull distribution assuring median life time. This motivated the researcher to design a skip lot sampling plan with single sampling plan as reference plan for odd Weibull distribution by assuring median life time.

This paper proposes the designing of a skip-lot sampling plan for time truncated life test based on Odd Weibull distribution. The minimum sample size necessary to ensure the specified median life time at the specified consumer's confidence level is computed using binomial model in section 3. Operating characteristic values are calculated and given in section 4. Minimum median ratio for the specified producer's risk are calculated and presented in section 5. A numerical example is provided in section 6 to illustrate the selection of life test plan.

II. ODD WEIBULL DISTRIBUTION

Cooray (2006) derived Odd Weibull distribution which is a generalization of Weibull family by considering the odds of the Weibull and inverse Weibull families. Odd Weibull family is convenient for modeling survival processes accommodating various shapes in the hazard function.

For modeling lifetime data, the Weibull distribution which has exponential distribution as sub model does not render a reasonable good fit for some real time applications where the hazard function is of bath-tub or unimodel shapes. In order to accommodate these behaviors in a single distribution Stacy (1962) proposed generalized Gamma distribution and Mudholker et al. (1995) proposed exponential Weibull family. Murthy et al. (2004) discussed a variety of bath-tub shaped lifetime distribution and their applications. Kimball (1960) applied odd Weibull distribution to represent the ages of death.

The probability density function of odd- Weibull distribution is given by

$$f(t) = \left(\frac{\rho^{\theta}}{t}\right) \left(\frac{t}{\sigma}\right)^{\rho} \exp\left(\left(\frac{t}{\sigma}\right)^{\rho}\right) \left(\exp\left(\left(\frac{t}{\sigma}\right)^{\rho} - 1\right)^{\theta-1} \left(1+\left(\exp\left(\frac{t}{\sigma}\right)^{\rho} - 1\right)^{\theta}\right)^{-2} \qquad t > 0;\ \sigma, \rho, \theta > 0 \qquad (1)$$

where t is a lifetime variable, ρ, θ are the shape parameter and σ is the scale parameter.

The cumulative distribution function of odd Weibull distribution is given by

$$F(t) = 1-\left(1+ \left(\exp\left(\frac{t}{\sigma}\right)^{\rho} - 1\right)^{\theta}\right)^{-1}, \quad t \geq 0 \qquad (2)$$

$$= 0 \qquad t < 0$$

Odd Weibull distribution tends to Weibull distribution when θ = 1 and tends to exponential distribution when θ = ρ = 1.

The median of odd Weibull distribution is given by

$$m = \sigma (\ln 2)^{1/\rho}$$

$$= \sigma b \qquad \text{by letting } b = (\ln 2)^{1/\rho} \qquad (3)$$

Expression 3 shows that the median is directly proportional to σ, when ρ is fixed.

III. DESIGN OF THE PROPOSED SKIP LOT SAMPLING PLAN

Assume that the quality of a product is represented by its median lifetime, m. The lot will be accepted if the submitted lot has a good quality, when the experimental data supports the null hypothesis, H_0: $m \geq m_0$ against the alternative hypothesis, H_1: $m<m_0$ where m_0 is a specified median lifetime. The significance level for the test is used through $1-P^*$, where P^* is the consumer's confidence level.

The designing of Skip lot sampling plan for the truncated life test consists of obtaining (i) sample size (ii) acceptance number (iii) the ratio of true median life to the specified median life m/m_0. The consumer risk, the probability of accepting a bad lot which has the true median life below the specified life m_0, is fixed as not to exceed $1-P^*$.

Skip-lot sampling plan is followed under the assumption that there is a continuous flow of lots from the production process and lots are offered for inspection one by one in the order of production and the production process is capable of producing units whose process quality level is stable.

The operating procedure of Skip Lot sampling plan for the truncated life test has the following steps

Step 1: Start with normal inspection, using reference plan. (Single sampling plan is used as reference plan).

Step 2: When i consecutive lots are accepted on normal inspection, switch to skipping inspection, and inspect only a fraction, f of the lots.

Step 3: When a lot is rejected on skipping inspection, switch to normal inspection.

Step 4: Screen each rejected lot and correct or replace all defective units found.

It is convenient to set the termination time as a multiple of the specified lifetime m_0.Let $t_0 = \delta_0 m_0$ where δ_0 is a specified multiplier. Then the proposed sampling plan is characterized by five parameters (i, f, δ_0, c, n), where i, c and n are an integers and f and δ_0 are fractions.

The probability of acceptance of the lot for Skip lot sampling plan is

$$P_a = \frac{fP + (1-f)P^i}{f + (1-f)P^i} \quad (4)$$

under the binomial model $P = \sum_{x=0}^{c} nc_x p^x q^{n-x}$

where P is the probability of acceptance of the reference single sampling plan and p is the probability of a failure observed during the experimental time which is given by

$$p = 1-(1+(\exp(\delta_0 b/(m/m_0))-1)^{\theta})^{-1} \quad (5)$$

where p at $m=m_0$ is given by

$$p = 1-(1+(\exp(\delta_0 b/1)-1)^{\theta})^{-1} \quad (6)$$

Therefore, the minimum sample size n ensuring $m \geq m_0$ at the consumer's confidence level P^* may be found as the solution to the following inequality

$$P_a \leq 1-P^* \quad (7)$$

There may be multiple solutions for the sample size n satisfying equation (7). Minimization of average sample number is incorporated to find the optimal sample size for the stated specifications.

The ASN for our Skip-lot sampling plan is given by

$$ASN = F.ASN(R) \quad (8)$$

where F is a average fraction defective and ASN(R) is the average sample number of the reference sampling plan.

Determination of minimum sample size reduces to the following optimization problem

$$\text{Minimize} \quad ASN = \frac{nf}{f + (1-f)P^i} \quad (9)$$

subject to $P_a \leq 1-P^*$, $n \geq 1$

where n is an integer. From Table 1 and 2, Minimum sample size may be obtained by varying n values which satisfying (7).Using the minimum n values, Average sample Numbers are computed from (9). Minimum sample sizes are obtained for various values of consumers confidence level P^* (= 0.75, 0.90, 0.95, 0.99) and the parameters (i, f, δ_0, c, θ) where i (= 5), f (= 0.5, 0.2), c (= 0 to 10), δ_0 (= 0.2, 0.4, 0.6, 0.8, 1).

Numerical results in Table 1 and 2 reveal that increase in confidence level increases the sample size quite rapidly when the test time is short.

Table 1: **Minimum sample size and ASN of Skip lot sampling plan assuring median life time Under Odd Weibull distribution with (θ, ρ) as (2,1) and f = 0.5**

P^*	δ_0	c										
		0	1	2	3	4	5	6	7	8	9	10
0.75	0.2	64 63.9	124 123.8	181 180.8	236 235.7	290 289.7	343 342.6	395 394.6	447 446.5	499 498.5	550 549.4	601 600.4
	0.4	15 14.9	29 28.9	42 41.9	55 54.9	67 66.9	80 79.9	92 91.9	104 103.9	116 115.9	128 127.8	140 139.8
	0.6	6 5.9	12 11.9	18 17.9	24 23.9	29 28.9	35 34.9	40 39.9	45 44.9	51 50.9	56 55.9	61 60.9
	0.8	4 3.9	7 6.9	11 10.9	14 13.9	17 16.9	20 19.9	23 22.9	26 25.9	29 28.9	33 32.9	36 35.9
	1	3 2.9	5 4.9	7 6.9	10 9.9	12 11.9	14 13.9	16 15.9	18 17.9	21 20.9	23 22.9	25 24.9
0.90	0.2	106 105.9	179 178.9	245 244.9	308 307.9	368 367.9	427 426.9	485 484.9	543 542.9	599 598.9	655 654.9	710 709.9
	0.4	24 23.9	41 40.9	56 55.9	71 70.9	85 84.9	98 97.9	112 111.9	125 124.9	138 137.9	151 150.9	164 163.9
	0.6	10 9.9	18 17.9	24 23.9	30 29.9	36 35.9	42 41.9	48 47.9	54 53.9	60 59.9	65 64.9	71 70.9
	0.8	6 5.9	10 9.9	14 14	17 16.9	21 20.9	24 23.9	28 27.9	31 30.9	34 33.9	38 37.9	41 40.9
	1	4 4	7 6.9	9 8.9	12 12	14 13.9	17 16.9	19 18.9	21 20.9	24 23.9	26 25.9	28 27.9
0.95	0.2	138 138	218 218	290 290	357 357	421 421	484 484	545 545	605 605	664 664	723 723	781 781
	0.4	31 31	50 50	66 66	82 82	97 97	111 111	125 125	139 139	153 153	167 167	180 180
	0.6	13 13	21 21	28 28	35 35	41 41	48 48	54 54	60 60	66 66	72 72	78 78
	0.8	7 7	12 12	16 16	20 20	24 24	27 27	31 31	34 34	38 38	41 41	45 45
	1	5 5	8 8	11 11	13 13	16 16	18 18	21 21	23 23	26 26	28 28	30 30
0.99	0.2	211 211	305 305	385 385	462 462	533 533	603 603	670 670	736 736	801 801	864 864	927 927
	0.4	48 48	69 69	87 87	105 105	122 122	138 138	154 154	169 169	184 184	198 198	213 213
	0.6	20 20	29 29	36 36	45 45	52 52	59 59	65 65	72 72	78 78	85 85	91 91
	0.8	11 11	16 16	20 20	25 25	29 29	33 33	37 37	41 41	45 45	48 48	52 52
	1	7 7	11 11	13 13	17 17	19 19	22 22	25 25	27 27	30 30	33 33	35 35

Table 2: Minimum sample size and ASN of Skip lot sampling plan assuring median lifetime under Odd Weibull distribution with (θ, ρ) as (2,1) and f = 0.2

P^*	δ_0	c										
		0	1	2	3	4	5	6	7	8	9	10
0.75	0.2	64 63.7	125 124.5	182 181.3	237 236.1	291 289.9	344 342.7	396 394.6	448 446.3	500 498.1	551 548.9	603 600.8
	0.4	15 14.9	29 28.9	42 41.8	55 54.8	67 66.7	80 79.9	92 91.7	104 103.6	116 115.6	128 127.5	140 139.5
	0.6	6 5.9	13 12.9	18 17.9	24 23.9	29 28.9	35 34.9	40 39.8	45 44.8	51 50.8	56 55.8	61 60.8
	0.8	4 3.9	7 6.9	11 10.9	14 13.9	17 16.9	20 19.9	23 22.9	26 25.9	30 29.9	33 32.9	36 35.9
	1	3 2.9	5 4.9	7 6.9	10 9.9	12 11.9	14 13.9	16 15.9	18 17.9	21 20.9	23 22.9	25 24.9
0.90	0.2	106 105.9	179 178.9	245 244.9	308 307.9	368 367.9	427 426.9	485 484.9	543 542.9	599 598.9	655 654.9	710 709.9
	0.4	24 23.9	41 40.9	56 55.9	71 70.9	85 84.9	98 97.9	112 111.9	125 124.9	138 137.9	151 150.9	164 163.9
	0.6	10 9.9	18 17.9	24 23.9	30 29.9	36 35.9	42 41.9	48 47.9	54 53.9	60 59.9	65 64.9	71 70.9
	0.8	6 5.9	10 9.9	14 13.9	17 16.9	21 20.9	24 23.9	28 27.9	31 30.9	34 33.9	38 37.9	41 40.9
	1	4 3.9	7 6.9	9 8.9	12 11.9	14 13.9	17 16.9	19 18.9	21 20.9	24 23.9	26 25.9	28 27.9
0.95	0.2	138 138	218 218	290 290	357 357	421 420.9	484 483.9	545 544.9	605 604.9	664 663.9	723 722.9	781 780.9
	0.4	31 31	50 50	66 65.9	82 81.9	97 96.9	111 111	125 125	139 139	153 153	167 167	180 180
	0.6	13 13	21 21	28 28	35 35	41 41	48 48	54 54	60 59.9	66 66	72 71.9	78 77.9
	0.8	7 6.9	12 12	14 13.9	20 20	24 24	27 27	31 31	34 34	38 38	41 41	45 45
	1	5 5	8 8	11 11	13 13	16 16	18 18	21 21	23 23	26 26	28 28	30 30
0.99	0.2	211 211	305 305	386 386	462 462	533 533	603 603	670 670	736 736	801 801	864 864	927 927
	0.4	48 48	69 69	88 88	105 105	122 122	138 138	154 154	169 169	184 184	198 198	213 213
	0.6	20 20	29 29	37 37	45 45	52 52	59 59	65 65	72 72	78 78	85 85	91 91
	0.8	11 11	16 16	21 21	25 25	29 29	33 33	37 37	41 41	45 45	48 48	52 52
	1	7 7	11 11	14 14	17 17	19 19	22 22	25 25	27 27	30 30	33 33	35 35

IV. OPERATING CHARACTERISTIC (OC) VALUES

The performance of the sampling plan according to the submitted quality of the product is represented by the operating characteristics values. The probability of acceptance will increase if the true life increase beyond the specified life. Obviously, a plan will be more desirable if its OC values increase more sharply to one. Therefore, we need to know the operating characteristic values for the proposed plan according to the ratio m/m_0 of the true median life to the specified life.

The OC values as a function of the ratio m/m_0, $P^*$and δ_0 are calculated and presented in Table 3 when m/m_0 = (2, 4, 6, 8, 10), δ_0 = (0.2, 0.4, 0.5, 0.6, 0.7), f = 0.5, c = 1, i = 5, P^* = (0.75, 0.90, 0.95, 0.99) for the considered life test Skip lot sampling plans under Odd Weibull distribution.

Table 3: The OC values for Skip lot sampling plan assuring median life time under Odd Weibull distribution with i=5

f=0.5, c=1							
P*	**n**	δ_0	**m/m$_0$**				
			2	4	6	8	10
0.75	124	0.2	0.9104	0.9945	0.9989	0.9996	0.9998
	29	0.4	0.9136	0.9950	0.9990	0.9996	0.9998
	12	0.6	0.9205	0.9956	0.9991	0.9997	0.9998
	7	0.8	0.9118	0.9953	0.9991	0.9997	0.9998
	5	1.0	0.8888	0.9942	0.9989	0.9996	0.9998
0.90	179	0.2	0.8151	0.9888	0.9977	0.9992	0.9997
	41	0.4	0.8271	0.9901	0.9980	0.9993	0.9997
	18	0.6	0.8177	0.9900	0.9981	0.9994	0.9997
	10	0.8	0.8134	0.9901	0.9981	0.9994	0.9997
	7	1.0	0.7727	0.9882	0.9977	0.9993	0.9997
0.95	218	0.2	0.7350	0.9835	0.9966	0.9989	0.9995
	50	0.4	0.7494	0.9853	0.9971	0.9991	0.9996
	21	0.6	0.7559	0.9864	0.9973	0.9991	0.9996
	12	0.8	0.7355	0.9857	0.9972	0.9991	0.9996
	8	1.0	0.7077	0.9843	0.9971	0.9991	0.9996
0.99	305	0.2	0.5564	0.9681	0.9936	0.9979	0.9991
	69	0.4	0.5814	0.9723	0.9945	0.9982	0.9992
	29	0.6	0.5872	0.9741	0.9950	0.9984	0.9993
	16	0.8	0.5786	0.9743	0.9951	0.9984	0.9993
	11	1.0	0.5219	0.9696	0.9943	0.9982	0.9992

From numerical values represented in Table 3, it is seen that the OC values increases to one more rapidly to move from lower value to the higher value of the ratio m/m_0. It is seen that

(i) increase in confidence level decreases the operating characteristic values for a given ratio m/m_0

(ii) increase in confidence level decreases the operating characteristic value for any given δ_0.

V. MINIMUM MEDIAN RATIOS

The producer may be interested to know what will be the minimum product quality level to be maintained in order to keep the producer's risk at the specified level. At the specified producer's risk of α the minimum ratio m/m_0 can be obtained by solving

$$P_a \geq 1-\alpha \qquad (10)$$

Corresponding to $\alpha = 0.05$, P^* (=0.75, 0.90, 0.95, 0.99), i (=5), f (=0.5, 0.2), c (=0, 1) and δ_0 = (0.2, 0.4, 0.5, 0.6, 0.7) the minimum median ratios are computed and presented in Table 4 using the equation (10). Numerical values in table reveal that

(i) increase in consumer's confidence level increases the minimum median ratios at the specified δ_0 and producer's risk

(ii) increase in c decreases the minimum median ratios at the specified consumer's confidence level, producer's risk.

Table 4: Minimum median ratio to the specified life at α=0.05, θ=2, i=5

f	**c**	P^*	δ_0				
			0.2	0.4	0.6	0.8	1
0.5	0	0.75	3.84	3.79	3.66	4.03	4.39
		0.90	4.93	4.76	4.67	4.88	5.03
		0.95	5.61	5.39	5.31	5.25	5.59
		0.99	6.93	6.68	6.53	6.52	6.57
	1	0.75	2.30	2.27	2.22	2.27	2.40
		0.90	2.75	2.69	2.71	2.71	2.84

		0.95	3.03	2.96	2.92	2.96	3.04
		0.99	3.58	3.46	3.41	3.41	3.55
0.2	0	0.75	2.89	2.86	2.78	3.06	3.35
		0.90	3.71	3.59	3.54	3.71	3.83
		0.95	4.21	4.06	4.01	3.99	4.25
		0.99	5.20	5.02	4.93	4.94	4.98
	1	0.75	1.96	1.94	1.98	1.95	2.06
		0.90	2.34	2.29	2.32	2.32	2.44
		0.95	2.58	2.52	2.49	2.54	2.61
		0.99	3.04	2.95	2.91	2.92	3.04

VI. SELECTION OF LIFETEST PLANS

Example: Assume that lifetime of a product under consideration follows an Odd Weibull distribution and life test is made using Skip lot plan with parameters i=5, f=0.5, c=5.Suppose also that the producer wants to establish that the true median life is greater than or equal to 2000 hours with the consumer's confidence level 0f 0.75.The experimenter wants to stop the experiment at 1200 hours. This leads to the experiment termination multiplier as $\delta_0 = 0.6$. Table (1) gives the minimum sample size n = 35. This sampling plan is put into operation as follows.

Start with normal inspection using the reference single sampling plan. Select 35 items, put on test for 500 hours and accept the lot if the sample contains less than or equal to 5 defectives. Otherwise reject the lot. If i=5 lots are accepted continuously on normal inspection, switch to skipping inspection for the submitted lots at the rate of f=0.5, if the lot is rejected on skipping inspection, switch to normal inspection.

Suppose that the producer is interested in knowing what quality level will lead to the producer's risk of less than or equal to 0.05. From table 4, the minimum median ratio for δ_0=0.6, P^*=0.75, c=0 and f=0.5 is 3.663. Therefore, the true median required for the lot of product should be at least 3666 hours.

VII.CONCLUSION

A Skip lot sampling plan for truncated life test is proposed in order to make a decision on the submitted lot under the assumption that the life time of the products follows Odd Weibull distribution, which is useful in system reliability analysis because its pattern of failure rate is quite versatile.

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