P-Transform Associated with General Class of Multivariable Polynomials and Multivariable H-Function

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Abstract

In the present paper, the author isto derive the P-transform of Multivariable H-function and general polynomials.P-transform is useful in reaction theory in astrophysics. P-transform is generalization of many integral transforms, Multivariable H-function and general Multivariable polynomials are general in nature. These results discussed here can be used to investigate wide class of new and known results, hitherto scattered in the literature. For the sake of illustration, some special case have also been mentioned here of our finding.

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I. INTRODUCTION

The P-transform is defined and represented in Kumar and Kilbas [1] as

$$\left(\mathcal{P}_{\tau}^{\rho,\mu,\beta} f\right)(x) = \int_0^{\infty} D_{\rho,\mu}^{\tau,\beta}\,(xt)dt, x > 0 \tag{1.1}$$

where$D_{\rho,\mu}^{\tau,\beta}(x)$ represents the kernel-function

$$D_{\rho,\mu}^{\tau,\beta}(x) = \int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\rho}}} y^{\tau-1}\,[1-a(1-\beta)y^{\rho}]^{\frac{1}{1-\tau}} e^{-xy^{-\mu}}\,dy, \qquad x > 0, \tag{1.2}$$

with $\tau \in \mathbb{C}, \mu > 0, \rho > 0, a > 0, \beta < 1$, when $D_{\rho,\beta}^{\tau,\mu}(x)$ is given by (1.2), P-transform is called type-1 P-transform. If we use

$$D_{\rho,\mu}^{\tau,\beta}(x) = \int_0^{\infty} y^{\tau-1}\,[1-a(\beta-1)y^{\rho}]^{\frac{1}{1-\tau}} e^{-xy^{-\mu}}\,dy, \qquad x > 0, \tag{1.3}$$

for$\tau \in \mathbb{C}, \mu > 0, \rho > R, a > 0, \beta > 1$, in (1.1) then we obtain a type-2 P-transform. The P-transform of both types are defined in the space $L_{\tau,r}(0,\infty)$ consisting of the Lebesgue measurable complex valued functions f for which

$$\parallel f \parallel_{\tau,r} = \left\{\int_0^{\infty} |t^{\tau} f(t)|^r \frac{dt}{t}\right\}^{\frac{1}{r}} < \infty, \tag{1.4}$$

for $1 \le r < \infty, \tau \in R$. The P-transform of both the types is obtained by using the pathway model of Mathai [5], Mathai and Haubold [6]. If $\mu = 1, a = 1, \beta \to 1$, we have

$$\lim_{\beta\to 1} D_{\rho,1}^{\tau,\beta}(x) = Z_{\rho}^{\tau}(x) \tag{1.5}$$

where$Z_{\rho}^{\tau}(x)$ is the kernel function of theKrätzel transform, introduced by Krätzel [12] and given by

$$K_{\rho}^{\tau}(x) = \int_0^{\infty} Z_{\rho}^{\tau}(xt)\, f(t)dt, \quad x > 0 \tag{1.6}$$

where

$$Z_{\rho}^{\tau}(x) = \int_0^{\infty} y^{\tau-1} e^{-y^{\rho}-xy^{-1}}\, dy \tag{1.7}$$

The transform in (1.6) and its several modifications were considered by many authors. Glaeske et al. [14] considered a generalized version of the Krätzel transform and its compositions with fractional calculus operators on the spaces of $F_{p,\theta}$ and$F'_{p,\theta}$. Bonilla et al. [7, 8] studies the Krätzel transform in the spaces $F_{p,\theta}$ and$F'_{p,\theta}$. Kilbas et al. [2] obtained the asymptotic representation for the modified Kratzel function. Klibas et al. [3] studied the Krätzel function in (1.7) for all values of ρ and established it in the terms of Fox's H-function, when $\mu = 1,\ \rho = 1,\ a = 1, and\ \ \beta \to 1$P-transform of both types reduces to the Meijer transform. For $\mu = 1, \rho = 1, a = 1, and\ \ \beta \to 1$, along with x replaced by $\frac{t^z}{4}$ in (1.2) and (1.3), we get

$$\lim_{\beta\to 1} D_{\rho,1}^{\tau,\beta}(x) = Z_\rho^\tau(x) \tag{1.8}$$

where$K_{-\tau}(t)$ is modified Bessel function of the third kind or the Mc-Donald function (see [4], sect. 7.2.2). Kilbas and Kumar [1] considered (1.3) for $\mu = 1$ and established its composition with fractional operators and represented it in terms of various generalized special functions.

The H-function of several complex variables, defined H. M. Srivastava and R. Panda [10], we will define and represent it in the following from [1, page 252, equation (C.1)]

$$H[z_1,\dots,z_r] = H_{p,q:\{p_r,q_r\}}^{0,n:\{m_r,n_r\}}\left[\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix}\left| \begin{matrix} (a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}:\left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\} \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q}:\left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r} \psi(\xi_1,\dots,\xi_r)\prod_{i=1}^r\left(\phi_i(\xi_i) z_i^{\xi_i} d\xi_i\right) \tag{1.9}$$

where

$$\psi(\xi_1,\dots\dots,\xi_r) = \frac{\prod_{j=1}^n \Gamma(1-a_j+\sum_{i=1}^r \alpha_j^i\xi_i)}{\prod_{j=m+1}^{p_i}\Gamma(a_j-\sum_{i=1}^r\alpha_j^i\xi_i)\prod_{j=1}^q\Gamma(1-\ b_j+\sum_{i=1}^r\beta_j^i\xi_i)} \tag{1.10}$$

$$\phi_i\xi_i = \frac{\prod_{j=1}^{n_i}\Gamma(1-c_j^i+\gamma_j^i\xi_i)\prod_{j=1}^{m_i}\Gamma(d_j^i-\delta_j^i\xi_i)}{\prod_{j=n+1}^{p_i}\Gamma\left(c_j^i-\gamma_j^i\xi_i\right)\prod_{j=m+1}^{q_i}\Gamma(1-d_j^i+\delta_j^i\xi_i)} \qquad (i\in\{1,2,3..\}) \tag{1.11}$$

Here, $\{m_r,n_r\}$ stands for $m_1,n_1,\dots,m_r,n_r$ and $\left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\}$ stands for the sequence of r ordered pairs$\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1},\dots,\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}$.

The general multivariable polynomials introduced and defined by Srivastava [15] in the following form:

$$S_{N_1,\dots,N_s}^{M_1,\dots,M_s}[x] = \sum_{k_1=0}^{N_1/M_1}\cdots\sum_{k_s=0}^{N_s/M_s}\frac{(-N_1)_{M_1,k_1}}{k_1!}Y_1^{(k_1)}\cdots\frac{(-N_s)_{M_s,k_s}}{k_s!}Y_s^{(k_s)}A[N_1,k_1;\dots;N_s,k_s] \tag{1.12}$$

where$N_i = 0,1,2,\dots; M_i(i = 1,..,k)$ are non-zero arbitrary positive integer. The coefficients$A[N_1,k_1;\dots;N_s,k_s]$ being arbitrary (real or complex) constants.

II. MAIN RESULTS

Theorem 1: If$f \in L_{\tau,r}(0,\infty),\ z_1,\dots,z_r,\tau \in \mathbb{C}, \mu > 0, h > 0, b > 0, \beta < 1$, be such $\rho > 0$ in the case of a type-1P-transform then

$$P_\tau^{\rho,\mu,\beta}\left[H[Z_1t^{h_1},\dots\dots,Z_rt^{h_r}]S_{N_1,\dots,N_s}^{M_1,\dots,M_s}[Y_1t^{b_1},\dots\dots,Y_st^{b_s}]\right]$$

$$= \sum_{k_1=0}^{N_1/M_1}\cdots\sum_{k_s=0}^{N_s/M_s}\frac{(-N_1)_{M_1,k_1}}{k_1!}\cdots\frac{(-N_s)_{M_s,k_s}}{k_s!}A[N_1k_1;\dots;N_sk_s]\frac{{Y_1}^{k_1}}{k_1!},\dots,\frac{{Y_s}^{k_s}}{k_s!}\frac{1}{\rho X^{\sum_{i=1}^s b_ik_i+1}}$$

$$\frac{1}{[a(1-\beta)]^{\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho}}}\Gamma\left(\frac{1}{1-\beta}+1\right)H^{0,N+2:\,M_1,N_1;\ldots\ldots;M_r,N_r}_{A+2,B+1:\,P_1,Q_1;\ldots\ldots;P_r,Q_r}\times$$

$$\left[\begin{array}{c}\frac{z_1X^{-h_1}}{[a(1-\beta)]^{\sum_{i=1}^{r}\frac{\mu h_i}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_rX^{-h_r}}{[a(1-\beta)]^{\sum_{i=1}^{r}\frac{\mu h_i}{\rho}}}\end{array}\middle|\begin{array}{l}(-\sum_{i=1}^{s}b_ik_i,h_1,\ldots,h_r)\ \left(1-\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho},\frac{\mu h_1}{\rho},\ldots,\frac{\mu h_r}{\rho}\right)(a_j,\vartheta_j^{(1)},\ldots,\vartheta_j^{(r)})_{1,A}\\ \left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho},\frac{\mu h_1}{\rho},\ldots,\frac{\mu h_r}{\rho}\right)(b_j,\psi_j^{(1)},\ldots,\psi_j^{(r)})_{1,B}\\ :\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P^{(1)}};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P^{(r)}}\\ :\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q^{(1)}};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q^{(r)}}\end{array}\right] \tag{2.1}$$

Where $\mathbb{R}\left(\frac{1}{1-\beta}+1\right)>0$ and the coefficients $A[N_1k_1;\ldots;N_sk_s>0]\ \forall\ i\in(1,\ldots,s)$ are arbitrary constants, real or complex.

Theorem 2: If $f\in L_{\tau,r}(0,\infty)$, $z_1,\ldots,z_r,\tau\in\mathbb{C},\mu>0,h>0,b>0,\beta>1$, be such $\rho\in R,\rho\neq 0$ in the case of a type-2 P-transform then

$$P_\tau^{\rho,\mu,\beta}\left[H[Z_1t^{h_1},\ldots\ldots,Z_rt^{h_r}]S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}[Y_1t^{b_1},\ldots\ldots,Y_st^{b_s}]\right]$$

$$=\sum_{k_1=0}^{N_1/M_1}\cdots\sum_{k_s=0}^{N_s/M_s}\frac{(-N_1)_{M_1,k_1}}{k_1!}\cdots\frac{(-N_s)_{M_s,k_s}}{k_s!}A[N_1k_1;\ldots;N_sk_s]\frac{Y_1^{\,k_1}}{k_1!},\ldots,\frac{Y_s^{\,k_s}}{k_s!}\frac{1}{\rho X^{\sum_{i=1}^{s}b_ik_i+1}}$$

$$\frac{1}{[a(\beta-1)]^{\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho}}}\Gamma\left(\frac{1}{\beta-1}+1\right)H^{0,N+2:\,M_1,N_1;\ldots\ldots;M_r,N_r}_{A+2,B+1:\,P_1,Q_1;\ldots\ldots;P_r,Q_r}\times$$

$$\left[\begin{array}{c}\frac{z_1X^{-h_1}}{[a(\beta-1)]^{\sum_{i=1}^{r}\frac{\mu h_i}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_rX^{-h_r}}{[a(\beta-1)]^{\sum_{i=1}^{r}\frac{\mu h_i}{\rho}}}\end{array}\middle|\begin{array}{l}(-\sum_{i=1}^{s}b_ik_i,h_1,\ldots,h_r)\ \left(1-\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho},\frac{\mu h_1}{\rho},\ldots,\frac{\mu h_r}{\rho}\right)(a_j,\vartheta_j^{(1)},\ldots,\vartheta_j^{(r)})_{1,A}:\\ \left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu\sum_{i=1}^{s}b_ik_i}{\rho},\frac{\mu h_1}{\rho},\ldots,\frac{\mu h_r}{\rho}\right)(b_j,\psi_j^{(1)},\ldots,\psi_j^{(r)})_{1,B}:\\ \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P^{(1)}};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P^{(r)}}\\ \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q^{(1)}};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q^{(r)}}\end{array}\right] \tag{2.2}$$

Where $\mathbb{R}\left(\frac{1}{\beta-1}\right)>0$ and the coefficients $A[N_1k_1;\ldots;N_sk_s>0]\ \forall\ i\in(1,\ldots,s)$ are arbitrary constants, real or complex.

Proof: To prove (2.1), we consider type-1 P-transform using (1.1) express H- function multivariable and general multivariablepolynomial with the help of (1.9), (1.12), we obtain

$$P_\tau^{\rho,\mu,\beta}\left[H[Z_1t^{h_1},\ldots\ldots,Z_rt^{h_r}]S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}[Y_1t^{b_1},\ldots\ldots,Y_st^{b_s}]\right]$$

$$=\int_0^\infty D^{\tau,\beta}_{\rho,\mu}(xt)H[Z_1t^{h_1},\ldots\ldots,Z_rt^{h_r}]S_L^{k_1;\ldots\ldots;k_s}[Y_1t^{b_1},\ldots\ldots,Y_st^{b_s}]dt$$

$$= \frac{1}{(2\pi i)^r} \int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\rho}}} y^{\tau-1} \left[1 - a(1-\beta) y^{\rho}\right]^{\frac{1}{1-\tau}} e^{-xty^{-\mu}}$$

$$\int_{L_1} \dots \int_{L_r} \left(\prod_{i=1}^{r} \phi_i(\xi_i) z_i^{\xi_i} \right) \psi(\xi_1, \dots, \xi_r) \sum_{k_1=0}^{N_1/M_1} \dots \sum_{k_s=0}^{N_s/M_s} \frac{(-N_1)_{M_1,k_1}}{k_1!} \dots \frac{(-N_s)_{M_s,k_s}}{k_s!}$$

$$A[N_1 k_1; \dots; N_s k_s] \frac{Y_1^{k_1}}{k_1!}, \dots, \frac{Y_s^{k_s}}{k_s!} d\xi_1 \dots d\xi_r \, dy \, dt$$

Changing the order of integrations and series, taking $-xty^{-\mu} = \upsilon$ and using gamma function, we get

$$= \sum_{k_1=0}^{N_1/M_1} \dots \sum_{k_s=0}^{N_s/M_s} \frac{(-N_1)_{M_1,k_1}}{k_1!} \dots \frac{(-N_s)_{M_s,k_s}}{k_s!} A[N_1 k_1; \dots; N_s k_s] \frac{Y_1^{k_1}}{k_1!}, \dots, \frac{Y_s^{k_s}}{k_s!}$$

$$\frac{1}{(2\pi i)^r} \int_{L_1} \dots \int_{L_r} \left(\prod_{i=1}^{r} \phi_i(\xi_i) z_i^{\xi_i} \right) \psi(\xi_1, \dots, \xi_r) \frac{\Gamma\left(\sum_i^r h_i \xi_i + \sum_i^s b_i k_i + 1\right)}{X^{\sum_i^r h_i \xi_i + \sum_{i=1}^s b_i k_i + 1}}$$

$$\left(\int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\rho}}} y^{\tau + \mu \sum_i^r h_i \xi_i + \mu \sum_i^s b_i k_i + \mu - 1} \left[1 - a(1-\beta) y^{\rho}\right]^{\frac{1}{1-\tau}} dy \right) d\xi_1 \dots d\xi_r$$

Now, solving inner integral with the help of beta function formula and then reinterpreting the result Mellin-Barnes contour integral in terms of h-function multivariable, we get the desired result.

On the similar lines, the proof of result (2.2) can be established using the definition of multivariable H-function (1.9) and type-2 P-transform (1.1).

III. SPECIAL CASES

On taking $a = 1, \mu = 1$ and $\beta \to 1$ in result (2.1), we have

(i) $\lim_{\beta \to 1} P_{\tau}^{\rho,1,\beta} \left[H[Z_1 t^{h_1}, \dots \dots, Z_r t^{h_r}] S_{N_1,\dots,N_s}^{M_1,\dots,M_s} [Y_1 t^{b_1}, \dots \dots, Y_s t^{b_s}] \right]$

$$= \sum_{k_1=0}^{N_1/M_1} \dots \sum_{k_s=0}^{N_s/M_s} \frac{(-N_1)_{M_1,k_1}}{k_1!} \dots \frac{(-N_s)_{M_s,k_s}}{k_s!} A[N_1 k_1; \dots; N_s k_s] \frac{Y_1^{k_1}}{k_1!}, \dots, \frac{Y_s^{k_s}}{k_s!}$$

$$\frac{1}{\rho X^{\sum_{i=1}^s b_i k_i + 1}} H_{A+2,B:\, P_1,Q_1;\dots\dots;P_r,Q_r}^{0,N+2:M_1,N_1;\dots\dots;M_r,N_r} \times$$

$$\left[\begin{matrix} Z_1 X^{-h_1} \\ \cdot \\ \cdot \\ \cdot \\ Z_1 X^{-h_1} \end{matrix} \middle| \begin{matrix} \left(-\sum_{i=1}^{s} b_i k_i, h_1, \dots, h_r\right) \left(1 - \frac{\tau + \sum_{i=1}^s b_i k_{i+1}}{\rho}, \frac{h_1}{\rho}, \dots, \frac{h_r}{\rho}\right) (a_j, \vartheta_j^{(1)}, \dots, \vartheta_j^{(r)})_{1,A} : \\ (b_j, \psi_j^{(1)}, \dots, \psi_j^{(r)})_{1,B} : \end{matrix} \right.$$

$$\left. \begin{matrix} \left(c_j^{(1)}, \gamma_j^{(1)}\right)_{1,P^{(1)}}; \dots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,P^{(r)}} \\ \left(d_j^{(1)}, \delta_j^{(1)}\right)_{1,Q^{(1)}}; \dots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,Q^{(r)}} \end{matrix} \right] \quad (3.1)$$

The general multivariable polynomial reduce to general class of polynomial and multivariable H-function to Fox's H-function, then applying u=2 in (2.1), (2.2) and (3.1), general class of polynomial reduces to Hermite polynomials [13] by setting

$S_V^U[X] = X^{V/2} H_V\left(\frac{1}{2\sqrt{X}}\right)$, in this case $A_{V,k} = (-1)^k$, we get

(ii) $P_{\tau}^{\rho,\mu,\beta} \left[H_{A,B}^{M,N}(Zt^h)(Yt^b) H_V\left(\frac{1}{2\sqrt{Yt^b}}\right) \right] = \sum_{k=0}^{V/2} \frac{(-V)_{2k}}{k!} (-Y)^k \frac{1}{\rho X^{bk+1}}$

$$\frac{1}{[a(1-\beta)]^{\frac{\tau+\mu+\mu bk}{\rho}}}\Gamma\left(\frac{1}{1-\beta}+1\right)$$

$$H_{A+2,B+1}^{M,N+2}\left[\frac{ZX^{-h}}{[a(1-\beta)]^{\frac{\mu h}{\rho}}}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right)(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right)\end{matrix}\right] \quad (3.2)$$

(iii) $P_\tau^{\rho,\mu,\beta}\left[H_{A,B}^{M,N}(Zt^h)(Yt^b)H_V\left(\frac{1}{2\sqrt{Yt^b}}\right)\right]=\sum_{k=0}^{V/2}\frac{(-V)_{2k}}{k!}(-Y)^k\frac{1}{\rho X^{bk+1}}$

$$\frac{1}{[a(\beta-1)]^{\frac{\tau+\mu+\mu bk}{\rho}}}\frac{1}{\Gamma\left(\frac{1}{\beta-1}\right)}$$

$$H_{A+2,B+1}^{M+1,N+2}\left[\frac{ZX^{-h}}{[a(\beta-1)]^{\frac{\mu h}{\rho}}}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right),(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right),(b_j,B_j)_{M+1,B}\end{matrix}\right] \quad (3.3)$$

(iv) $\lim_{\beta\to 1}P_\tau^{\rho,1,\beta}\left[H_{A,B}^{M,N}(Zt^h)(Yt^b)H_V\left(\frac{1}{2\sqrt{Yt^b}}\right)\right]=\sum_{k=0}^{V/2}\frac{(-V)_{2k}}{k!}(-Y)^k\frac{1}{\rho X^{bk+1}}$

$$H_{A+2,B+1}^{M,N+2}\left[\frac{Z}{X^h}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+bk+1}{\rho},\frac{h}{\rho}\right)(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\end{matrix}\right] \quad (3.4)$$

Results in (ii), (iii) and (iv) are valid under the same conditions as are required for (2.1), (2.2) and (3.1) respectively. Putting u=1 in (2.1), (2.2) and (3.1), general class of polynomial reduces to Laguerrre polynomials [13] by setting

$S_V^U[X]=L_V^U(X)$, in this case $A_{V,k}=\binom{V+S}{V}\frac{1}{(S+1)_k}$, we get

(v) $P_\tau^{\rho,\mu,\beta}[H_{A,B}^{M,N}(Zt^h)L_V^U(Yt^b)]=\sum_{k=0}^{[V]}\binom{V+s}{V-k}\frac{(-Y)^k}{k!}\frac{1}{\rho X^{bk+1}}$

$$\frac{1}{[a(1-\beta)]^{\frac{\tau+\mu+\mu bk}{\rho}}}\Gamma\left(\frac{1}{1-\beta}+1\right)$$

$$H_{A+2,B+1}^{M,N+2}\left[\frac{ZX^{-h}}{[a(1-\beta)]^{\frac{\mu h}{\rho}}}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right)(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right)\end{matrix}\right] \quad (3.5)$$

(vi) $P_\tau^{\rho,\mu,\beta}[H_{A,B}^{M,N}(Zt^h)L_V^U(Yt^b)]=\sum_{k=0}^{[V]}\binom{V+s}{V-k}\frac{(-Y)^k}{k!}\frac{1}{\rho X^{bk+1}}$

$$\frac{1}{[a(\beta-1)]^{\frac{\tau+\mu+\mu bk}{\rho}}}\frac{1}{\Gamma\left(\frac{1}{\beta-1}\right)}$$

$$H_{A+2,B+1}^{M+1,N+2}\left[\frac{ZX^{-h}}{[a(\beta-1)]^{\frac{\mu h}{\rho}}}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right),(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\left(\frac{1}{\beta-1}-\frac{\tau+\mu+\mu bk}{\rho},\frac{\mu h}{\rho}\right),(b_j,B_j)_{M+1,B}\end{matrix}\right] \quad (3.6)$$

(vii) $$\lim_{\beta\to 1} P_{\tau}^{\rho,1,\beta}\left[H_{A,B}^{M,N}(Zt^h)L_V^U(Yt^b)\right] = \sum_{k=0}^{[V]}\binom{V+s}{V-k}\frac{(-Y)^k}{k!}\frac{1}{\rho X^{bk+1}}$$

$$H_{A+2,B+1}^{M,N+2}\left[\frac{Z}{X^h}\middle|\begin{matrix}(a_j,A_j)_N,(-bk,h)\left(1-\frac{\tau+bk+1}{\rho},\frac{h}{\rho}\right)(a_j,A_j)_{N+1,A}\\(b_j,B_j)_{1,M}\end{matrix}\right] \tag{3.7}$$

Results in (v), (vi) and (vii) are valid under the same conditions as are required for (2.1), (2.2) and (3.1) respectively. The results derived in this paper would at once yield a very large number of results, involving a large variety of polynomials and various special functions.

IV. CONCLUSION

In the present paper, we have obtained the results have been developed in terms of the product of multivariable H-function and a general class of polynomials in a compact and elegant with the help of P-transform. Most of the results obtained have been put in a compact form, avoiding the occurrence of problems of mathematics and thus making them useful in applications.

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