Type of Graph Labeling

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Abstract

Any tree n vertices is connected to ge graceful if it is verities can be labeled using integers 0,1…….n-1 such that each vertex label as well as the corresponding edge label is distinct throughout the tree. Three has multiple attempts with different approaches to prove this conjecture but it remains the same. Here as well as discuss the methods used to solve this problem and types of graceful labeling.

I. INTRODUCTION

In the mathematical discipline of graph theory, a graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices or both of a graph.

A graph G= (V, E), a vertex labeling is a function of V to a set of labels. A graph with such a function defined is called a vertex – labeled graph an edge labeling is a function of E to a set of labels. In this case the graph is called edge – labeled graph

II. HARMONIOUS LABELING

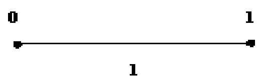
A harmonious labeling on a graph G is an injection from the vertices of G to the group of integers modulo K. where K is the number of edges of G. That induces a bijection between the edges of G and the numbers modulo K by taking the edge label for an edge (X, Y)

Theorem:-
The graph $B^2(n,n)$ is harmonious $\forall n$

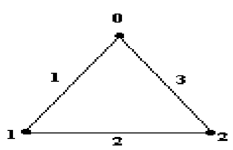
Proof:-
Consider $B^2(n,n)$ with the vertex set
$\{u, v, u_i, v_i, 1 \le i \le n\}$
Where u_i, v_i are the pendent vertices?
Let G be the graph $B^2(n,n)$ then $|\mathrm{V(G)} - 2\mathrm{n} + 2|$ and $|\mathrm{E(G)} - 4\mathrm{n} + 1|$
We define the vertex labeling
$f\colon \mathrm{V(G)} \longrightarrow \{0,1,2,3 \ldots (\mathrm{q}-1)\}$ as follows
v = 0, $u = 2n + 1$
$v_i = i, 1 \le i \le n$
$u_i = n + i, 1 \le i \le n$

Let A,B,C,D, denote edge set
A = $\{e_i = vv_i / e_i = i : 1 \le i \le n\}$
B = $\{e_i = uv_i / e_i = (2n + i + 1), 1 \le i \le n\}$
C = $\{e_i = uv_i / e_i = (n + i) : 1 \le i \le n\}$
D = $\{e_i = uu_i / e_i = (3n + i + 1), 1 \le i \le n\}$
It is clear that vertex set labeling and edge set labeling is distinct
Example:

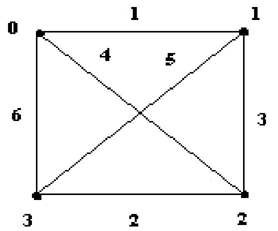
K_2



K_3



K_4

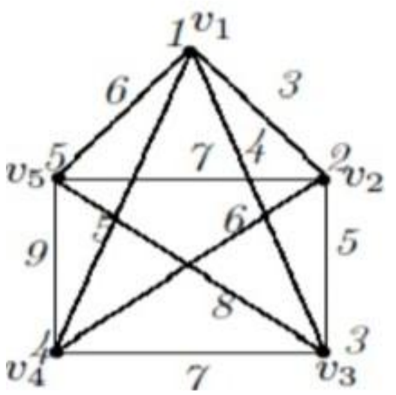


III. LUCKY LABELING

A lucky labeling of a graph G is an assignment of positive integers to the vertices of G such that if S(V) denotes the sum of the labels on the neighbours of V then S is a vertex coloring of G. the lucky number of G is the least K such that G has a lucky labeling with the integers $\{1 \ldots K\}$.

Theorem:-
Lucky number of complete graph K_n is $\eta(K_n) = 2\text{n} - 1$.
Proof:-
Let $V_{1,}V_2 \ldots . V_n$ be the vertices of K_n.
Then $|v(K_n)| = n$ and $|v(K_n)| = \binom{n}{2}$
Then the vertex and edge labeling are defined as follows:
$f(v_i) = i$, $1 \leq i \leq n$
$f * (v_i v_j) = i + j, 1 \leq i, j \leq n$
∴ Therefore, lucky number of K_n is $n(K_n) = 2n - 1$

Example:



IV. GRAPH COLORING

A graph coloring is a subclass of graph labeling. A vertex coloring assigns different labels to adjacent vertices; an edge coloring assigns different labels to adjacent edges

If we define, for any edge $e = \{u, v\} \epsilon\, E(G)$
Then value of $\varphi(e) = |\varphi(u) - \varphi(v)|$. φ is a one-to-one mapping of the set E(G) onto se $\{1,2 \ldots n\}$.

Theorem:
For each integer $n \geq 3$,

$$\mathcal{X}'_m(K_n) = \begin{cases} n+1 & \text{if } n \equiv (\text{mod } 4) \\ n & \textit{otherwise} \end{cases}$$

Proof.

Let $G = K_n$, where $V(G)$ ={$v_1, v_2, \dots, v_n$}. if n is odd, then let $c_1: E(G) \to Z_n$ be an edge coloring given by

$$c_1(e) = \begin{cases} i & \text{if } e = u_1u_2 (1 \le \mathrm{i} \le -1) \\ 0 & otherwise \end{cases}$$

Then $s(v_1) = i$ for $(1 \le \mathrm{i} \le -1)$, implying that c_1 is a modular n-edge coloring of G. It then follows by Proposition 2.1.2. that $\mathcal{X}'_m(G) = n$ if n is odd. If n is even, then we consider tow cases

Case 1, $n \equiv 0$ (mod 4). Let $n = 4p$ for some positive integer p. Define an edge coloring $c_2: E(G) \longrightarrow Z_{4p}$ by

$$c_2(e) = \begin{cases} p & \text{if } e \in \{v_iv_{i+1}: 1 \le \mathrm{i} \le -4\mathrm{p} - 2\} \cup \{u_1u_{4p-1}\} \\ i & \text{if } e = \{v_iv_{4p}: and\ 1 \le \mathrm{i} \le -4\mathrm{p} - 1\} and\ i \ne 2p \\ 0 & otherwise \end{cases}$$

Thus for $1 \le \mathrm{i} \le -4\mathrm{p}$

$$s(v_i) = \begin{cases} 2p & \text{if } i = 2p \\ 0 & \text{if } i = 4p \\ 2p + i & otherwise \end{cases}$$

in Z_{4p}. Hence c_2 is a modular $4p$ edge coloring of G. the result now follows by Proposition.

Case 2. $n \equiv 2$ (mod 4). Let $n = 4p + 2$ for some positive integer p. Define an edge coloring $c_3: E(G) \longrightarrow Z_{4p+3}$ by

$$c_3(e) = \begin{cases} i - 1 & \text{if } e = v_iv_{4p+2} and\ 2 \le \mathrm{i} \le -2\mathrm{p} - 1 \\ i + 1 & \text{if } e = v_iv_{4p+2} and\ 2\mathrm{p} + 2 \le \mathrm{i} \le -4 + -1 \\ 1 & \text{if } e \in \{v_iv_{i+1}: 1 \le \mathrm{i} \le 2\mathrm{p}\} \\ 0 & otherwise \end{cases}$$

in Z_{4p+3} and so c_3 is a modular $(4p + 3)$ – edge coloring of G. Thus, $\mathcal{X}'_m(G) \le n + 1$ if $n \equiv 2$(mod 4). On the other hand, assume, to the contrary, that there exists a modular $(4p + 2)$ edge coloring c' of G. Then by Observaiton

$$2 \sum_{e \in E(G)} c'(e) = \sum_{i=1}^{4p+2} s(v_i) = 0 + 1 + \cdots + (4p + 1) = 2p + 1$$

In Z_{4p+3}, which is impossible. Therefore, $\mathcal{X}'_m(G) \le n + 1$, which in turn implies that $\mathcal{X}'_m(G) \le n + 1$ if $n \equiv 2$(mod 4).

It is well known that if v is a vertex in a nontrivial graph G, then either

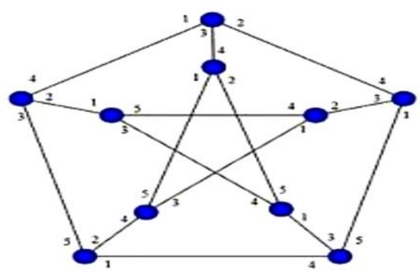
$\mathcal{X}(G - v) = \mathcal{X}(G)$ or $\mathcal{X}(G - v) = \mathcal{X}(G) - 1$

Also, if an edge e is deleted from an nonempty graph G, then

$\mathcal{X}(G - e) = \mathcal{X}(G)$ or $\mathcal{X}(G - e) = \mathcal{X}(G) - 1$

This, however, is not the case for the modular chromatic index of a graph. For example, let $G = K_n$ with $n \equiv 2$(mod 4). By theorem 2.2.2, $\mathcal{X}'_m(G) = n + 1$, while $\mathcal{X}'_m(G - v) = \mathcal{X}'_m(K_{n-1}) = n - 1$ as $n - 1 \not\equiv 2$ (mod 4) , implying that $\mathcal{X}'_m(G - v) = \mathcal{X}'_m(G) - 2$ for each $v \in V(G)$. Furthermore $\mathcal{X}'_m(G - e) = \mathcal{X}'_m(G) - 2$ for each $e \in V(G)$, as we show next. It is known that $\chi(K_n - e) = n - 1$for each integer $n \ge 3$.

Example:



V. EDGE – GRACEFUL LABELING

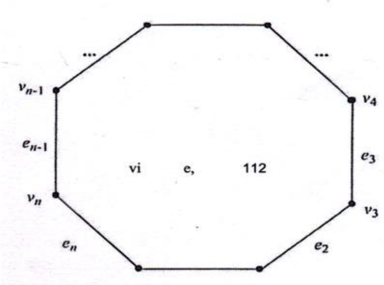
And edge – graceful labeling on a simple graph on p vertices and q edges is a labeling of the edges by distinct integers in $\{1 \ldots q\}$. such that the labeling on the vertices induced by labeling a vertex with the sum of the incident edges taken modulo p assigns all values from 0 to p-1 to the vertices.

Theorem

$C_n\,(n \geq 3)$ is strong edge – graceful for all n when n is odd.

Proof

Let $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3 \ldots, \mathrm{v}_\mathrm{n}\}$ be the vertices of C,, and $\{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3 \ldots, \mathrm{e}_\mathrm{n}\}$ be the edges of C_n which are denoted as



C_n with ordinary labeling

We first label the edges of G as ıuııuws.

$f(e_1) = i \qquad 1 \leq i \leq n$

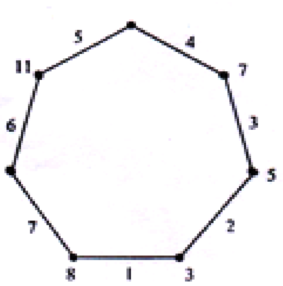
Then the induced vertex labels are:

$f^+(v_1) = n + 1$

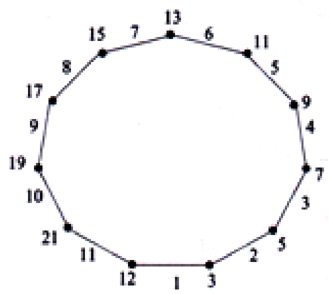
$f^+(v_i) = 2i - 12 \leq i \leq n$

Clearly, the vertex labels are all distinct. Hence, $C_n\,(n \geq 3)$ is strong edge graceful for all n when n is odd.

The SEGL of C_7 and C_{11} are illustrated



C_7 with SEG"



C_{11} with SEGL

VI. CONCLUSION

One of the important areas of graph theory in graph labeling used in many applications like coding theory, X-ray crystallography, radar, astronomy, data base management. This paper gives an overview of labeling of graphs in communication networks.

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I have declared this article my own preparation. If found any corrections i'll rectified.

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