

Competition between the Two Species using the Predator Prey Model

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Abstract

Predator Prey models are useful and often used in the environmental science field because they allow researchers to both observe the dynamics of animal populations and make predictions of this project as to how they will develop over time. The objective of this project was to create 3 projections of animal population based on a simple Predator- prey model and explore the trend visible. Each case began with a set of initial conditions that produced different outcomes for the function of the population of Foxes and Rabbits over a 30 months' time span. Using Euler's method an Excel spreadsheet was developed to produce the values for the population of Rabbits and Foxes over the span of 30 months.

Keywords: *Predator prey model, Foxes, Rabbits, Euler's method*

I. INTRODUCTION

The function of predation performs in the dynamics of prey populations is debatable. Our understanding of predator- prey relationships is complex by means of a multitude of factors inside the environment and a widespread lack of expertise of most ecological structures. Various other elements, except predation can also alter or restrict prey populations and various factors have an effect on the degree to which predation impacts prey populations. Furthermore somefactors can also create time logs or even motive generational outcomes that pass disregarded.

Predation has been defined as individual of one species consuming residing people of every other species[1]. The function of predators in the population dynamics of prey species has been investigated for decades, yet figuring out whether or not no longer predators restrict or alter a prey population, stays debatable in the scientific career [2][3][4][5]. Much of the debate consequences from the multitude of competing variables, together with predation that have an impact on demographics of prey species and the difficulty of conducting big-scale, long time research with a few degree of control or replication.

II. CONSTRAINS OF PREDATOR AND PREY

Evaluation has located constraints of both predator and prey species with obvious implications for the relationships between them. In widespread, comparative body size, electricity, speed and agility dictate a predators capacity to kill specific prey, whilst comparable constrains on prey outline which predators pose a danger to them.. Such physical and behavioraltraits or constraints have developed over sizeable intervals and represent an “evolutionary race” between predator and prey. Some physical capabilities,examples, the spread of pronghorn antelope may additionally evenrepresent residual tendencies from interaction with predators now extinct [6].

III. FORMULATION OF THE SCIENTIFIC PROBLEM

Predator-prey models are used by scientists to predict or explain trends in animal populations. There are many instances in nature where one species of animal feeds on another species of animal, which in turn feeds on another thing. The first species is called predator and the second is called the prey.

Theoretically the predator can destroy all the prey so that the latter become extinct. However if this happens the predator will also become extinct since, as we assume, it depends on the prey for its existence.

What actually happens in nature is that a cycle develops where at some time prey maybe abundant and the predators few. Because of the abundance of prey, the predator population grows and reduces the population of prey. This results in a reduction of predators and consequent increase of prey and the cycle continues. An important problem of ecology, the science which studies the interrelationships of organisms and their environment, is to investigate the question of coexistence of the two species.

To this end, it is natural to seek a mathematical formulation of this predator- prey problem and to use it to forecast the behavior of populations of various species at different times [7].

Consider a population of foxes, the predator and rabbits, the prey. Examine whether the two species can survive in the long term. So, to simply the model, we use the following assumptions [8].

1. The only source of food for the foxes is rabbits and the only predator of the rabbits is foxes.
2. Without rabbits present, foxes would die out.
3. Without foxes present, the population of rabbits would grow.
4. The presence of rabbits increases the rate at which the population of foxes grows.
5. The presences of foxes decrease the rate at which the population of rabbits grows.

We will model these populations using a discrete dynamic model. Each state of the system consists of the populations of foxes and rabbits at a point in time. Since this state consists of two components, this is a two dimensional discrete dynamic system.

To create our model we first need to define some variables. Let

F_n = Population of foxes at the end of month n

R_n = Population of rabbits at the end of month n

As in this model, the assumptions are stated in terms rates of change,

$$\Delta F_n = F_{n+1} - F_n \text{ and}$$

$$\Delta R_n = R_{n+1} - R_n .$$

There are many ways we could model these rates of change with the assumptions.

Assumptions (2) and(3) deal with the rates of change of each population in the absence of the other. A reasonable way to model these is to say that the rates are proportional to the populations. This yields the difference equations

$$\Delta F_n = F_{n+1} - F_n = -a F_n \text{ (1)}$$

$$\Delta R_n = R_{n+1} - R_n = d R_n \text{ (2)}$$

Where $0 < a, d \leq 1$. Note that the coefficient of proportionality in equation (1) is negative to reflect the fact that the foxes would die out (a negative rate of change) without the rabbits. The co-efficient in equation (2) is positive because the population of rabbits grows (a positive rate of change) without the foxes.

Now assumptions (4) and (5) say that these rates in equations (1) and (2) either increase or decrease in the presences of the other species. So to incorporate these assumptions. We simply add one term to each of equations (1) and (2), yielding

$$F_{n+1} - F_n = -a F_n + b R_n \text{ (3)}$$

$$R_{n+1} - R_n = -c F_n + d R_n \text{ (4)}$$

Where $0 \leq b, c$. Note that the added term in equation (3) is positive to reflect the fact that the presence of rabbits of foxes grows. The added term in equation (4) is negative because the presence of foxes decreases the rate at which rabbit grow.

Rewriting equations (3) and(4) yields our model in the forms of a system of linear equations

$$F_{n+1} = (1 -a) F_n + b R_n \text{ (5)}$$

$$R_{n+1} = -c F_n + (1+ d) R_n \text{ (6)}$$

Because our model has the form of a system of linear equations, it is called a two dimensional linear discrete dynamical system.

The parameters (1-a) and b will generically be called the fox death and birth factors respectively, while the parameters -c and (1+d) will be called the rabbit deaths and birth factors respectively.

Set up an Excel spread sheets to solve the two differential equations using a first order Euler method. The initial values of the parameters and populations are shown in the figure -1.Copy row 8 down to row 37 to model 30 months. Apply it to the following 3 cases (Table 1) each of which is a different set if initial conditions.

Table: 1

Case	Starting populations	
	Rabbits	Foxes
1.	200	500
2.	550	220
3.	500	200

Case 1: Foxes-500; Rabbits -200

Table: 2

	A	B	C
1			
2		Deaths	Births
3	Foxes	0.5	0.4
4	Rabbits	-0.75	1.1
5			
6	Months	Foxes	Rabbits
7	0	500	200
8	=A7+1	=B\$3*B7+C7*\$C\$3	=B7*\$B\$4+C7*\$C\$4

Table :3

Months	Foxes	Rabbits
0	500	200
1	330	135
2	219	92

3	146	64
4	99	46
5	68	34
6	47	26
7	34	20
8	25	16
9	19	14
10	15	12
11	12	10
12	10	9
13	9	9
14	8	8
15	7	7
16	7	7
17	6	7
18	6	6
19	5	6
20	5	5
21	5	5
22	4	5
23	4	5
24	4	4
25	4	4
26	4	4
27	3	4
28	3	4
29	3	3
30	3	3

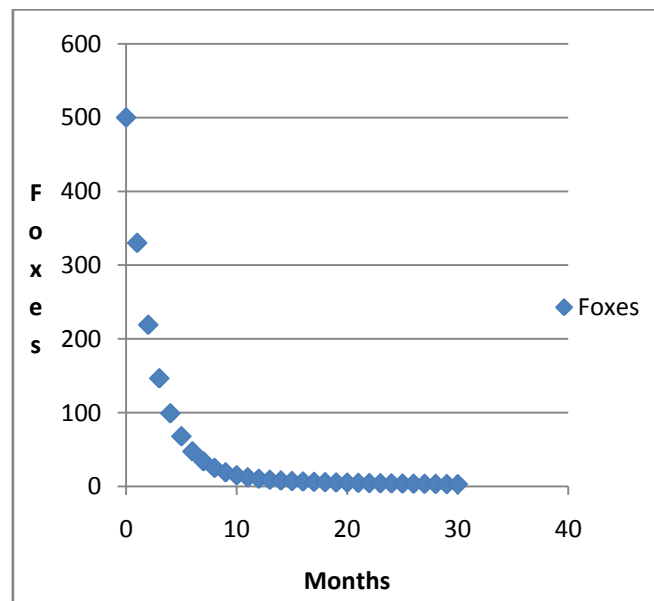


Figure 1: Fox populations over 30 month span starting with 500 foxes

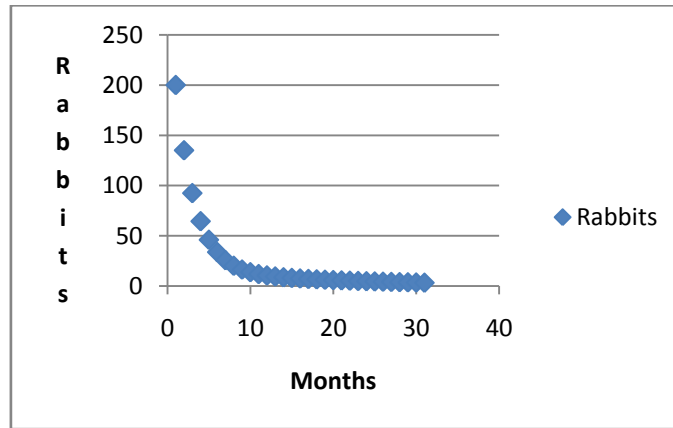


Figure 2: Rabbit populations over 30 months span starting with 200 rabbits

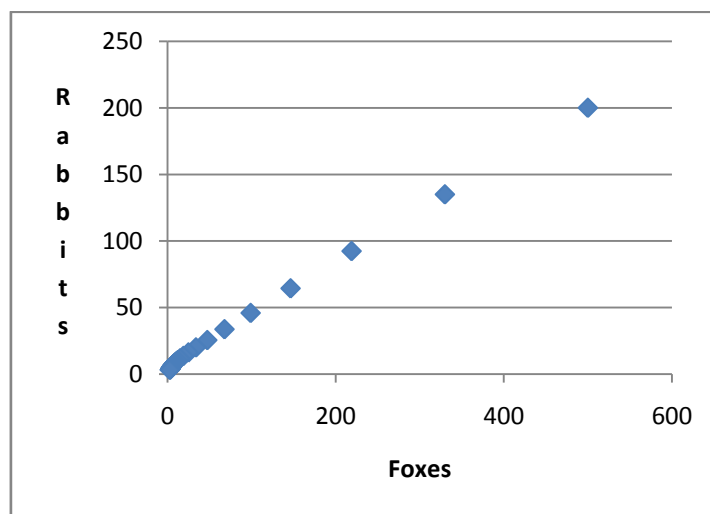


Figure 3: Rabbit and Fox population over 30 months span starting with 200 rabbits and 500 foxes

Case : 2 Foxes-220 ; Rabbits-500

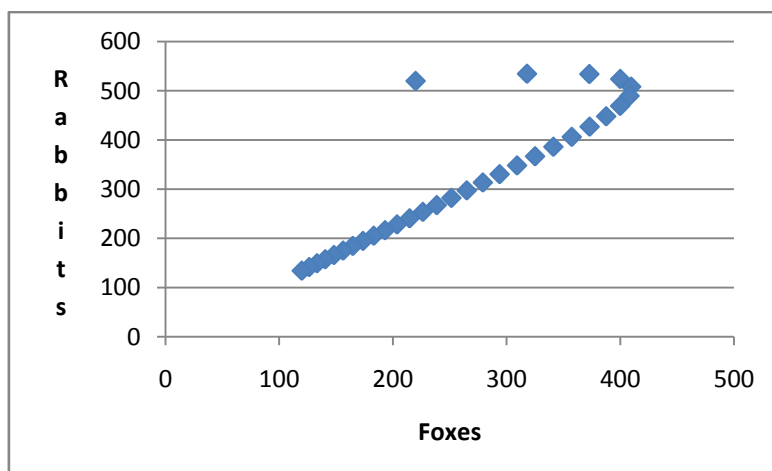


Figure 4: Rabbit and Fox population over 30 months span starting with 220 rabbits and 550 foxes

Case:3 Foxes- 200;Rabbits-500

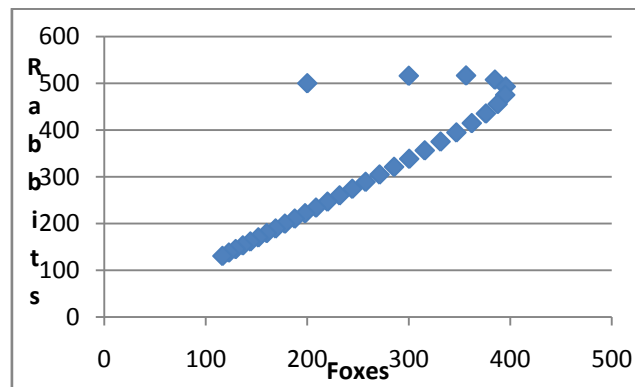


Figure 5: Rabbit and Fox population over 30 months span starting with 500 rabbits and 200 foxes

IV. RESULTS AND DISCUSSION

In case 1 we started with a rabbit population prey of 200 and a predator population foxes of 500. In figure 3 the curve formed in the graph of rabbits versus foxes is called a trajectory of the system. Notice that the curve tends towards the origin (0 foxes and 0 rabbits). This means that both species eventually die out. This also shown in the figures 4 and 5 .Change the initial populations from case 2 and case 3 and observe that the trajectories also tend towards the origin.

Over all these various scenarios, using the Euler method, certain overriding trends can be seen. The graph and figures support the notion that the rabbit population and fox population are codependent on one another .The graphs show the number of rabbits present at any moment is related to the number of foxes present at any moment and vice versa. The periodicity of the population depends on the initial values but is regular and consistent in all cases.

V. CONCLUSION

This predator prey model is a rudimentary model of the complex ecology of this model world;it assumes just one prey for the predator and vice versa. It also assumes no outside influences like diseases, changing conditions, populations and so on.

This model is an excellent tool to teach principles involved in ecology and to show some rather counter-initiative results. It also shows a special relationship betweenbiology and mathematics.

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