Some Fractional Derivatives of the Multivariable Gimel-Function

Frédéric Ayant

Teacher in High School , France

ABSTRACT
In this present paper we derive a number of main formulae involving fractional derivarives of the generalized multivariable Gimel-function. We also make use of the generalized Leibnitz's theorem for fractional derivatives in order to obtain results which involve a product of two multivariable Gimel-function. These results are shown to apply to obtain many new results.

KEYWORDS : Generalized multivariable Gimel-function, multiple integral contours, .fractional derivative, generalized Leibnitz's theorem.

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1. Introduction and preliminaries.

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

We define a generalized transcendental function of several complex variables.

$$\gimel(z_1, \cdots, z_r) = \gimel^{m_2,n_2;m_3,n_3;\cdots;m_r,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array} \right|$$

$$\begin{array}{l} [(\mathrm{a}_{2j}; \alpha^{(1)}_{2j}, \alpha^{(2)}_{2j}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha^{(1)}_{2ji_2}, \alpha^{(2)}_{2ji_2}; A_{2ji_2})]_{n_2+1,p_{i_2}}, [(a_{3j}; \alpha^{(1)}_{3j}, \alpha^{(2)}_{3j}, \alpha^{(3)}_{3j}; A_{3j})]_{1,n_3}, \\ [(\mathrm{b}_{2j}; \beta^{(1)}_{2j}, \beta^{(2)}_{2j}; B_{2j})]_{1,m_2}, [\tau_{i_2}(b_{2ji_2}; \beta^{(1)}_{2ji_2}, \beta^{(2)}_{2ji_2}; B_{2ji_2})]_{m_2+1,q_{i_2}}, [(b_{3j}; \beta^{(1)}_{3j}, \beta^{(2)}_{3j}, \beta^{(3)}_{3j}; B_{3j})]_{1,m_3}, \end{array}$$

$$\begin{array}{l} [\tau_{i_3}(a_{3ji_3}; \alpha^{(1)}_{3ji_3}, \alpha^{(2)}_{3ji_3}, \alpha^{(3)}_{3ji_3}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots; [(\mathrm{a}_{rj}; \alpha^{(1)}_{rj}, \cdots, \alpha^{(r)}_{rj}; A_{rj})_{1,n_r}], \\ [\tau_{i_3}(b_{3ji_3}; \beta^{(1)}_{3ji_3}, \beta^{(2)}_{3ji_3}, \beta^{(3)}_{3ji_3}; B_{3ji_3})]_{m_3+1,q_{i_3}}; \cdots; [(\mathrm{b}_{rj}; \beta^{(1)}_{rj}, \cdots, \beta^{(r)}_{rj}; B_{rj})_{1,m_r}], \end{array}$$

$$\begin{array}{l} [\tau_{i_r}(a_{rji_r}; \alpha^{(1)}_{rji_r}, \cdots, \alpha^{(r)}_{rji_r}; A_{rji_r})_{n_r+1,p_r}] : \ [(\mathrm{c}^{(1)}_j, \gamma^{(1)}_j; C^{(1)}_j)_{1,n^{(1)}}], [\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}}, \gamma^{(1)}_{ji^{(1)}}; C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}] \\ [\tau_{i_r}(b_{rji_r}; \beta^{(1)}_{rji_r}, \cdots, \beta^{(r)}_{rji_r}; B_{rji_r})_{m_r+1,q_r}] : [(\mathrm{d}^{(1)}_j), \delta^{(1)}_j; D^{(1)}_j)_{1,m^{(1)}}], [\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}, \delta^{(1)}_{ji^{(1)}}; D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}] \end{array}$$

$$\left. \begin{array}{l} ;\cdots; [(c^{(r)}_j, \gamma^{(r)}_j; C^{(r)}_j)_{1,m^{(r)}}], [\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}}, \gamma^{(r)}_{ji^{(r)}}; C^{(r)}_{ji^{(r)}})_{m^{(r)}+1,p^{(r)}_i}] \\ ;\cdots; [(d^{(r)}_j, \delta^{(r)}_j; D^{(r)}_j)_{1,n^{(r)}}], [\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}}, \delta^{(r)}_{ji^{(r)}}; D^{(r)}_{ji^{(r)}})_{n^{(r)}+1,q^{(r)}_i}] \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \, \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{m_2} \Gamma^{B_{2j}}(b_{2j} - \sum_{k=1}^{2} \beta^{(k)}_{2j} s_k) \prod_{j=1}^{n_2} \Gamma^{A_{2j}}(1 - a_{2j} + \sum_{k=1}^{2} \alpha^{(k)}_{2j} s_k)}{\sum_{i_2=1}^{R_2} [\tau_{i_2} \prod_{j=n_2+1}^{p_{i_2}} \Gamma^{A_{2ji_2}}(a_{2ji_2} - \sum_{k=1}^{2} \alpha^{(k)}_{2ji_2} s_k) \prod_{j=m_2+1}^{q_{i_2}} \Gamma^{B_{2ji_2}}(1 - b_{2ji2} + \sum_{k=1}^{2} \beta^{(k)}_{2ji2} s_k)]}$$

$$\frac{\prod_{j=1}^{m_3}\Gamma^{B_{3j}}(b_{3j}-\sum_{k=1}^{3}\beta_{3j}^{(k)}s_k)\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=m_3+1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

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$$\frac{\prod_{j=1}^{m_r}\Gamma^{B_{rj}}(b_{rj}-\sum_{k=1}^{r}\beta_{rj}^{(k)}s_k)\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=m_r+1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)})]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $m_2,n_2,\cdots,m_r,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2\leqslant q_{i_2},0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant m_r\leqslant q_{i_r},0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=m^{(k)}+1,\cdots,p^{(k)});(k=1,\cdots,r);$

$\mathrm{D}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=n^{(k)}+1,\cdots,q^{(k)});(k=1,\cdots,r).$

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k).$

$\beta_{kj}^{(l)},B_{kj}\in\mathbb{R}^+;(j=1,\cdots,m_k);(k=2,\cdots,r);(l=1,\cdots,k).$

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k).$

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k).$

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r).$

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r).$

5) $c_j^{(k)}\in\mathbb{C};(j=1,\cdots,n_k);(k=1,\cdots,r);d_j^{(k)}\in\mathbb{C};(j=1,\cdots,m_k);(k=1,\cdots,r).$

$a_{kji_k} \in \mathbb{C}; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r).$

$b_{kji_k} \in \mathbb{C}; (j = m_k + 1, \cdots, q_{i_k}); (k = 2, \cdots, r).$

$d^{(k)}_{ji^{(k)}} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r).$

$\gamma^{(k)}_{ji^{(k)}} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r).$

The contour L_k is in the $s_k(k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha^{(k)}_{2j} s_k\right) (j = 1, \cdots, n_2), \Gamma^{A_{3j}}\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha^{(k)}_{3j} s_k\right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}}\left(1 - a_{rj} + \sum_{i=1}^{r} \alpha^{(i)}_{rj}\right) (j = 1, \cdots, n_r), \Gamma^{C^{(k)}_j}\left(1 - c^{(k)}_j + \gamma^{(k)}_j s_k\right) (j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$to the right of the contour L_k and the poles of $\Gamma^{B_{2j}}\left(b_{2j} - \sum_{k=1}^{2} \beta^{(k)}_{2j} s_k\right) (j = 1, \cdots, m_2), \Gamma^{B_{3j}}\left(b_{3j} - \sum_{k=1}^{3} \beta^{(k)}_{3j} s_k\right) (j = 1, \cdots, m_3)$ $, \cdots, \Gamma^{B_{rj}}\left(b_{rj} - \sum_{i=1}^{r} \beta^{(i)}_{rj}\right) (j = 1, \cdots, m_r), \Gamma^{D^{(k)}_j}\left(d^{(k)}_j - \delta^{(k)}_j s_k\right) (j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2} A^{(k)}_i \pi$ where

$$A^{(k)}_i = \sum_{j=1}^{m^{(k)}} D^{(k)}_j \delta^{(k)}_j + \sum_{j=1}^{n^{(k)}} C^{(k)}_j \gamma^{(k)}_j - \tau_{i^{(k)}} \left(\sum_{j=m^{(k)}+1}^{q^{(k)}_i} D^{(k)}_{ji^{(k)}} \delta^{(k)}_{ji^{(k)}} + \sum_{j=n^{(k)}+1}^{p^{(k)}_i} C^{(k)}_{ji^{(k)}} \gamma^{(k)}_{ji^{(k)}} \right) +$$

$$\sum_{j=1}^{n_2} A_{2j} \alpha^{(k)}_{2j} + \sum_{j=1}^{m_2} B_{2j} \beta^{(k)}_{2j} - \tau_{i_2} \left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha^{(k)}_{2ji_2} + \sum_{j=m_2+1}^{q_{i_2}} B_{2ji_2} \beta^{(k)}_{2ji_2} \right) + \cdots +$$

$$\sum_{j=1}^{n_r} A_{rj} \alpha^{(k)}_{rj} + \sum_{j=1}^{m_r} B_{rj} \beta^{(k)}_{rj} - \tau_{i_r} \left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha^{(k)}_{rji_r} + \sum_{j=m_r+1}^{q_{i_r}} B_{rji_r} \beta^{(k)}_{rji_r} \right) \tag{1.4}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$ where $i = 1, \cdots, r$:

$\alpha_i = \min_{\substack{1 \leqslant k \leqslant m_i \\ 1 \leqslant j \leqslant m^{(i)}}} Re \left(\sum_{h=2}^{r} \sum_{h'=1}^{h} B_{hj} \frac{b_{hj}}{\beta^{h'}_{hj}} + D^{(i)}_k \frac{d^{(i)}_k}{\delta^{(i)}_k} \right)$ and $\beta_i = \max_{\substack{1 \leqslant k \leqslant n_i \\ 1 \leqslant j \leqslant n^{(i)}}} Re \left(\sum_{h=2}^{r} \sum_{h'=1}^{h} A_{hj} \frac{a_{hj} - 1}{\alpha^{h'}_{hj}} + C^{(i)}_k \frac{c^{(i)}_k - 1}{\gamma^{(i)}_k} \right)$

Remark 1.
If $m_2 = n_2 = \cdots = m_{r-1} = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = B_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = B_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the generalized multivariable Gimel-function reduces in the generalized multivariable Aleph- function (extension of multivariable Aleph-function defined by Ayant [1]).

Remark 2.
If $m_2 = n_2 = \cdots = m_r = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in a generalized multivariable I-function (extension of multivariable I-function defined by Prathima et al. [5]).

Remark 3.

If $A_{2j} = B_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = B_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in generalized of multivariable I-function (extension of multivariable I-function defined by Prasad [6].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the generalized multivariable H-function (extension of multivariable H-function defined by Srivastava and Panda [7,8].
In your investigation, we shall use the following notations.

$$\mathbb{A} = [(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A} = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A = [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B} = [(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{m_2+1,q_{i_2}},[(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\mathrm{b}_{(r-1)j};\beta_{(r-1)j}^{(1)},\cdots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,m_{r-1}}],$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{m_{r-1}+1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B} = [(\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)};B_{rj})_{1,m_r}],[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{m_r+1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U = m_2,n_2;m_3,n_3;\cdots;m_{r-1},n_{r-1};V = m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)} \tag{1.11}$$

$$X = p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y = p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)} \tag{1.12}$$

The fractional derivative of a function $f(x)$ of complex order μ ([4], p.49 and [3] p. 181) is defined by

$${}_{\alpha}D_x^{\mu}[f(x)] = \left[\begin{array}{l} \frac{1}{\Gamma(-u)}\int_{\alpha}^{x}(x-y)^{-\mu-1}f(y)\mathrm{d}y; Re(\mu)<0,\alpha\in\mathbb{R} \\ . \\ \frac{d^m}{dx^m}\,{}_{\alpha}D_x^{\mu-m}[f(x)];0\leqslant Re(\mu)<m,m\in\mathbb{N} \end{array}\right. \tag{1.13}$$

where m is a positive integer.

The special cases of the fractional derivarive operator ${}_{\alpha}D_x^{\mu}$ when $\alpha = 0$ will be written as D_x^{μ}.

In this present paper we shall derive several fractional derivative formulae involving the generalized multivariable Gimel-function which will be represented as above. In your investigation, we shall adopt the contracted notations cited above.

2. Fractional derivative formulae.

In this section, we shall prove the following fractional derivative formulae involved the generalized multivariable Gimel-function defined in section I.

Theorem 1.

$$D_x^{\mu}\left[x^k(x+a)^{a_1}(a-x)^{-a_2}\gimel\left(z_1x^{\rho_1}(x+a)^{b_1}(a-x)^{-c_1},\cdots,z_1x^{\rho_r}(x+a)^{b_r}(a-x)^{-c_r}\right)\right]=$$

$$a^{a_1-a_2}x^{k-\mu}\sum_{l,m=0}^{\infty}\frac{(-)^m\left(\frac{x}{a}\right)^{l+m}}{l!m!}\gimel^{U;m_r+1,n_r+2:V}_{X;p_{i_r}+3,q_{i_r}+3,\tau_{i_r}:R_r:Y}$$

$$\left(\begin{array}{c|c} z_1x^{\rho_1}a^{b_1-c_1} & \mathbb{A};(-a_1;b_1,\cdots,b_r;1),(-l-m-k;\rho_1,\cdots,\rho_r;1),\mathbf{A},(1-m-a_2;c_1,\cdots,c_r;1);A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_1x^{\rho_1}a^{b_1-c_1} & \mathbb{B};(1-a_2;c_1,\cdots,c_r;1),\mathrm{B},(l-a_1;b_1,\cdots,b_r;1),(\mu-l-m-k;\rho_1,\cdots,\rho_r;1):B \end{array}\right) \tag{3.1}$$

provided

$\mu,k,a_i,b_i,\rho_i,c_i,b_i-c_i,a_1-a_2,k-\mu>0(i=1,\cdots r)$, $\left|arg\left(\frac{x}{a}\right)\right|<\pi$

$Re(-a_1),Re(1-a_2),Re(1-m-a_2),Re(\mu-l-m-k)>0$

$$Re\,(k)+\sum_{i=1}^{r}(b_i+\rho_i-c_i)\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$|arg\left(z_kx^{\rho_i}(x+a)^{b_i}(a-x)^{-c_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To prove the theorem 1, expressing the generalized multivariable Gimel-function with the help of (1.1), applying the binomial expansion and formule ([4], p.67)

$$(x+a)^{\lambda}=a^{\lambda}\sum_{m=0}^{\infty}\binom{\lambda}{a}\left(\frac{x}{a}\right)^m,\left|\frac{x}{a}\right|<1\text{ and}$$

$$D_x^{\mu}(x^{\lambda})=\frac{\Gamma(\lambda+1)x^{\lambda-\mu}}{\Gamma(\lambda-\mu+1)},Re(\lambda)>-1$$

and interpreting the resulting expression with the help of (1.1), we obtain the desired theorem 1.

Theorem 2.

$$D_x^{\mu}\left[x^k(x+a)^{a_1}(a-x)^{-a_2}\gimel\left(z_1x^{\rho_1}(x+a)^{b_1}(a-x)^{-c_1},\cdots,z_1x^{\rho_r}(x+a)^{b_r}(a-x)^{-c_r}\right)\gimel^*(\omega_1x^{\alpha_1},\cdots,\omega_rx^{\alpha_r})\right]=$$

$$a^{a_1-a_2}x^{k-\mu}\sum_{l,m,s=0}^{\infty}\frac{(-)^m\binom{\mu}{s}\left(\frac{x}{a}\right)^{l+m}}{l!m!}\gimel^{U;m_r+1,n_r+2:V}_{X;p_{i_r}+3,q_{i_r}+3,\tau_{i_r}:R_r:Y}$$

$$\left(\begin{array}{c|c} z_1 x^{\rho_1} a^{b_1-c_1} & \mathbb{A}; (-a_1; b_1, \cdots, b_r; 1), (-l-m; \rho_1, \cdots, \rho_r; 1), \mathbf{A}, (1-m-a_2; c_1, \cdots, c_r; 1); A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_1 x^{\rho_1} a^{b_1-c_1} & \mathbb{B}; (1-a_2; c_1, \cdots, c_r; 1), \mathbf{B}, (l-a_1; b_1, \cdots, b_r; 1), (-l-m+s; \rho_1, \cdots, \rho_r; 1) : B \end{array}\right)$$

$$\daleth^{U^*;0,n_r^*+1:V^*}_{X^*;p_{i_r}^*+1,q_{i_r}^*+1,\tau_{i_r}^*:R_r^*:Y^*}\left(\begin{array}{c|c} \omega_1 x^{\alpha_1} & \mathbb{A}^*; (-k; \alpha_1, \cdots, \alpha_r; 1), \mathbf{A}^*; A^* \\ \cdot & \cdot \\ \cdot & \cdot \\ \omega_1 x^{\alpha_1} & \mathbb{B}^*; \mathbf{B}^*, (-k+s+\mu-s; \alpha_1, \cdots, \alpha_r; 1) : B^* \end{array}\right) \tag{2.2}$$

Provided

$\mu, k, \alpha_i, a_i, b_i, \rho_i, c_i, b_i - c_i, a_1 - a_2, k - \mu > 0 (i = 1, \cdots r)$, $\left|arg\left(\frac{x}{a}\right)\right| < \pi$

$$Re(\mu - k - s) > 0, Re(k) + \sum_{i=1}^{r}(b_i + \rho_i - c_i) \min_{\substack{1\leqslant j\leqslant m_i \\ 1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h} B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}} + D_j^{(i)}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) + 1 > 0$$

$$Re(k) + \sum_{i=1}^{r}\alpha_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0.$$

$|arg\left(z_k x^{\rho_i}(x+a)^{b_i}(a-x)^{-c_i}\right)| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).$|arg\left(\omega_i x^{\alpha_i}\right)| < \frac{1}{2}A_i^{*(k)}\pi$ where $A_i^{*(k)}$ is defined by (1.4)

To prove the theorem 2, we use the generalized Leibnitz's rule

$$D_x^{\mu}\left[f(x)g(x)\right] = \sum_{m=0}^{\infty}\binom{\mu}{m}D_x^{\mu-m}[f(x)]D_x^m[g(x)]$$

Theorem 3.

$$D_x^{\mu_1}D_y^{\mu_2}\left[x^{k_1}y^{k_2}(x+a)^{a_1}(a-x)^{-a_2}(y+c)^{d_1}(c-y)^{-d_2}\right.$$

$$\daleth\left(z_1 x^{\rho_1}y^{\omega_1}(x+a)^{b_1}(a-x)^{-c_1}(y+d)^{e_1}(d-y)^{-f_1}x^{\rho_1}y^{\omega_1}\cdots, z_r x^{\rho_r}y^{\omega_r}(x+a)^{b_r}(a-x)^{-c_r}(y+d)^{e_r}(d-y)^{-f_r}\right) =$$

$$a^{a_1-a_2}c^{d_1-d_2}x^{k_1-\mu_1}y^{k_2-\mu_2}\sum_{l,m,p,q=0}^{\infty}(-)^{m+q}\left(\frac{x}{a}\right)^{l+m}\left(\frac{y}{c}\right)^{p+q} \daleth^{U;m_r+2,n_r+4:V}_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}$$

$$\left(\begin{array}{c|c} a^{b_1-c_1}d^{e_1-f_1}x^{\rho_1}y^{\omega_1} & \mathbb{A}; (\text{-k}_1 - l + m; \rho_1, \cdots, \rho_r; 1), (-k_2 - p + q; \omega_1, \cdots, \omega_r; 1), (-a_1; b_1, \cdots, b_r; 1), \\ \cdot & \cdot \\ \cdot & \cdot \\ a^{b_r-c_r}d^{e_r-f_r}x^{\rho_r}y^{\omega_r} & \mathbb{B}; (\text{1-a}_2; c_1, \cdots, c_r; 1), (1-d_2; f_1, \cdots, f_r), \mathbf{B}, \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-d}_1; e_1, \cdots, e_r; 1), \mathbf{A}, (1-a_2+m; c_1, \cdots, c_r; 1), (1-d_2-q; f_1, \cdots, f_r : 1) : A \\ \cdot \\ \cdot \\ (\mu_1 - k_1 - l - m; \rho_1, \cdots, \rho_r; 1), (\mu_2 - k_2 - p - q; \omega_1, \cdots, \omega_r; 1), (l - a_1; b_1, \cdots, b_r; 1), (p - d_1; e_1, \cdots, e_r; 1) : B \end{array}\right) \tag{2.3}$$

Provided

$\mu_1,\mu_2,k_1,k_2,\alpha_i,a_i,b_i,\rho_i,c_i,b_i-c_i,a_1-a_2,k_1-\mu_1,k_2-\mu_2>0(i=1,\cdots r)$, $\max\left\{\left|arg\left(\frac{x}{a}\right)\right|,\left|arg\left(\frac{y}{c}\right)\right|\right\}<\pi$

$Re(-k_1-l-m)>0, Re(-k_2-p-q)>0, Re(1-a_2)>0, Re(1-d_2)>0, Re(\mu_1-k_1-l-m)>0$

$Re(1-a_2-m)>0, Re(1-d_2-q)>0, Re(\mu_2-p-q-k_2)>0, Re(l-a_1)>0, Re(p-d_1)>0$

$$Re\,(k_1)+\sum_{i=1}^{r}(\rho_i+b_i-c_i)\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$$Re\,(k_2)+\sum_{i=1}^{r}(\omega_i+e_i-f_i)\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$|arg\left(z_k x^{\rho_i}y^{\omega_i}(x+a)^{b_i}(a-x)^{-c_i}(y+d)^{e_i}(d-y)^{-f_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

To prove the theorem 3, we use the theorem 1 twice with respect to the variable y , and then with respect to the variable x , here x and y are two independent variables.

3. Special cases.

In this section, we give several particular cases.

Taking $b_i=0(i=1,\cdots,r)$, the fractional derivative formula (2.1) reduces to

Cororally 1.

$$D_x^{\mu}\left[x^k(x+a)^{a_1}(a-x)^{-a_2}\daleth\left(z_1x^{\rho_1}(a-x)^{-c_1},\cdots,z_1x^{\rho_r}(a-x)^{-c_r}\right)\right]=a^{a_1-a_2}x^{k-\mu}\sum_{l,m=0}^{\infty}\frac{(-)^m\left(\frac{x}{a}\right)^{l+m}}{l!m!}$$

$$\daleth_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}^{U;m_r+1,n_r+1:V}\left(\begin{array}{c|c}z_1x^{\rho_1}a^{-c_1} & \mathbb{A};(-l-m-k;\rho_1,\cdots,\rho_r;1),\mathbf{A},(1-m-a_2;c_1,\cdots,c_r;1);A\\ \cdot & \cdot\\ \cdot & \cdot\\ z_1x^{\rho_1}a^{-c_1} & \mathbb{B};(1-a_2;c_1,\cdots,c_r;1),\mathrm{B},(\mu-l-m-k;\rho_1,\cdots,\rho_r;1):B\end{array}\right) \quad (4.3)$$

provided

$\mu,k,a_i,\rho_i,c_i,a_1-a_2,k-\mu>0(i=1,\cdots r)$, $\left|arg\left(\frac{x}{a}\right)\right|<\pi$

$Re(-a_1), Re(1-a_2), Re(1-m-a_2), Re(\mu-l-m-k)>0$

$$Re\,(k)+\sum_{i=1}^{r}(\rho_i-c_i)\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$|arg\left(z_k x^{\rho_i}(a-x)^{-c_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

In the above equation, taking $a_2=c_i=0(i=1,\cdots,r)$, we obtain
Corollary 2.

$$D_x^{\mu}\left[x^k(x+a)^{a_1}\beth\left(z_1x^{\rho_1},\cdots,z_1x^{\rho_r}\right)\right]=a^{a_1}x^{k-\mu}\sum_{l=0}^{\infty}\binom{a_1}{l}\frac{\left(\frac{x}{a}\right)^l}{l!}$$

$$\beth_{X;p_{i_r}+1,q_{i_r}+1,\tau_{i_r}:R_r:Y}^{U;m_r,n_r+1:V}\left(\begin{array}{c|c} z_1x^{\rho_1} & \mathbb{A};(-l-k;\rho_1,\cdots,\rho_r;1),\mathbf{A},;A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_1x^{\rho_1} & \mathbb{B};\mathrm{B},(\mu-l-k;\rho_1,\cdots,\rho_r;1):B \end{array}\right) \tag{3.2}$$

provided

$\mu,k,a_i,\rho_i,k-\mu>0(i=1,\cdots r),\left|arg\left(\frac{x}{a}\right)\right|<\pi \quad Re(-a_1),Re(\mu-m-k)>0$

$$Re\left(k\right)+\sum_{i=1}^{r}\rho_i\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$|arg\left(z_kx^{\rho_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

In (4.2) if $a_1=0$, we het

Corollary 3.

$$D_x^{\mu}\left[x^k\beth\left(z_1x^{\rho_1},\cdots,z_1x^{\rho_r}\right)\right]=x^{k-\mu}\ \beth_{X;p_{i_r}+1,q_{i_r}+1,\tau_{i_r}:R_r:Y}^{U;m_r,n_r+1:V}\left(\begin{array}{c|c} z_1x^{\rho_1} & \mathbb{A};(-k;\rho_1,\cdots,\rho_r;1),\mathbf{A},;A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_1x^{\rho_1} & \mathbb{B};\mathrm{B},(\mu-k;\rho_1,\cdots,\rho_r;1):B \end{array}\right) \tag{3.3}$$

provided

$\mu,k,\rho_i,k-\mu>0(i=1,\cdots r),\left|arg\left(\frac{x}{a}\right)\right|<\pi \quad Re(\mu-k)>0$

$$Re\left(k\right)+\sum_{i=1}^{r}\rho_i\min_{\substack{1\leqslant k\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$|arg\left(z_kx^{\rho_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Taking $b_i=0(i=1,\cdots,r)$ in theorem 2, we get

Corollary 4.

$$D_x^{\mu}\left[x^k(x+a)^{a_1}(a-x)^{-a_2}\beth\left(z_1x^{\rho_1}(a-x)^{-c_1},\cdots,z_1x^{\rho_r}(a-x)^{-c_r}\right)\beth^*(\omega_1x^{\alpha_1},\cdots,\omega_rx^{\alpha_r})\right]=$$

$$a^{a_1-a_2}x^{k-\mu}\sum_{l,m,s=0}^{\infty}(-)^m\binom{\mu}{\gamma}\binom{a_1}{l}\left(\frac{x}{a}\right)^{l+m}\beth_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}$$

$$\left(\begin{array}{c|c} z_1x^{\rho_1}a^{b_1-c_1} & \mathbb{A};(-l-m;\rho_1,\cdots,\rho_r;1),\mathbf{A},(1-m-a_2;c_1,\cdots,c_r;1);A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_1x^{\rho_1}a^{b_1-c_1} & \mathbb{B};(1-a_2;c_1,\cdots,c_r;1),\mathbf{B},(-l-m+s;\rho_1,\cdots,\rho_r;1):B \end{array}\right)$$

$$\gimel^{U^*;0,n_r^*+1:V^*}_{X^*;p^*_{i_r}+1,q^*_{i_r}+1,\tau^*_{i_r}:R^*_r:Y^*}\left(\begin{array}{c|c} \omega_1 x^{\alpha_1} & \mathbb{A}^*;(-k;\alpha_1,\cdots,\alpha_r;1),\mathbf{A}^*;A^* \\ \cdot & \cdot \\ \cdot & \cdot \\ \omega_1 x^{\alpha_1} & \mathbb{B}^*;\mathbf{B}^*,(-k+s+\mu-s;\alpha_1,\cdots,\alpha_r;1):B^* \end{array}\right) \tag{3.4}$$

Provided

$\mu, k, \alpha_i, a_i, \rho_i, c_i, a_1-a_2, k-\mu > 0 (i=1,\cdots r)$, $\left|arg\left(\frac{x}{a}\right)\right| < \pi$

$$Re(\mu-k-s)>0, Re\,(k)+\sum_{i=1}^{r}(\rho_i-c_i)\min_{\substack{1\leqslant k\leqslant m_i\\ 1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_k^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right)+1>0$$

$$Re(k)+\sum_{i=1}^{r}\alpha_i\min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$$

$|arg\left(z_k x^{\rho_i}(a-x)^{-c_i}\right)| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).$|arg\left(\omega_i x^{\alpha_i}\right)| < \frac{1}{2}A_i^{*(k)}\pi$ where $A_i^{*(k)}$ is defined by (1.4)

Remark 6.
If $m_2=n_2=\cdots=m_{r-1}=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$, then we can obtain the same fractional derivatives formulae in the generalized multivariable Aleph- function (extension of multivariable Aleph-function defined by Ayant [1])

Remark 7.
If $m_2=n_2=\cdots=m_r=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2= =\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then we can obtain the same fractional derivatives formulae in a generalized multivariable I-function (extension of multivariable I-function defined by Prathima et al. [6]).

Remark 8.
If$A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then we can obtain the same fractional derivatives formulae in generalized of multivariable I-function (extension of multivariable I-function defined by Prasad [5]).

Remark 9.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the generalized multivariable H-function (extension of multivariable H-function defined by Srivastava and Panda [7,8] and then we can obtain the same fractional derivatives formulae.

4. Conclusion.

The fractional derivative formulae involving in this paper are double fold generality in term of variables. By specializing the various parameters and variables involved, these formulae can suitably be applied to derive the corresponding results involving wide variety of useful functions (or product of several such functions) which can be expressed in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several intersting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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