Two-Phase Magneto Hydro Dynamic Flow and Heat Transfer through a Horizontal Channel in Presence of Uniform Magnetic Field

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Abstract

This study dealt with a theoretical analysis of two-phase flow and heat transfer in a horizontal channel in the presence of an applied electric field and inclined magnetic field has been investigated. The inclined magnetic field is a strong magnetic field and the induced magnetic field is produced along the flow direction. The fluids in both phases were incompressible and the flow was assumed to be steady, one-dimensional and fully developed. Further it is also assumed that the two fluids have different viscosities and thermal conductivities. In this paper, we have investigated the upper phase of the two fluids which is assumed to be electrically conducting and that of the lower phase is electrically non-conducting. The transport properties of the two fluids were taken to be constant and equal temperature. The analytical solutions for velocities, induced magnetic field and temperature distributions are obtained and are computed numerically for different heights and viscosity ratios for two fluids and for two values of electric load parameter R_e .The computed results of velocity distributions, induced magnetic field strength and the temperature distributions are depicted graphically for distances from the fixed horizontal plates and for different angle of inclinations.

Keywords: *Induced magnetic field, horizontal channel, incompressible, electrically conducting fluid, Hartmann number.*

Mathematics Subject Classification 2000 (MSC2000) 76W05

I. INRTODUCTION

The flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators and flowmeters and have applications in nuclear reactors, filtration, geothermal systems and others. Heat transfer in developing magnetohydrodynamic Poiseuille flow and variable transport properties carried out by Alireza, S. and Sahai, V.[1]

The magnetohydrodynamic (MHD) heat transfer in a two-phase flow with the fluid in one phase being electrically conducting has been studied by Lohrasbi and Sahai [2]. The flow through the generator channel has two regions: (a) gaseous suspension region containing slag and seed particles and (b) liquid phase consisting of condensed slag. Both phases are electrically conducting and have variable transport properties.

The coal-fire MHD generator channel is subjected to an unusually sever thermal environment. Postlethwaite and Sluyter [3] present an overview of the heat transfer problems associated with a MHD generator. The fluid mechanics and the heat transfer characteristics of generator channel are significantly influenced by the presence of magnetic field. The slag layers in the walls of the channel complicate the problem. Further the thermal energy is supplied to the conducting fluid through Ohmic heating. Thus the temperature field in an electrically conducting fluid in presence of a magnetic field differs from that in a non-conducting fluid.

Malashetty and Leela [4] have studied the problem of magnetohydrodynamic flow and heat transfer in a horizontal channel in a two-phase flow. He observed that in case of open circuit problem for negative values of the electric load parameter R_e , the effect of increasing the Hartmann number M is to accelerate the velocity and to increase the temperature field in contrast to the short circuit case. Recently there have been some theoretical and experimental works on the stratified laminar flow of two immiscible liquids in a horizontal pipe. The interest in these types of problems stems from the possibility of reducing the power required to pumping oil in a pipeline by suitable addition of water. Hartmann flow of a conducting fluid in a channel between two horizontal insulating plates of infinite extent with a layer of non-conducting fluid between upper channel wall and the conducting fluid has been studied by Shail [5]. He found that an increase of the order of 30% can obtained in the flow rate of the conducting fluid for suitable ratios of the depths and viscosities of the two fluids and realistic value of the Hartmann number. Setayesh and Sahai [6] analyzed the magnetohydrodynamic heat transfer problem with variable transport properties. Malashetty and Umavathi [7] have studied the Two-Phase Magnetohydrodynamic flow and heat transfer in an inclined channel in which one phase being electrically conducting. He assumed that the transport properties of both fluids are constant. It is found that the velocity and temperature can be increased or decreased with suitable values of the ratios of viscosities, thermal conductivities, the heights and the angle of inclination. Singha and Deka [8] studied the laminar convection flow of a viscous electrically conducting incompressible fluid between two heated vertical parallel plates in presence of a uniform inclined magnetic field. It is found that with the increase of angle of inclination, velocity increases with different magnetic field strength. Further it is also observed that with the bigger strength of magnetic field variation of velocity with angle of inclination is bigger. So it is concluding that with large magnetic field strength if inclination of field is slightly changed we can expect large change in fluid velocity. The two-phase problem of magnetohydrodynamic flow and heat transfer in a parallel-plate channel is studied analytically by Singha and Deka [10]. It is observed that in case of open circuit problem for negative values of the electric load parameter R_e, the effect of increasing the Hartmann number M is to accelerate the velocity and is to increase the temperature in contrast to the short case. P. Sri Ramachandra Murty and G. Balaji Prakash [11] have done a detailed analysis on MHD Two-Fluid Flow and Heat Transfer between Two Inclined Parallel Plates in a Rotating System. Recently Hasan Nihal Zaidi, Naseem Ahmad [12] have studied MHD Convection Flow of Two Immiscible Fluids in an Inclined Channel with Heat Generation / Absorption.

In the present paper, we have studied the Magnetohydrodynamics two-phase flow and heat transfer problem in a horizontal channel and it is considered in presence of a uniform inclined magnetic field. The fluids in the two-phase were assumed to be immiscible, incompressible, steady, one-dimensional and fully developed. Further it is also assumed that the two fluids have different viscosities and thermal conductivities. In this problem, we have investigated the upper phase of the two fluids which is assumed to be electrically conducting and that of the lower phase is electrically non-conducting. The transport properties of the two fluids were taken to be constant and equal temperature. The analytical solutions for velocities, induced magnetic field and temperature distributions are obtained and are computed numerically for different heights and viscosity ratios for two fluids and for two values of electric load parameter R_e . The computed results of velocity distributions, induced magnetic field strength and the temperature distributions are plotted distances from the fixed horizontal plates and for different angle of inclinations. Analytical solutions for velocity, induced field and the temperature distributions are obtained; skin frictional factors are computed for different angle of inclinations and for different magnetic field strengths.

Our study is motivated by the work of Singha and Deka [8] who investigated one- phase flow of a viscous electrically conducting incompressible fluid between two heated vertical parallel plates in presence of a uniform inclined magnetic field. In the present work, we have studied two-phase MHD flow and heat transfer in an inclined magnetic field. In our studies it has found that the volumetric flow rate in a channel may be controlled by the orientation of applied magnetic field.

II. FORMULATION OF THE PROBLEM

The physical model shown in Fig.1, consists of two infinite parallel plates extending in the X and Z - direction. The region $0 \le y \le h_1$, is occupied by a fluid of viscosity μ_1, electrical conductivity σ_1 , and thermal conductivity k_1 and the region $-h_2 \le y \le 0$, is occupied by a layer of different (immiscible) fluid of viscosity μ_2 and thermal conductivity k_2 .The transport properties of both fluids are assumed to be constant. The fluid flows in the X direction. A uniform magnetic field of strength B_0 is applied in the direction making an angle θ to the vertical line which in turn another magnetic field $B_x(y)$ along the lines of motion.

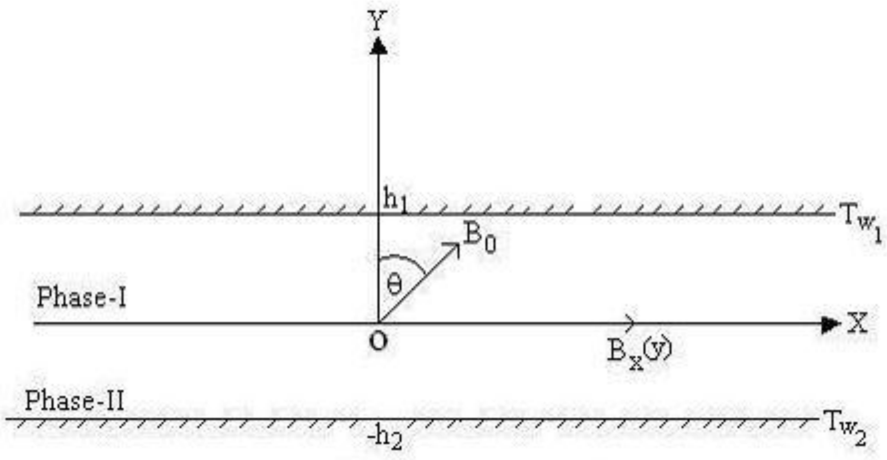

Fig. 1: Physical Configuration

The fluid velocity and the magnetic field distributions are

$$\vec{V} = [u(y), 0, 0] \text{ and}$$

$$\vec{B} = \left[B_x(y) + B_0 \cos\left(\frac{\pi}{2} - \theta\right), B_0 \cos\theta, 0 \right]$$

$$= [B_x(y) + B_0 \sin\theta, B_0 \cos\theta, 0]$$

$$= [B_x(y) + B_0 \sqrt{1-\lambda^2}, B_0 \lambda, 0]$$

where $\cos\theta = \lambda$ .The two bounding walls are maintained at constant temperature T_w .

The flow is assumed to be steady, laminar, incompressible and fully developed. The flow of both phases is assumed to be at a constant pressure gradient $P = -\frac{\partial p}{\partial x}$.

Under these assumptions as stated above, the governing equation of motion, magnetic field and equation of energy for the two phases are

$$\nabla \cdot \vec{V} = 0 . \tag{1}$$

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\left(\frac{1}{\rho}\right)\nabla p + \mu \nabla^2 \vec{V} + \frac{1}{\rho}\left(\vec{J} \times \vec{B}\right) + \vec{Z} . \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{V} \times \vec{B}) - \left(\frac{1}{\sigma \mu_e}\right)\nabla^2 \vec{B} = 0 . \tag{3}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T\right) = \kappa \nabla^2 T + \rho \nu \Phi + \frac{J^2}{\sigma} . \tag{4}$$

Here Φ represents the dissipation function given by

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{3}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

and the third term in the right hand side of equation (2) is the magnetic body force and $\vec{J}$ is the current density due to the magnetic field defined by

$$\vec{J} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right) \tag{5}$$

$\vec{Z}$ is the force due to buoyancy Z= $\beta g\left(T_0 - T\right)$ (6)

The gravitational body force $\vec{Z}$ has been neglected in the equation (2).

Using the velocity and magnetic field distribution as stated above, the equations (1) to (4) are as follows:

The equation of motion and energy for the two-phases reduces to

$$\frac{1}{\rho}P + \mu\frac{d^2u}{dy^2} - \frac{\sigma}{\rho}(E_z + uB_0\lambda)B_0\lambda = 0. \tag{7}$$

$$B_0\lambda\frac{du}{dy} + \left(\frac{1}{\sigma\mu_e}\right)\frac{d^2B_x}{dy^2} = 0. \tag{8}$$

$$\rho c_p u\frac{\partial T}{\partial x} = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma\left(E_z + uB_0\lambda\right)^2. \tag{9}$$

III. BOUNDARY CONDITIONS

The fluid and thermometric boundary conditions are unchanged by the addition of electromagnetic fields. The no slip condition requires that the velocity must vanish at the wall boundaries. In addition, the fluid velocity, sheering stress, temperature and heat flux must be continuous across the interface $y = 0$. Boundary and interface conditions on u_1, u_2 and B_x are

$$u_1(h_1) = 0,\ u_2(-h_2) = 0,\ u_1(0) = u_2(0),\ \mu_1\frac{du_1}{dy_1} = \mu_2\frac{du_2}{dy_2} \quad \text{at } y=0$$

$$B_x = 0 \quad \text{at } y = \pm h \tag{10}$$

If the walls are maintained at constant temperatures the boundary conditions on T_1 and T_2 are given by

$$T_1(h_1) = T_{w_1},\quad T_2(-h_2) = T_{w_2},\quad T_1(0) = T_2(0),$$

$$k_1\frac{dT_1}{dy_1} = k_2\frac{dT_2}{dy_2} \quad \text{at } y=0 \tag{11}$$

IV. VELOCITY AND MAGNETIC FIELD DISTRIBUTIONS

The governing equation for the velocity u_1 of phase-I can be written as

$$\frac{1}{\rho}P + \mu_1 \frac{d^2 u_1}{dy_1^2} - \frac{1}{\rho}\sigma_1 (E_z + u_1 B_0 \lambda) B_0 \lambda = 0 . \tag{12}$$

The equation governing the velocity u_2 of the non-conducting lower phase-II may be obtained by setting $\sigma_2 = 0$ in equation (7) and is given by

$$\frac{d^2 u_2}{dy_2^2} = -\frac{1}{\rho}\left(\frac{P}{\mu_2}\right). \tag{13}$$

The equation governing the magnetic field of the conducting upper phase-I can be written as

$$B_0 \lambda \frac{du_1}{dy_1} + \left(\frac{1}{\sigma \mu_e}\right)\frac{d^2 B_x}{dy_1^2} = 0 . \tag{14}$$

It is convenient to non-dimensionalize the above equations the following transformations will be used here:

$$u_1^* = \left(\frac{u_1}{\overline{\overline{u}}_1}\right), \qquad y_1^* = \frac{y_1}{h_1}, \qquad u_2^* = \left(\frac{u_2}{\overline{\overline{u}}_1}\right), \qquad y_2^* = \frac{y_2}{h_2},$$

$$G = \frac{P}{\left(\mu_1 \overline{u}_1 / h_1^2\right)}, \qquad R_e = \frac{E_z}{\overline{u}_1 B_0}, \qquad b = \frac{B_x}{B_0} . \tag{15}$$

Equations (12)-(14) becomes respectively,

$$G + \frac{d^2 u_1}{dy_1^2} - M^2 (R_e + u_1 \lambda)\lambda = 0 . \tag{16}$$

$$\left(\frac{G\alpha}{\beta^2}\right) + \frac{d^2 u_2}{dy_2^2} = 0 . \tag{17}$$

$$\frac{d^2 b}{dy_1^2} + \lambda R_m \frac{du_1}{dy_1} = 0 . \tag{18}$$

(The subscripts 1 and 2 refer to the upper and lower phases respectively).

Where M is the Hartmann number, which is the measure of the strength of the applied magnetic field,

$$M = B_0 h_1 \sqrt{\frac{\sigma_1}{\mu_1}}$$

R_m is the magnetic Reynolds number, $R_m = \overline{u_1} h_1 \sigma_1 \mu_e$

R_e is the electric field loading parameter.

α is the ratio of the viscosities of the two fluids, $\alpha = \dfrac{\mu_1}{\mu_2}$.

β is the ratio of the height of the two fluids, $\beta = \dfrac{h_1}{h_2}$.

G is the non-dimensional pressure gradient.

The boundary conditions (10) reduces to

$$u_1(+1) = 0, \qquad u_2(-1) = 0, \qquad u_1(0) = u_2(0),$$
$$\frac{du_1}{dy_1} = \left(\frac{\beta}{\alpha}\right)\frac{du_2}{dy_2} \quad \text{at} \quad y = 0$$
$$b = 0 \quad \text{at} \quad y = \pm 1 \tag{19}$$

Equations.(16)-(18) can be solved easily subject to the boundary conditions (19). The exact solution for the velocity of the electrically conducting phase is given by

$$u_1\{y_1) = U + k_1 \cosh[My_1\lambda] + k_2 \sinh[My_1\lambda]. \tag{20}$$

The corresponding solution for the velocity of the non-conducting fluid is given by

$$u_2(y_2) = k_3 + k_4 y_2 - \frac{G\alpha y_2^2}{2\beta^2}. \tag{21}$$

And the solution for the magnetic field of the electrically conducting phase is given by

$$b(y_1) = \frac{R_m\left((\cosh[M\lambda] - \cosh[M\lambda y_1])k_2 - k_1(\sinh[M\lambda y_1] - y_1\sinh[M\lambda])\right)}{M}. \tag{22}$$

Where

$$U = \frac{G}{M^2\lambda^2} - \frac{R_e}{\lambda},$$
$$M_1 = 2\beta\left(M\gamma\lambda\cosh[M\lambda] + \beta\sinh[M\lambda]\right),$$
$$k_1 = \frac{-2MU\beta\gamma\lambda + \left(G\alpha - 2U\beta^2\right)\sinh[M\lambda]}{M_1},$$
$$k_2 = -\frac{2U\beta^2 + \left(G\alpha - 2U\beta^2\right)\cosh[M\lambda]}{M_1},$$

$$k_3 = \frac{2MU\beta\gamma\lambda\left(\cosh[M\lambda]-1\right)+G\alpha\sinh[M\lambda]}{M_1},$$

$$k_4 = \frac{M\gamma\lambda\left(-2U\beta^2-(G\alpha-2U\beta^2)\cosh[M\lambda]\right)}{\beta M_1},$$

V. TEMPERATURE DISTRIBUTION

Once the velocity distributions are known the temperature distributions for the two regions are determined by solving the energy equations subject to the appropriate boundary and interface conditions.

In the present problem, it is assumed that the two walls are maintained at constant and equal temperatures. The term involving $\frac{\partial T}{\partial x}$ in the energy equation (9) drops out for such a condition. The governing equation for the temperature T_1 of the conducting phase-I is then given by

$$k_1\frac{d^2T_1}{dy_1^2}+\mu_1\left(\frac{du_1}{dy_1}\right)^2+\sigma\left(E_z+u_1B_0\lambda\right)^2=0\ . \tag{23}$$

The temperature T_2 of the non-conducting fluid in phase-II is similarly governed by the equation

$$k_2\frac{d^2T_2}{dy_2^2}+\mu_2\left(\frac{du_2}{dy_2}\right)^2=0\ . \tag{24}$$

To non-dimensionalize the equations (23) and (24) the following transformations are used

$$u_1^*=\left(\frac{u_1}{\bar{u}_1}\right),\quad y_1^*=\frac{y_1}{h_1},\quad u_2^*=\left(\frac{u_2}{\bar{u}_1}\right),\quad y_2^*=\frac{y_2}{h_2},\quad G=\frac{P}{\left(\mu_1\bar{u}_1/h_1^2\right)},$$
$$R_e=\frac{E_z}{\bar{u}_1B_0},\quad \theta_1=\frac{T_1-T_w}{\bar{u}_1^2\,\mu_1/k_1},\quad \theta_2=\frac{T_2-T_w}{\bar{u}_1^2\,\mu_1/k_1}, \tag{25}$$

Here T_w is the common wall temperature. Equations (23) and (24) then reduces to the following form

$$\frac{d^2\theta_1}{dy_1^2}+\left(\frac{du_1}{dy_1}\right)^2+M^2\left(R_e+\lambda u_1\right)^2=0\ . \tag{26}$$

$$\frac{d^2\theta_2}{dy_2^2}+\frac{\gamma}{\alpha}\left(\frac{du_2}{dy_2}\right)^2=0\ . \tag{27}$$

Here γ is the ratio of the thermal conductivities of the two fluids, $\gamma=\frac{k_1}{k_2}$,

α is the ratio of the viscosities of the two fluids, $\alpha=\frac{\mu_1}{\mu_2}$, M is the non-dimensional Hartmann number, $M = B_0 h_1 \sqrt{\frac{\sigma_1}{\mu_1}}$

In the non-dimensional form, the boundary conditions of the continuity of temperature and heat flux at the interface $y = 0$ becomes

$$\theta_1(+1) = 0\,, \quad \theta_2(-1) = 0\,, \quad \theta_1(0) = \theta_2(0)\,,$$
$$\frac{d\theta_1}{dy_1} = \frac{\beta}{\gamma}\left(\frac{d\theta_2}{dy_2}\right) \quad \text{at} \quad y = 0 \tag{28}$$

In the present case, it is assumed that $T_{w_1} = T_{w_2} = T_w$

Once the velocity distributions are known the temperature distributions for the two phases are determined using equations (26)-(28) and (20)-(21).

The solutions of equations (26) and (27) is given by

$$\theta_1(y_1) = -\frac{1}{4\lambda}(\lambda\cosh[2M\lambda y_1]k_1^2 + \lambda\cosh[2M\lambda y_1]k_2^2 + 8\sinh[M\lambda y_1]k_2k_5$$
$$+ 4\cosh[M\lambda y_1]k_1(\lambda\sinh[M\lambda y_1]k_2 + 2k_5) - 2\lambda(2k_7 + 2k_8y_1 - M^2k_5^2y_1^2)) \tag{29}$$

$$\theta_2(y_2) = k_9 + k_{10}y_2 - \frac{\gamma k_4^2 y_2^2}{2\alpha} + \frac{G\alpha\gamma k_4 y_2^3}{3\alpha\beta^2} - \frac{G\alpha^2\gamma y_2^4}{12\alpha\beta^4}\,. \tag{30}$$

Where

$$k_5 = R_e + \lambda U \quad , \quad k_6 = \frac{1}{12\alpha\beta^3(\beta+\gamma)\lambda}\,,$$

$$k_7 = k_6(3\alpha\beta^3\lambda(\beta + \gamma\cosh[2M\lambda])k_1^2 + 3\alpha\beta^3\lambda(\beta+\gamma\cosh[2M\lambda])k_2^2$$
$$+ 24\alpha\beta^3\gamma(\sinh[M\lambda] - M\lambda)k_2k_5 + 6\alpha\beta^3k_1(\gamma\lambda(\sinh[2M\lambda] - 2M\lambda)k_2$$
$$+ 4(\beta + \gamma\cosh[M\lambda]k_5) + \gamma\lambda(G\alpha^2 + 4G\alpha\beta^2k_4 + 6\beta^4k_4^2 + 6M^2\alpha\beta^3k_5^2))$$

$$k_8 = k_6(6\alpha\beta^4\lambda\sinh[M\lambda]^2k_1^2 + 6\alpha\beta^4\lambda\sinh[M\lambda]^2k_2^2 + 24\alpha\beta^3(M\gamma\lambda + \beta\sinh[M\lambda]k_2k_5$$
$$+ 6\alpha\beta^3k_1(\lambda(2M\gamma\lambda + \beta\sinh[2M\lambda])k_2 + 4\beta(\cosh[M\lambda] - 1)k_5)$$
$$- \lambda(G\alpha^2\gamma + 4G\alpha\beta^2\gamma k_4 + 6\beta^4\gamma k_4^2 - 6M^2\alpha\beta^4k_5^2))$$

$$k_9 = \gamma(6\alpha\beta^3\lambda\sinh[M\lambda]^2k_1^2 + 6\alpha\beta^3\lambda\sinh[M\lambda]^2k_2^2 + 24\alpha\beta^3(-M\lambda + \sinh[M\lambda])k_2k_5$$
$$+ 6\alpha\beta^3k_1(\lambda(-2M\lambda + \sinh[2M\lambda])k_2 + 4(\cosh[M\lambda] - 1)k_5)$$
$$+ \lambda(G\alpha^2 + 4G\alpha\beta^2k_4 + 6\beta^4k_4^2 + 6M^2\alpha\beta^3k_5^2))k_6$$

$$k_{10} = -\frac{k_6}{\beta}(\gamma(-6\alpha\beta^4\lambda\sinh[M\lambda]^2k_1^2 - 6\alpha\beta^4\lambda\sinh[M\lambda]^2k_2^2 - 24\alpha\beta^4(-M\lambda + \sinh[M\lambda])k_2k_5$$
$$- 6\alpha\beta^4k_1(\lambda(-2M\lambda + \sinh[2M\lambda])k_2 + 4(\cosh[M\lambda] - 1)k_5)$$

$$+\lambda(G\alpha^2\gamma+4G\alpha\beta^2\gamma k_4+6\beta^4\gamma k_4^2-6M^2\alpha\beta^4 k_5^2)))$$

VI. DISCUSSION OF THE RESULTS

The velocity and temperature distributions are now plotted graphically against the distances between the two plates. Both the cases are taken in to account, the case of electrical load parameter $R_e=0$ and $R_e=-1.0$. $R_e=0$ represents the short circuit case in which there is no electric field, and the flow is driven entirely by the applied pressure gradient.

In figure 2 the velocity distribution is plotted against the distances from the solid boundaries. It is clear from the figure 2 for the Phase-I(conducting fluid) at liquid boundary $y=0$ the fluid velocity $u>0$ where as at solid boundary $y=1$ it is zero. This is the evidence that there is an enhancement volumetric flow rate in the channel due to the presence of liquid boundary. This flow rate will be increased when the angle of inclination of imposed magnetic field with vertical line of the channel is increased. This observation is important for the probable application in the pumping of conducting liquid along with the immiscible liquid. This shows that beside the minimizing power requirement for pumping the question of controlling flow rate by changing imposed magnetic field direction. Here some effect is established in fig.4 for different electrical load parameter R_e .

In earlier work [3] it has been pointed out that, the pressure requirement to pump electrically conducting liquid can be reduced. In our paper it has found that the volumetric flow rate in pipe in a channel may be controlled by the orientation of applied magnetic field. The temperature distribution is shown in figure 3 and figure 5. It should be noted that the temperature distributions for the conducting phase in both cases i.e. $R_e=0$ and $R_e=-1$ have qualitatively the same characteristics as they exhibited by classical one-phase Hartmann flow [8].

The induced magnetic field distribution is shown in figure 6 and figure 7 for $(R_e=0$ and $R_e=-1)$ in conductor strength of magnetic field decreases almost linearly as we approach to the boundary wall of the channel Rate of change of induced magnetic field gradients are strong for the case $R_e=-1.0$ than for $R_e=0$.

In figure 8 and figure 9 for $(R_e=0$ and $R_e=-1)$ same effect has been observed as shown in figure 6 and figure 7.

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NOMENCLATURE

B_0 Uniform applied magnetic field

$B_x(y)$ Induced magnetic field

E_z Constant electric field in the direction of z

h_1, h_2 Heights of the two regions

$M = B_0 h_1 \sqrt{\frac{\sigma_1}{\mu_1}}$, Hartmann number

$G = \frac{P}{\left(\mu_1 \bar{u}_1 / h_1^2\right)}$, Non-dimensional Pressure gradient

T_w Common wall temperature

$R_e = \frac{E_z}{\bar{u}_1 B_0}$ Electric load parameter

u_1, u_2 Velocity of the two fluids

$\bar{u}_1$ Average velocity

x, y, z Co-ordinates in space

Greek letters

α $\frac{\mu_1}{\mu_2}$, ratio of the viscosities

β $\frac{h_1}{h_2}$, ratio of heights of two fluid

γ $\frac{k_1}{k_2}$, ratio of the thermal conductivities

$\theta_1 = \frac{(T_1 - T_w)}{(\bar{u}_1^2 \mu_1 / k_1)}$, non- dimensional temperature in phase-I

$\theta_2 = \frac{T_2 - T_w}{\bar{u}_1^2 \mu_1 / k_1}$, non- dimensional temperature in phase-II

μ_1, μ_2 Viscosities of the two fluids

σ_1, σ_2 Electrical conductivity of the two fluids

Subscripts

1, 2 refer to the upper and lower phase

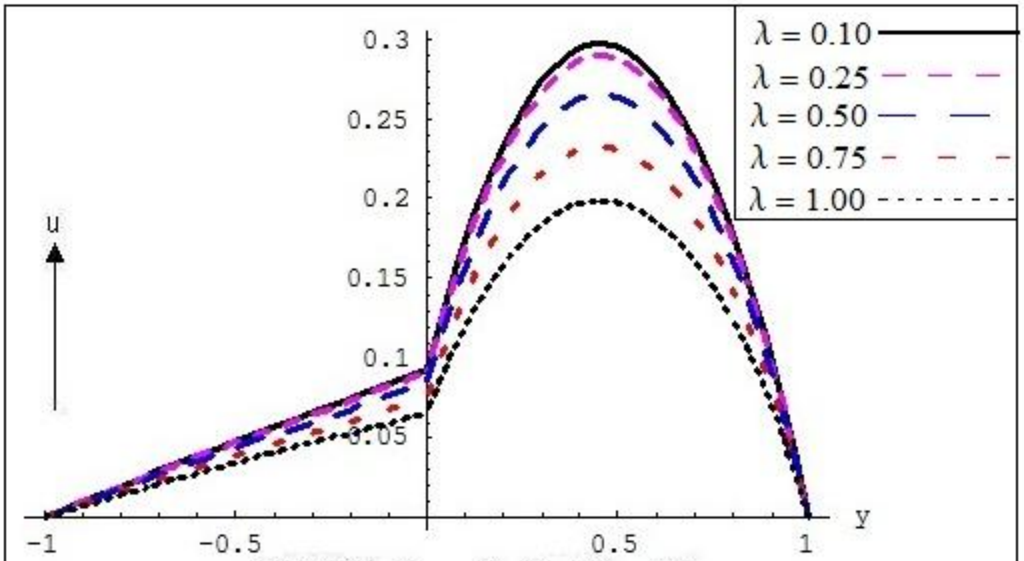

Fig .2 Velocity profiles for different λ
($R_e = 0.0,\ \alpha = 1/3;\ \beta = 10;\ \gamma = 1.0$)

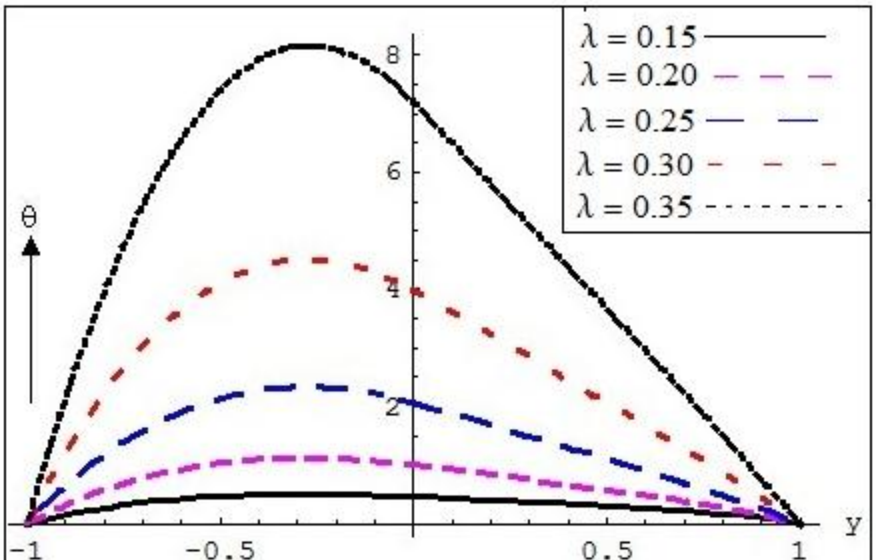

Fig .3 Temperature profiles for different λ
($R_e = 0.0,\ \alpha = 1/3;\ \beta = 10;\ \gamma = 1.0$)

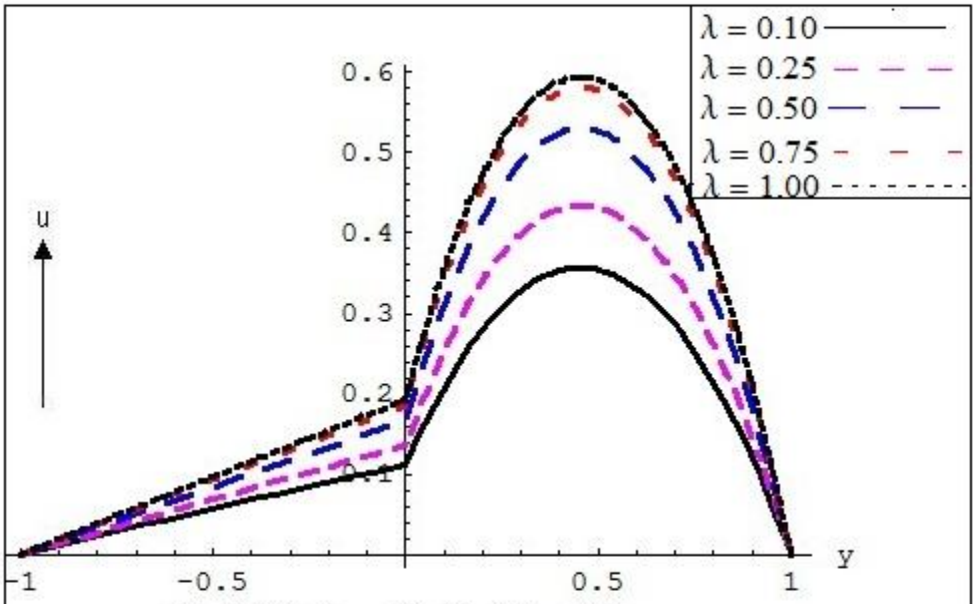

Fig .4 Velocity profiles for different λ
($R_e = -1.0,\ \alpha = 1/3;\ \beta = 10;\ \gamma = 1.0$)

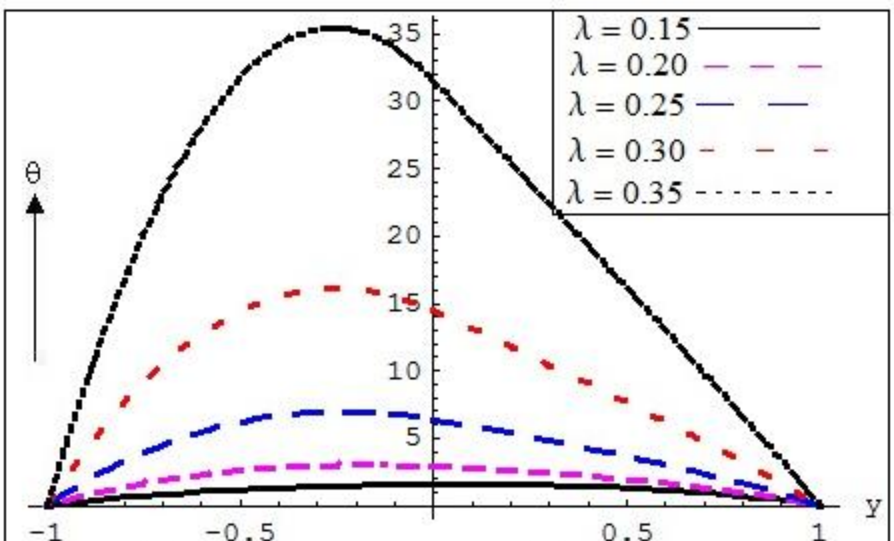

Fig .5 Temperature profiles for different λ
($R_e = -1.0,\ \alpha = 1/3;\ \beta = 10;\ \gamma = 1.0$)

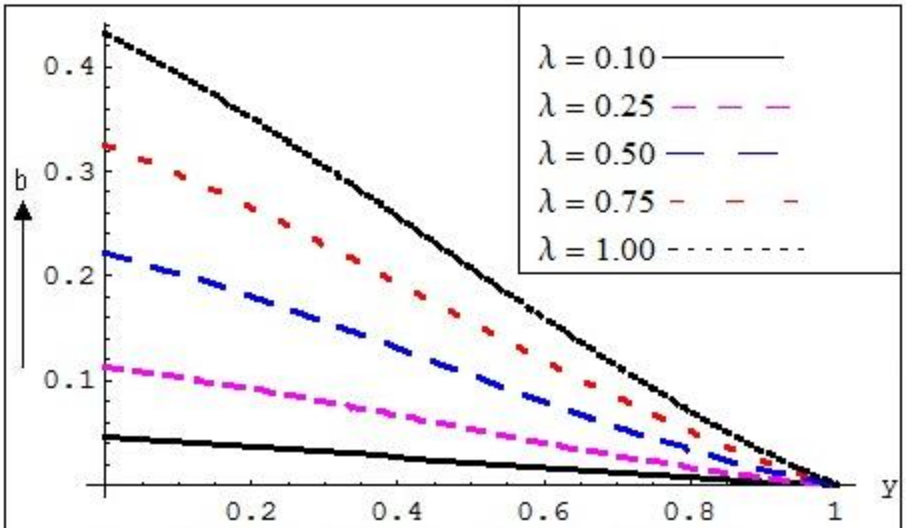

Figure 6 : Magnetic field profiles for different λ for $R_e = 0.0$, $\alpha = \frac{1}{3}$, $\beta = 10.0$, $\gamma = 1.0$, $R_m = 1.0$

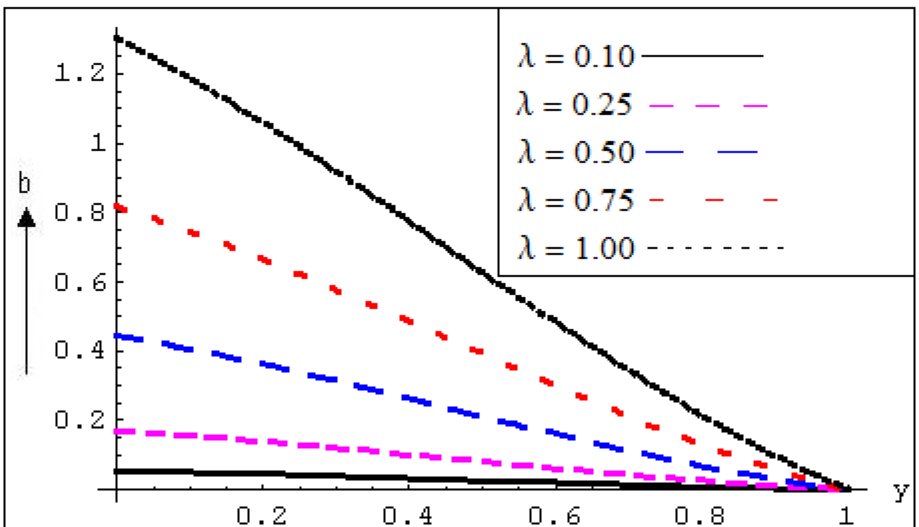

Fig .7 Magnetic field profiles for different λ $\alpha = \frac{1}{3}; \beta = 10; R_e = -1.0; R_m = 1.0$

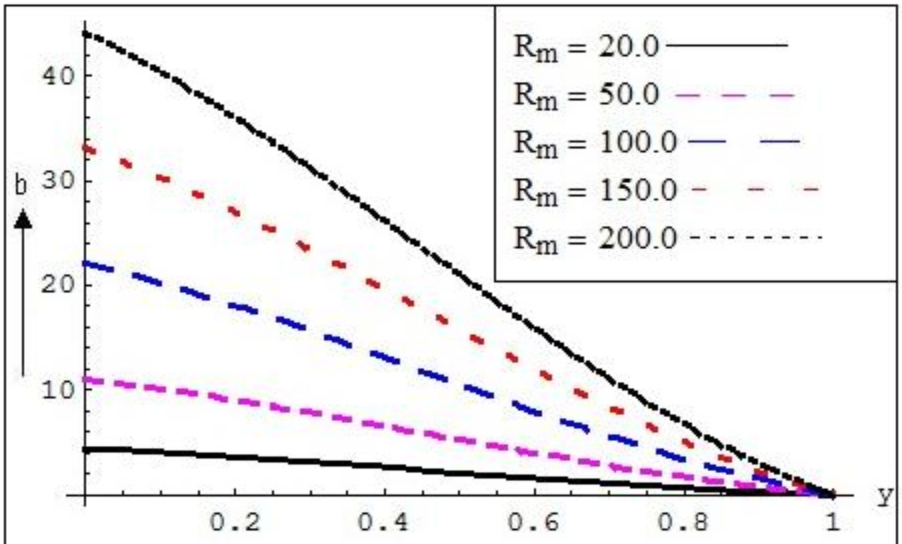

Figure 8 : Magnetic field profiles for large R_m

for $R_e = 0.0,\ \alpha = \frac{1}{3},\ \beta = 10.0,\ \gamma = 1.0,\ \lambda = 0.5$

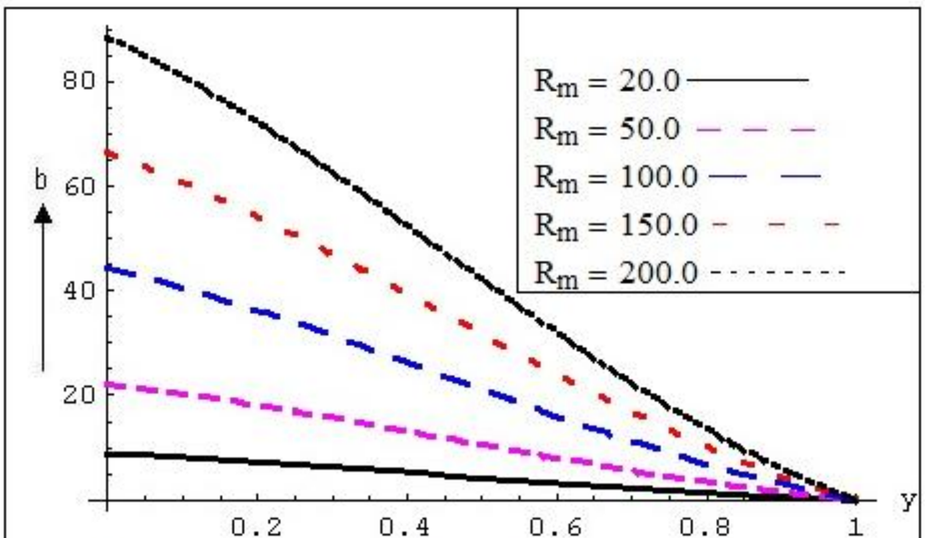

Figure 9 : Magnetic field profiles for large R_m

for $R_e = -1.0,\ \alpha = \frac{1}{3},\ \beta = 10.0,\ \gamma = 1.0,\ \lambda = 0.5$