Total Magic Cordial Labeling of Split Graph of Cycle, Wheel and Fan Graph

Parameswari.R[*1], Rajeswari.R[2]
**Research Scholar,Sathyabama University, Chennai, Tamilnadu, India*
Professor SathyabamaUniversityChennai, Tamilnadu, India

Abstract

A graph is defined to be comprised of two finite sets namely the vertex set and the edge set. Graphs are often denoted by G and we may write G = (V, E), where V represents the vertex set (which is nonempty) and E represents the edge set (which may be empty). A graph labeling is defined as a mapping from set of graph elements to set of non negative integers. Cordial labeling was introduced as a variation of graceful and harmonious labeling. Total magic cordial labeling was introduced as a weaker version of edge magic and cordial labeling and is defined as, a graph G with V as vertex set and E as edge set is defined as a mapping from the set of vertices and edges to the integers {0,1} such that a sum of terminal vertices and edges is congruent to C (mod2) provided with the condition that the absolute value of f_0 *and* f_1 *differ at most by one, where* f_0 *is the sum of vertices and edges labeled zero and* f_1 *is the sum of vertices and edges labeled one. In this paper the total magic cordial labelling of split graph of cycle, wheel, fan and triangular book graph has been discussed.*

AMS Subject Classification: ***05C78***

Keywords: *Graph labeling, Total Magic Cordial labeling (TMC),Split graph, Cycle, Wheel, Fan.*

I. INTRODUCTION

All graphs in this paper are finite, undirected and simple. To avoid repetition, unless specified otherwise, G has Vas the set of vertices and E as the set of edges. Here C_n $F_{1,n}$ and W_n denotes Cycle, Fan and Wheel graph of order n respectively. A split graph is a graph in which the vertices can be partitioned into a clique and an independent set. Split graphs were first studied by Foldes and Hammer, and independently introduced by Tyshkevich and Chernyak. If the vertices or edges or both of the graph are assign integers subject to certain conditions then it is known as graph labeling. For extensive survey on graph labeling we refer to [3]. Enough literature is available in printed as well as electronic form on different types of graph labeling. Labeling of graph is the potential area of research and more than 2000 research papers have been published so far in past five decades.

The concept of cordial labeling was introduced by Cahit [1]. After Cahit, the meaning of cordiality in the graph labeling problems is well understood and studied [4], [5]. A binary vertex labeling $f : V(G) \rightarrow \{0, 1\}$ induces an edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ defined by $f^*(uv) = |f(u)- f(v)|$. Such labeling is called cordial if the conditions$|v_f(0) - v_f(1)| \leq 1$ and $|e_f^*(0) - e_f^*(1)| \leq 1$ are satisfied, where $v_f(i)$ and $e_f^*(i)$, where i = 0, 1 are the number of vertices and edges with label i respectively. A graph is called cordial if it admits a cordial labeling. Kotzig and Rosa introduced the concept of edge-magic total labeling in [6]. We studied the new construction of magic graphs and its properties [7] [8] [9]. Based on edge magic total labeling and cordial labeling a new labeling called total magic cordial labeling was introduced by Cahit [2]. In [11, 12] TMC for some undirected graph and special graph called duplicate graph have been investigated. Also TMC for directed graphs have been defined and proved the same for Paley digraph [10]. Some labeling with variations in cordial theme have also been introduced such as prime cordial labeling, A-cordial labeling, E-cordial labeling, H-cordial labeling, Product cordial labeling, EP cordial labeling etc. The wide range of applications arising from the area, for instance, X-ray, Crystallographic analysis, Design communication Network addressing systems, in determining optimal circuit layouts and Radio-astronomy etc. Qualitative labeling of graph elements have inspired research in diverse field of human enquiry such as conflict resolution in social psychology, electrical circuit theory and energy crisis etc. Quantitative labeling of graphs led to quite intricate fields of application such as coding theory, Synch-set codes, Missile guidance codes and convolution codes with optimal auto correlation properties.

Traditional network representations are usually global in nature. Enormous graphs are everywhere, from social and communication networks to the World Wide Web. In sensor networks, it provides fast communication with the help of radio labeling. Automatic channel allocation is also possible in wireless networks such as Cellular telephony, Wi-Fi, Security systems and many more. In social networks, it is most efficient and provides effective communication. It provides effective communication by using certificates and key graphs. An important contribution to social network analysis came from sociograms. A sociogram can be seen as a graphical representation of a network: people are represented by dots (called vertices) and their relationships by lines connecting those dots (called edges). These are some types of graph labeling that plays a role and it has other types also like ad hoc networks, sensor networks, compression networks etc… In this paper the split graph of cycle, wheel and fan are proved as total magic cordial graphs.

II. PRELIMINARIES

This section gives brief summary of definitions which are useful for the present exploration.

Definition 2.1: Let G (V, E) be an undirected graph. A labeling is called *Total Magic Cordial labeling*, if there exists a mapping f : V∪E→{0,1} such that f(u) + f(v) + f(uv) = C(mod 2) for all (u, v)∈E provided the condition $|f(0) – f(1)| \leq 1$ is hold, where $f(0) = v_f(0) + e_f(0)$ and $f(1) = v_f(1) + e_f(1)$ and $v_f(i)$, $e_f(i)$, i∈{0, 1} are, respectively, the number of vertices and edges labeled with i.

Definition 2.2: For a graph G the *Split graph* is obtained by adding to each vertex v a new vertex v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted by Spl(G).

Definition 2.3: A *cycle* graph C_n, sometimes simply known as an n-cycle, is a graph on n nodes containing a single cycle through all nodes.

Definition 2.4: The *wheel graph* W_{n+1} can be defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph.

Definition 2.5: *Fan graph* $F_{1,n}$ is defined as the graph K_1+P_n where K_1 is the core singleton graph and Pn is a path graph with n vertices.

III. MAIN RESULT

Theorem 3.1: The Split of Cycle graph $Spl(C_n)$ admits total magic cordial labeling.

Proof: Let Spl(Cn) be a split graph of cycle Cn. the vertex set be $V = \{v_i, v_i' \,/\, 1 \leq i \leq n\}$ and the edge set $E = \{v_iv_{i+1}, v_iv_{i+1}', v_i'v_{i+1} \,/ 1 \leq i \leq n-1\} \cup \{v_nv_1, v_n'v_1, v_nv_1'\}$. To maintain a constant C congruent to (mod 2), for every n, define a mapping f: V∪E →{0, 1}

Let us define f: V∪E →{0, 1} as follows.

$$f(v_i) = 0 \quad \text{for } 1 \leq i \leq n$$
$$f(v_i') = \begin{cases} 1 & \text{i odd} \\ 0 & \text{i even} \end{cases}$$
$$f(v_i v_{i+1}) = 1 \quad \text{for} \quad 1 \leq i \leq n-1$$
$$f(v_n v_1) = 1$$
$$f(v_i v_{i+1}') = \begin{cases} 1 & \text{i odd} \\ 0 & \text{i even} \end{cases}$$
$$f(v_n v_1') = 0$$
$$f(v_i' v_{i+1}) = \begin{cases} 0 & \text{i odd} \\ 1 & \text{i even} \end{cases}$$
$$f(v_n' v_1) = 1$$

Case1: When n is even

$$f_0 = \left(n + \frac{n}{2} + \frac{n}{2} + \frac{n}{2} + 1\right)$$

$$f_1 = \left(\frac{n}{2} + n - 1 + 1 + \frac{n}{2} + \frac{n}{2} + 1\right)$$

then

$$|\, f_0 - f_1 \,| = |\, \frac{5n}{2} + 1 - \frac{5n}{2} - 1 \,| = 0.$$

Case 2: When n is odd

$$f_0 = \left(n + \frac{n-1}{2} + \frac{n-1}{2} + 1 + \frac{n+1}{2} + 1\right)$$

$$f_1 = \left(\frac{n+1}{2} + 1 + \frac{n+1}{2} + \frac{n-1}{2} + n - 1\right)$$

then

$$|\, f_0 - f_1 \,| = |\, \frac{5n+3}{2} - \frac{5n+1}{2} \,| = 0.$$

It shows that the number of vertices and edges labeled zero and one differ at most by one in both cases. Hence the graph Spl(Cn) admits Total magic cordial labeling.

Theorem 3.2: The Split of Wheel graph Spl (W_n) admits total magic cordial labeling.

Proof: Let G be a wheel graph with central vertex u and $\{v_1,v_2,\ldots v_n\}$ be the degree 3 vertices. The Split of G has 2n +2 vertices and 6n edges. Here we have to discuss two cases.

Case 1: When n is even

To maintain a constant C congruent to (mod 2), let us define f: V∪E →{0, 1} as follows.

$f(u) = 0$
$f(v_i) = 0 \quad \text{for } 1 \le i \le n$
$f(u') = 1$
$f(v_i{}') = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$
$f(uv_i) = 1 \quad \text{for } 1 \le i \le n$
$f(v_i v_{i+1}) = 1 \quad \text{for } 1 \le i \le n-1$
$f(v_n v_1) = 1$
$f(uv_i{}') = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$
$f(u'v_i) = 0 \quad \text{for } 1 \le i \le n$
$f(v_i v_{i+1}{}') = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$
$f(v_n v_1') = 0$
$f(v_i{}' v_{i+1}) = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$
$f(v_n' v_1) = 1$
$f_o = \left(1 + n + \frac{n}{2} + \frac{n}{2} + n + \frac{n}{2} - 1 + 1 + \frac{n}{2}\right)$
$f_1 = \left(1 + \frac{n}{2} + n + n - 1 + 1 + \frac{n}{2} + \frac{n}{2} + \frac{n}{2} - 1 + 1\right)$
$|\, f_o - f_1 \,| = |\, (4n+1) - (4n+1) \,| = 0.$

Case2: When n is odd

To maintain a constant C≡ 1 (mod 2), let us define f: V∪E →{0, 1} as follows.

$f(u) = 0$
$f(v_i) = 1 \quad \text{for } 1 \le i \le n$
$f(u') = 0$
$f(v_i{}') = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$
$f(uv_i) = 0 \quad \text{for } 1 \le i \le n$
$f(v_i v_{i+1}) = 1 \quad \text{for } 1 \le i \le n-1$
$f(v_n v_1) = 1$
$f(uv_i{}') = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$
$f(u'v_i) = 0 \quad \text{for } 1 \le i \le n$
$f(v_i v_{i+1}{}') = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$
$f(v_n v_1') = 1$
$f(v_i{}' v_{i+1}) = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$
$f(v_n' v_1) = 1$
$f_o = \left(1 + 1 + \frac{n-1}{2} + n + \frac{n+1}{2} + n + \frac{n-1}{2} + \frac{n-1}{2}\right)$

$f_1 = \left(n + \frac{n+1}{2} + n - 1 + 1 + \frac{n-1}{2} + \frac{n-1}{2} + 1 + \frac{n-1}{2} + 1\right)$

$| f_o - f_1 | = | (4n+1) – (4n+1) | = 0.$

It shows that the number of vertices and edges labeled zero and one differ at most by one in both cases. Hence the graph $Spl(W_n)$ admits total magic cordial labeling.

Theorem 3.3: The Split of Fan graph Spl $(F_{1,n})$ admits total magic cordial labeling.

Proof: Let Spl $(F_{1,n})$ be a graph with 2n +2 vertices and 6n-3 edges. Define
f: V∪E →{0,1} as

$f(u) = 0\ ;\ f(u') = 1$

$f(v_i) = 0 \quad \text{for } 1 \le i \le n$

$f(v_i') = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$

$f(uv_i) = 1 \quad \text{for } 1 \le i \le n$

$f(v_i v_{i+1}) = 1 \quad \text{for } 1 \le i \le n$

$f(uv_i') = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$

$f(v_i v_{i+1}') = \begin{cases} 1 & i \text{ odd} \\ 0 & i \text{ even} \end{cases}$

$f(v_{i+1} v_i') = \begin{cases} 0 & i \text{ odd} \\ 1 & i \text{ even} \end{cases}$

Case1: When n is even

$f_o = \left(1 + n + \frac{n}{2} + n + \frac{n}{2} + \frac{n-2}{2} + n\right)$

$f_1 = \left(1 + \frac{n}{2} + n + n - 1 + \frac{n}{2} + \frac{n}{2} + \frac{n-2}{2}\right)$

$| f_o - f_1 | = | (4n) - (4n - 1) | = 1.$

Case2: When n is odd

$f_o = \left(1 + n + \frac{n-1}{2} + n + \frac{n+1}{2} + \frac{n-1}{2} + \frac{n-1}{2}\right)$

$f_1 = \left(1 + \frac{n+1}{2} + n + n - 1 + \frac{n-1}{2} + \frac{n-1}{2} + \frac{n-1}{2}\right)$

$| f_o - f_1 | = | (4n) – (4n-1) | = 1.$

It shows that the number of vertices and edges labeled zero and one differ at most by one in both cases. Hence the graph $Spl(F_{1,}n)$ admits Total magic cordial labeling.

IV. CONCLUSION

It is fascinating to explore Total magic cordial graphs as all graph do not admit total magic cordial labeling. Here it has been proved that split of cycle, wheel and fan graphs are Total magic cordial graphs. In future to investigate more result for other graph families.

REFERENCES

[1] I.Cahit(1987), "Cordial graphs: A weaker version of graceful and harmonious graphs",Ars Combin.23, 201–208.
[2] I.Cahit(2002),, "Some totally modular cordial graphs", Discuss. Math. Graph Theory 22 247–258.
[3] Gallian (2017), J.A, "A dynamic survey of graph labeling", The Electronic Journal of Combinatory 18, #DS6.
[4] I.Cahit(1990), "On cordial and 3-equitable labeling of graphs", Util. Math., 37189 -198.
[5] Z.Szaniszl'o (1994),"k-equitable labeling of cycles and some other graphs", Ars Combin.37 49–63.
[6] A.Kotzig, A. Rosa (1970), "Magic valuations of finite graphs", Canad. Math. Bull. 13 4,451–461.
[7] M.Baca,Y. Lin, M. Miller, R. Simanjuntak (2001), "New constructions of magic and antimagic graph labelling", Utilitas Math. 60 229–239.
[8] W.D.Wallis,E.T.Baskoro,M.Miller, Slamin(2000), "Edge-magic total labelings", Austral. J. Combin. 22 177–190.
[9] W.D. Wallis(2001), "Magic Graphs, Birkhauser Boston,.
[10] R.Parameswari,R.Rajeswari (2013), "Total Bi magic labeling and Total Magic Cordial labeling of Paley Digraphs", IEEE Conference, PRIME.
[11] R.Parameswari,R. Rajeswari (2015), "Total Magic Cordial Labeling Of (Pn, K1), Gear And Shadow Graphs", International Journal of Pure and Applied Mathematics, 101 No. 6, 993-1001.
[12] R.Parameswari (2015), "Total magic cordial labeling of square and shadow graph", Applied Mathematical Sciences, Hikari Publication, 1312- 885X,No.9,25-28.