Complex Intuitionistic Flexible Fuzzy Soft Interior Ideals Over Semigroups

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Abstract

In this paper,we introduce the notion of complex intuitionistic flexible fuzzy soft semi groups and itsCartesian product with the help of suitable example. We define direct product, ideals and prove some results.Also we investigate complex intuitionistic flexible fuzzy soft left (respectively right) ideal in semi group and study some results prove on its.

Keywords :*Soft set, fuzzy soft set, intuition sticflexiblefuzzy set, left ideal, interior ideal, union and intersection.*

I. INTRODUCTION

Intuitionistic fuzzy sets has been successfully applied in the fields of modelling imprecision [3], decision making problems [4], pattern recognition [5], computational intelligence [3] and medical diagnosis [5]. Complex fuzzy set and logic, which is the extension of fuzzy sets and logic respectively, was first proposed by Ramot et.al [13,14]. Maji et.al [8,9,10] introduced the notion of fuzzy soft sets. This work were further revised and improved by Ahmad and Kharal [7].In similarity to the case of intuitionistic fuzzy set, a complex intuitionistic fuzzy set is characterized by a complex grade of membership and complex grade of non-membership [14]. . In 2011, Neog and Sut [12] put forward some propositions regarding fuzzy soft set theory. Complex intuitionistic fuzzy sets [13] have been applied in multi attribute decision making problems combining fuzzy sets with soft sets. They defined arbitrary fuzzy soft union and intersection and proved De Morgan inclusions and De Morgan laws in fuzzy soft set theoryIntuitionistic fuzzy set as a generalization of fuzzy set by adding the degree of non-membership into the fuzzy set [16]. Thus, an intuitionistic fuzzy set is characterized by a degree of membership and a degree of non-membership. According to their definition, a complex fuzzy set is characterized by a complex grade of membership which is a combination of a traditional fuzzy degree of membership referred to as the amplitude term with the addition of an extra term, the phase term. Thereafter, Maji and his co-author [10] notion of intuitionistic fuzzy soft set which is based on the combination of introduced the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set. M. Borah et.al [2] defined disjunctive sum and difference of two intuitionistic fuzzy soft sets and studied their basic properties. In thispaper,we introduce the notion of complex intuitionistic flexible fuzzy soft semi groups and its Cartesian product with the help of suitable example. we define direct product, ideals and prove some results.Also we investigate complex intuitionistic flexible fuzzy soft left (respectively right) ideal in semi group and study some results prove on its.

II. PRELIMINARIES

Definition 2.1[Zadeh]: Let X be a set. Then a mapping that is $\mu : X \rightarrow [0,1]$ is called a fuzzy subset of X.

Definition 2.2[Subbiah et.al]: Let X be a set. Then a mapping $\mu: X \rightarrow M^*([0,1])$ is called flexible subset of X, where $M^*([0,1])$ denotes the set of all non empty subset of [0,1]

Definition 2.3[Subbiah et.al]: Let X be a non empty set. Let μ and λ be two flexible fuzzy subsets of X. Then the intersection of μ and λ denoted by $\mu\cap\lambda$ and defined by $\mu\cap\lambda=\{\min\{a,b\}/\ a\in\mu(x), b\in\lambda(x)\}$ for all $x\in X$.The union of μ and λ and denoted by $\mu\cup\lambda$ and defined by $\mu\cup\lambda=\{\max\{a,b\}/\ a\in\mu(x), b\in\lambda(x)\}$ for all $x\in X$.

Definition 2.4[Molodtsov]: Let U be an initial universe.Let P (U) be the power set of U, E be the set of all parameters and $A\subseteq E$. A soft set (f_A, E) on the universe U is defined by the set of order pairs $(f_A, E) = \{(e, f_A$

(e)): e∈ E, f_A∈ P (U)} where f_A : E → P (U) such that f_A (e) = ϕif e ∉A.Heref_A is called an approximate function of the soft set.

Example 2.5: Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set of parameters. If A = {e_1 ,e_2} ⊆ E. Let $f_A(e_1)$ = {u_1, u_2 ,u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3}. Then we write the soft set(f_A , E)= {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{ u_1,u_2,u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy. We may represent the soft set in the following form:

U	e_1	e_2	e_3
u_1	1	1	0
u_2	1	1	0
u_3	1	1	0
u_4	1	0	0

Definition2.6 [NaimCagman]: Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A → $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U), where $\tilde{P}$(U) denotes the collection of all subsets of U.

Example 2.7: Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then
(f_A, E) = {{$f_A(e_1)$ = {(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)},
f_A (e_2) = {(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)}} is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

Definition 2.8: Let U be an initial universal set and let E be set of parameters and A⊆ E. Let P (U) denotes the set of all intuitionistic fuzzy sets of U. A pair (F, A) is called an intuitionistic fuzzy soft set over U if F is a mapping given by F: A → P (U).

Example 2.9: Suppose that there are four people in the universe given by,

U = {u_1, u_2 , u_3 , u_4}and E = {e_1 ,e_2,e_3}where $e_1$stands for young , $e_2$stands for smart, $e_3$stands for middle-aged. Suppose that F(e_1)= $\left\{\frac{u_1}{(0.5,0.2)},\frac{u_2}{(0.9,0.1)},\frac{u_3}{(0.4,0.1)},\frac{u_4}{(0.0,0.5)}\right\}$, F($e_2$) = $\left\{\frac{u_1}{(0.3,0.1)},\frac{u_2}{(0.8,0.2)},\frac{u_3}{(0.0,0.4)},\frac{u_4}{(0.5,0.3)}\right\}$,F($e_3$) = $\left\{\frac{u_1}{(0.4,0.2)},\frac{u_2}{(0.7,0.3)},\frac{u_3}{(0.4,0.3)},\frac{u_4}{(0.6,0.0)}\right\}$.

Thus intuitionistic fuzzy soft set is a parameterized family of all Intuitionistic fuzzy set of U and gives us a approximate description of the object. We may represent the soft set in the following:

U	e_1	e_2	e_3
u_1	(0.5,0.2)	(0.3,0.1)	(0.4,0.2)
u_2	(0.9,0.1)	(0.8,0.2)	(0.7,0.3)
u_3	(0.4,0.1)	(0.0,0.4)	(0.4,0.3)
u_4	(0.0,0.5)	(0.5,0.3)	(0.6,0.0)

Definition 2.10: A complex fuzzy subset A, defined on a universe of discourse X, is characterized by a membership function τ_A(x) that assigns any element x∈X a complex valued grade of membership in A. The values of τ_A(x) all lie within the unit circle in the complex plane and thus all of the form P_A(x) $e^{j\mu_A(x)}$ where P_A(x) and $e^{j\mu_A(x)}$ are both real valued andP_A(x) ∈[0,1]. Here P_A(x) is termed as amplitude term and $e^{j\mu_A(x)}$ is termed as phase term.

The complex fuzzy set may be represented in the set form as A = {(x,τ_A(x)) / x∈X }. It is denoted by CFS. The phase term of complex membership function belongs to (0,2π). Now we take those forms which Ramotet.al presented in [13] to define the game of winner, neutral and lose.

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & \text{if } p_A > p_B \\ \mu_B(x) & \text{if } p_A < p_B \end{cases}.$$

This is a novel concept and it is the generalization of the concept " winner take all"for the union of phase terms.

Example 2.11: Let $X = \{x_1, x_2, x_3\}$ be a universe of discourse. Let A and B be complex fuzzy sets in X as shown below.

$$A = \{0.1e^{i(0.2)}, 0.9e^{i\frac{3\pi}{4}}, 0.9e^{i(0.5)}\}$$

$$B = \{0.2e^{i(0.5)}, 0.5e^{i\frac{\pi}{4}}, 0.8e^{i(0.7)}\}$$

$$A \cup B = \{0.7e^{i(0.6)}, 0.2e^{i\frac{3\pi}{4}}, 0.5e^{i(0.9)}\}$$

We can easily calculate the phase terms $e^{i\mu_{A\cap B}(x)}$ on the same line by winner, neutral and loser game.

Definition 2.12: A complex intuitionistic flexible fuzzy subset A of a group G is said to be a intuitionistic complex fuzzy subgroup of G if for all x ,y ∈ G, $A(xy) \geq \min\{A(x), A(y)\}$, $A(x^{-1}) \geq A(x)$ where the product x and y is denoted by xy and the inverse of x by x^{-1}. It is well known and easy to see that a complex fuzzy subgroup G satisfies $A(x) \leq A(e)$ and $A(x^{-1}) = A(x)$ for all x ∈ G, where 'e' is the identity of G.

Definition 2.13: A Complex intuitionistic flexible fuzzy soft set C = { < Pc . $\mathbf{e}^{i\delta_c}$> } on a semi group S is known as a Complex intuitionistic flexible fuzzy soft left ideal of S, if C(xy) ≥ C(y) . i.e., Pc (xy) .$e^{i\delta_{c(xy)}} \geq$ Pc (y) .$e^{i\delta_{c(y)}}$ for all x,y ∈S.

Example 2.14: Let S = { 1,2,3,4 } be a semigroup with the following multiplication table.

●	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	2	1
4	1	1	2	2

Consider a Complex intuitionistic flexible fuzzy soft set C onS are:
C = { < 1, 0.3 $e^{0.7\pi i}$>, < 1, 0.3 $e^{0.8\pi i}$>,< 1, 0.3 $e^{0.6\pi i}$>,< 1, 0.3 $e^{0.3\pi i}$> }, then C is a complex intuitionistic flexible fuzzy left ideal of S.

Definition2.15: A Complex intuitionistic flexible fuzzy soft set C = { < Pc . $\mathbf{e}^{i\delta_c}$> } on a semi group S is known as a complex intuitionistic flexible fuzzy soft right ideal of S, if
C(xy) ≥ C(x) (i.e)Pc (xy) . $e^{i\delta_{c(xy)}} \geq$ Pc (x).$e^{i\delta_{c(x)}}$ for all x,y ∈ S.

Definition 2.16: A Complex intuitionistic flexible fuzzy soft set C = { < Pc . $\mathbf{e}^{i\delta_c}$> } on a semi group S is known as a complex intuitionistic flexible fuzzy soft right ideal of S, if it both are a complex intuitionistic flexible fuzzy soft left ideal and a complex intuitionistic flexible fuzzy soft right ideal of S.

Example 2.17: Let S = { 1,2,3 } be a semigroup with the followingCayle table.

●	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Consider a Complex intuitionistic flexible fuzzy soft set C on S are:
C = { < 1, 0.7 $e^{0.7\pi i}$>, < 2, 0.3 $e^{0.2\pi i}$>,< 3, 0.4 $e^{0.7\pi i}$> }, then obviously C is a complex intuitionistic flexible fuzzy soft ideal of S.

Remark 2.18: Every Complex intuitionistic flexible fuzzy soft left (resp., right) ideal is a complex intuitionistic flexible fuzzy soft semi group. But the converse may not be true as seen in the following example.

Example 2.19: Let S = { a,b,c,d } be a semigroup with the following Cayle table.

●	a	B	c	d
A	a	A	a	A
B	a	A	a	A
C	a	A	a	B
D	a	A	b	C

Consider a complex intuitionistic flexible fuzzy soft set C on S are:
C = { < a, 0.8 e $^{0.6\ \pi i}$>, < b, 0.6 e $^{0.6\ \pi i}$>,< c, 0.8 e $^{0.5\ \pi i}$>, < d, 0.4 e $^{0.4\ \pi i}$>}, then obviously C is a complex intuitionistic flexible fuzzy soft semi group , however it is not a right ideal of S, because $\mu(cd) = \mu_c(b) = 0.6\ e^{0.6\pi i}$ **(not ≥)** $0.8e^{0.5\pi i} = \mu_c(c)$.

Definition 2.20 : A Complex intuitionistic flexible fuzzy soft set on a semi group S is known as a complex intuitionistic flexible fuzzy soft interior ideal of S, if C (xky) ≥ C (k)
i.e.)$Pc\ (xky)\ .\ e^{i\delta\ c(xky)} \geq Pc\ (k)\ .e^{\ i\delta\ c(k)}$ for all x,k, y ∈S.

Example 2.21: Let S = { a,b,c,d } be a semigroup. Then

●	A	b	C	d
a	A	a	A	a
b	A	a	A	a
c	A	a	B	a
d	A	a	B	c

C = { < a, 0.7 e $^{0.6\ \pi i}$>, < b, 0.3 e $^{0.7\ \pi i}$>,< c, 0.3 e $^{0.7\ \pi i}$>, < d, 0.0 e $^{0.0\ \pi i}$>}, then obviously C is a complex intuitionistic flexible fuzzy soft interior ideal of S.

Remark 2.22: Every complex intuitionistic flexible fuzzy soft interior ideal is a complex intuitionistic flexible fuzzy soft interior ideal. But the converse may not be true as seen in the above example. For
Left ideal:$\mu_c(dc) = \mu_c(b) = 0$ (not ≥) $0.3\ e^{0.7\pi i} = \mu_c(c)$
Right ideal: $\mu_c(dc) = \mu_c(b) = 0 \geq \mu_c(d)$. So it is a complex intuitionistic flexible fuzzy soft right ideal but not a left ideal. Hence C is not a complex intuitionistic flexible fuzzy soft ideal.

Section-3.Results:
In this section, we will discuss some important results based on the above definitions**.**
Proposition3.1 : A complex intuitionistic flexible fuzzy soft set C on a semi group S, the following are equivalent;

(i) C is a complex intuitionistic flexible fuzzy soft ideal of S.

(ii) S $\otimes$ C $\subseteq$C.

Proof: (i) $\Rightarrow$ (ii)

Assume that C is a complex intuitionistic flexible fuzzy soft ideal of S.
Let x ∈S, such that (S $\otimes$ C) (x) =0; then it is clear S $\otimes$ C $\subseteq$C, whenever there exist any two element y,k∈S such that x = yk.
Then $(Is\ \bullet Pc.\ e^{\ i\delta\ c}\ )\ (x) = \max_{(x=yk)}\ [\ \inf\ \{\ Is(y)\ \bullet Pc(k)\bullet\ e^{\ i\delta\ c}(k)\ \}]$

$\leq \max_{(x=yk)}\ [\ \inf\ \{\ 1\bullet e^{i2\pi}\bullet Pc(yk)\bullet\ e^{\ i\delta\ c}(yk)\ \}]$

$= Pc\ (x)\ \bullet\ e^{\ i\delta\ c}(x)$

(ii) $\Rightarrow$ (i)

Suppose S $\otimes$ **C** $\subseteq$C. For any element y,k∈S, let x = yk, then

$Pc(yk)\bullet\ e^{\ i\delta\ c(yk)} = Pc(k)\bullet\ e^{\ i\delta\ c(x)}$

$\geq Is(x)\bullet Pc(x)\bullet\ e^{\ i\delta\ c}(x)$

$= \max_{(x=yk)}[\ \inf\ \{\ Is(y)\ \bullet Pc(k)\bullet\ e^{\ i\delta\ c}(k)\ \}]$

$= Pc\ (x)\ \bullet\ e^{\ i\delta\ c}(x)$
Hence C is a complex intuitionistic flexible fuzzy soft left ideal of S.

Proposition 3.2 : A complex intuitionistic flexible fuzzy soft set C on a semi group S, the following are equivalent. (i) C is a complex intuitionistic flexible right ideal of S.

(ii) C $\otimes$ S$\subseteq$ C.

Proof: Proof is similar to the proposition 3.1.

Theorem 3.3: If C is a complex intuitionistic flexible fuzzy soft set of a semi group S. Then S $\otimes$ **C (resp.** C $\otimes$ **S)** is a complex intuitionistic flexible fuzzy soft left (resp. right) ideal of S.

Proof: Since $S \otimes (S \otimes C) = (S \otimes S) \otimes C) \subseteq S \otimes C$

It follows from proposition 3.1, that $S \otimes C$ is a complex intuitionistic flexible fuzzy soft left ideal of S. Similarly $C \otimes S$ is a complex intuitionistic flexible fuzzy soft right ideal of S.

Theorem 3.4: Let S be a left zero semigroup of S. If C is a complex intuitionistic flexible fuzzy soft left ideal of S, then C(x) = C(y) for all x,y∈ S.

Proof: Let x,y∈S . Then xy=x and yx = y then

$Pc(x) \bullet e^{i\delta_c(x)} = Pc(xy) \bullet e^{i\delta_c(xy)}$

$\geq Pc(y) \bullet e^{i\delta_c(y)}$

$= Pc(yx) \bullet e^{i\delta_c(yx)}$

$= Pc(x) \bullet e^{i\delta_c(x)}$

Therefore C(x) = C(y) for all x,y∈S.

Theorem3.5 : Let S be a right zero semigroup of S. If C is a complex intuitionistic flexible fuzzy soft right ideal of S, then C(x) = C(y) for all x,y∈ S .

Proof: Proof is similar to the theorem 3.3.

Theorem 3.6: Let C be a complexintuitionistic flexible fuzzy soft left ideal of a semi group S. If the set of idempotent elements of S form a left zero semi group of S, then C(x) = C(y) for all idempotent elements x and y of S .

Proof: Let Idm (S) be the set of all idempotent elements of S and assume that Idm(S) is a left zero semigroup of S.

For any x,y∈ Idm (S) ,

We have xy = x and yx = y then

$Pc(x) \bullet e^{i\delta_c(x)} = Pc(xy) \bullet e^{i\delta_c(xy)}$

$\geq Pc(y) \bullet e^{i\delta_c(y)}$

$= Pc(yx) \bullet e^{i\delta_c(yx)}$

$= Pc(y) \bullet e^{i\delta_c(y)}$

Therefore C(x) = C(y) for all x,y ∈Idm (S).

Proposition 3.7: If S is a semi group**,** then the intersection of two complex intuitionistic flexible fuzzy soft left (respectively right) ideals of S is a complex intuitionistic flexible fuzzy soft left (respectively right) ideal of S.

Proof: Let C_1 and C_2 be any two complex intuitionistic flexible fuzzy soft left ideals of semigroup S, and x,y∈S . Then

$(Pc_1 \bullet e^{i\delta_{c1}} \bigcap Pc_2 \bullet e^{i\delta_{c2}})(xy) = \inf\{ Pc_1(xy) \bullet e^{i\delta_{c_1}(xy)}, Pc_2(xy) \bullet e^{i\delta_{c_2}(xy)}\}$

$\geq \inf\{ Pc_1(y) \bullet e^{i\delta_{c_1}(xy)}, Pc_2(xy) \bullet e^{i\delta_{c_2}(y)}\}$

$= (Pc_1 \bullet e^{i\delta_{c_1}} \bigcup Pc_2 \bullet e^{i\delta_{c_2}})(y).$

Thus $C_1 \bigcap C_2$ is a complex intuitionistic flexible fuzzy soft left ideal of S.

Proposition 3.8: If S is a semi group, then the union of two complex intuitionistic flexible fuzzy soft left (resply., right) ideals of S is a complex intuitionistic flexible fuzzy soft left (respectively right) ideal of S.

Proof: Let C_1 and C_2 be any two complex intuitionistic flexible fuzzy soft left ideals of S, and x,y∈S. Then

$(Pc_1 \bullet e^{i\delta_{c1}} \bigcup Pc_2 \bullet e^{i\delta_{c2}})(xy) = \sup\{ Pc_1(xy) \bullet e^{i\delta_{c_1}(xy)}, Pc_2(xy) \bullet e^{i\delta_{c_2}(xy)}\}$

$\geq \sup\{ Pc_1(y) \bullet e^{i\delta_{c_1}(xy)}, Pc_2(xy) \bullet e^{i\delta_{c_2}(y)}\}$

$= (Pc_1 \bullet e^{i\delta_{c_1}} \bigcap Pc_2 \bullet e^{i\delta_{c_2}})(y)$

Thus $C_1 \bigcup C_2$ is a complex intuitionistic flexible fuzzy soft left ideal of S.

Theorem 3.9: If C_1 and C_2 be a complex intuitionistic flexible fuzzy soft left and right ideals of a semi group S, respectively, then $C_1 \otimes C_2 \subseteq C_1 \bigcap C_2$.

Proof: Let C_1 is complex intuitionistic flexible fuzzy soft right ideal and C_2 is any complex intuitionistic flexible fuzzy soft left ideal of S. Then by proposition 3.7 and proposition 3.8, we have
$C_1 \otimes C_2 \subseteq C_1 \otimes S \subseteq C_1$ and $C_1 \otimes C_2 \subseteq S \otimes C_2$.

Section-4.Characterization of Complex intuitionistic flexible soft set on Regular Semi group

Theorem 4.1: Let C be a complex intuitionistic flexible soft left ideal of a semigroup S of the set of all idempotent elements of S form a left Zero Semi group of S.Then C(x) = C(y) for all idempotent elements of x and y of S.
Proof: Let us assume that Idm(s) be the set of all idempotent elements of S and is a left Zero Semi group of S. For any x,y∈ Idm (S), we have xy = x and yx = y, Thus

$$
\begin{aligned}
Pc(x)\bullet e^{i\,\delta c(x)} &= Pc(xy)\bullet e^{i\,\delta c(xy)} \\
&\geq Pc(y)\bullet e^{i\,\delta c(y)} \\
&= Pc(yx)\bullet e^{i\,\delta c(yx)} \\
&\geq Pc(x)\bullet e^{i\,\delta c(x)} \\
&= Pc(y)\bullet e^{i\,\delta c(y)} \quad \text{and}
\end{aligned}
$$

$$
\begin{aligned}
\Im_C(x)\bullet e^{i\,\Delta c(x)} &= \Im_C(xy)\bullet e^{i\,\Delta c(xy)} \\
&\leq \Im_C(y)\bullet e^{i\,\Delta c(y)} \\
&= \Im_C(yx)\bullet e^{i\,\Delta c(yx)} \\
&\leq \Im_C(x)\bullet e^{i\,\Delta c(x)} \\
&= \Im_C(y)\bullet e^{i\,\Delta c(y)}
\end{aligned}
$$

Thus C(x) = C(y) for all x,y∈ Idm (S).
Theorem 4.2 : Let C be a complex intuitionistic flexible soft right ideal of a semi group S. If the set of all idempotent elements of S from a right zero semigroup of S, then C(x) = C(y) for all idempotents elements of x and y of S.

Proof: Proof is similar to the theorem 4.1.

Proposition 4.3: If S is a semi group, then the following properties are hold.

(i) The intersection of two complex intuitionistic flexible soft semi groups of S is a complex intuitionistic flexible soft semi group of S.

(ii) The intersection of two complex intuitionistic flexible left (respectively right) ideals of S is a complex intuitionistic flexible soft left (respectively right) ideal of S.

Proof: Let
$C_1 = \{ C_{1T} = Pc_1 \bullet e^{i\,\delta c1}, C_{1F} = rc1 \bullet e^{i\,\Delta c1} \}$ and
$C_2 = \{ C_{2T} = Pc_2 \bullet e^{i\,\delta c2}, C_{2F} = rc2 \bullet e^{i\,\Delta c2} \}$ be any two complex intuitionistic flexible soft semi groups of S
Let x, y ∈S.Then

$$
\begin{aligned}
&(Pc_1\bullet e^{i\,\delta c_1} \cap Pc_2\bullet e^{i\,\delta c_2})(xy) \\
&= \inf\{Pc_1(xy)\bullet e^{i\,\delta c_1(xy)}, Pc_2(xy)\bullet e^{i\,\delta c_2(xy)}\} \\
&\geq \inf\{\inf\{Pc_1(x)\bullet e^{i\,\delta c_1(x)}, Pc_1(y)\bullet e^{i\,\delta c_1(y)}\}, \\
&\qquad \inf\{Pc_2(x)\bullet e^{i\,\delta c_2(x)}, Pc_2(y)\bullet e^{i\,\delta c_2(y)}\}\} \\
&= \inf\{\inf\{Pc_1(x)\bullet e^{i\,\delta c_1(x)}, Pc_2(x)\bullet e^{i\,\delta c_2(x)}\}, \\
&\qquad \inf\{Pc_1(y)\bullet e^{i\,\delta c_1(y)}, Pc_2(y)\bullet e^{i\,\delta c_2(y)}\}\} \\
&= \inf\{(Pc_1\bullet e^{i\,\delta c_1} \cap Pc_2\bullet e^{i\,\delta c_2})(x), \\
&\qquad (Pc_1\bullet e^{i\,\delta c_1} \cap Pc_2\bullet e^{i\,\delta c_2})(y)\}
\end{aligned}
$$

$$
\begin{aligned}
&(rc_1\bullet e^{i\,\Delta c_1} \cup rc_2\bullet e^{i\,\Delta c_2})(xy) \\
&= \sup\{rc_1(xy)\bullet e^{i\,\Delta c_1(xy)}, rc_2(xy)\bullet e^{i\,\Delta c_2(xy)}\} \\
&\leq \sup\{\sup\{rc_1(x)\bullet e^{i\,\Delta c_1(x)}, rc_1(y)\bullet e^{i\,\Delta c_1(y)}\}, \\
&\qquad \sup\{rc_2(x)\bullet e^{i\,\Delta c_2(x)}, rc_2(y)\bullet e^{i\,\Delta c_2(y)}\}\}
\end{aligned}
$$

$= \sup\{\sup\{rc_1(x) \bullet e^{i\Delta c_1(x)}, rc_2(x) \bullet e^{i\Delta c_2(x)}\},$
$\sup\{rc_1(y) \bullet e^{i\Delta c_1(y)}, rc_2(y) \bullet e^{i\Delta c_2(y)}\}\}$
$= \sup\{(rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(x),$
$(rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(y)\}$

Thus $C_1 \cap C_2$ is a complex intuitionistic flexible soft semi group of S.

(ii) Let C_1 and C_2 be any two complex intuitionistic flexible soft left ideals of semi group S, and x, y $\in$S. Then

Let x, y $\in$S, then

$(Pc_1 \bullet e^{i\delta c_1} \cap Pc_2 \bullet e^{i\delta c_2})(xy)$
$= \inf\{Pc_1(xy) \bullet e^{i\delta c_1(xy)}, Pc_2(xy) \bullet e^{i\delta c_2(xy)}\}$
$\geq \inf\{Pc_1(y) \bullet e^{i\delta c_1(y)}, Pc_2(y) \bullet e^{i\delta c_2(y)}\}$
$= (Pc_1 \bullet e^{i\delta c_1} \cap Pc_2 \bullet e^{i\delta c_2})(y)$.

and

$(rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(xy)$
$= \sup\{rc_1(y) \bullet e^{i\Delta c_1(y)}, rc_2(y) \bullet e^{i\Delta c_2(y)}\}$
$= (rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(y)$.

Thus $\mathbf{C_1 \cap C_2}$ is a complex intuitionistic flexible soft semi left ideal of S. The intersection of two complex intuitionistic flexible soft right ideals can be proved in a similar manner.

Proposition 4.4: If S be a semi group. Then the following properties are hold.

(i) The Union of two complex intuitionistic flexible soft semi groups of S is a complex intuitionistic flexible soft semi group of S.

(ii) The Union of two complex intuitionistic flexible left (respectively right) ideals of S is a Complex intuitionistic flexible soft left (respectively right) ideal of S.

Theorem 4.5: If C_1 and C_2 be a complex intuitionistic flexible soft left and right ideals of a semi group S, respectively, then $\mathbf{C_1 * C_2 \subseteq C_1 \cap C_2}$.

Proof: If C_1 is complex intuitionistic flexible soft right ideal and C_2 is any complex left intuitionistic flexible soft ideal of S.

We have $\mathbf{C_1 * C_2 \subseteq C_{1*S} \subseteq C_1}$ and $\mathbf{C_1 * C_2 \subseteq S * C_2 \subseteq C_2}$.

Hence $\mathbf{C_1 * C_2 \subseteq C_1 \cap C_2}$.

Theorem 4.6: If S is regular semi group, then $\mathbf{C_1 * C_2 \subseteq C_1 \cap C_2}$ for every complex intuitionistic flexible soft right ideal of C_1 and C_2 of S.

Proof: Let α be any element of S. Since S is regular, there exists an element x $\in$S such that α×α .

Hence we have

$(Pc_1 \bullet e^{i\delta c_1} \cap Pc_2 \bullet e^{i\delta c_2})(\alpha)$
$= (\max)_{\alpha=y}\rho\ \{\inf\{Pc_1(y) \bullet e^{i\delta c_1(y)}, Pc_2(\rho) \bullet e^{i\delta c_2(\rho)}\}\}$
$= (\max)_{\boldsymbol{\alpha x \alpha = y}}\rho\ \{\inf\{Pc_1(y) \bullet e^{i\delta c_1(y)}, Pc_2(\rho) \bullet e^{i\delta c_2(\rho)}\}\}$
$\geq \inf\{Pc_1(\alpha) \bullet e^{i\delta c_1(\alpha)}, Pc_2(\alpha) \bullet e^{i\delta c_2(\alpha)}\}$
$\geq \inf\{Pc_1(\alpha) \bullet e^{i\delta c_1(\alpha)}, Pc_2(\alpha) \bullet e^{i\delta c_2(\alpha)}\}$
$= (Pc_1 \bullet e^{i\delta c_1} \cap Pc_2 \bullet e^{i\delta c_2})(\alpha)$

and

$(rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(xy)$
$= (\min)_{\alpha=y}\rho\ \{\sup\{rc_1(y) \bullet e^{i\Delta c_1(y)}, rc_2(\rho) \bullet e^{i\Delta c_2(\rho)}\}\}$
$= (\min)_{\boldsymbol{\alpha x \alpha = y}}\rho\ \{\sup\{rc_1(y) \bullet e^{i\Delta c_1(y)}, rc_2(\rho) \bullet e^{i\Delta c_2(\rho)}\}\}$

$$\leq \sup \{rc_1(\alpha x) \bullet e^{i\Delta c_1(\alpha x)}, rc_2(\alpha) \bullet e^{i\Delta c_2(\alpha)}\}$$
$$\leq \sup \{rc_1(\alpha) \bullet e^{i\Delta c_1(\alpha)}, rc_2(\alpha) \bullet e^{i\Delta c_2(\alpha)}\}$$
$$= (rc_1 \bullet e^{i\Delta c_1} \cup rc_2 \bullet e^{i\Delta c_2})(\alpha).$$

We have $\mathbf{C_1 * C_2 \supseteq C_1 \cap C_2}$ andtheorem4.5, $C_1 * C_2 \subseteq C1 \cup C_2$ is true from theorem4.5.
Hence $\mathbf{C_1 * C_2 = C_1 \cap C_2}$.

Theorem 4.7: For any non-empty subset H of a semi group S, We have

- **(i)** H is a Semi group of S if and only if the characteristic complex intuitionistic flexible soft set $\mathbf{C_H}$ of H in S is a complex bi fuzzy soft semi group of S.
- **(ii)** H is a left (respectively right) ideal of S if and only if the characteristic complex intuitionistic flexible soft set $\mathbf{C_H}$ of H in S is a complex intuitionistic flexible soft left (respectively right) ideal of S.

Proof: The Proof is straight forward.

Theorem 4.8 : For every complex intuitionistic flexible soft right ideal C_1 and every Complex intuitionistic flexible soft left ideal C_2 of a semi group S, if $\mathbf{C_1 * C_2 = C_1 \cap C_2}$, then S is regular.

Proof: Assume that $\mathbf{C_1 * C_2 = C_1 \cap C_2}$ for every complex intuitionistic flexible soft right ideal C_1 and every complex intuitionistic flexible soft left ideal C_2 of a semi group S. Let M and N be any right and left ideal of S, respectively.

In order to see that $\mathbf{M \cap N \subseteq MN}$ holds. Let α be only element of $M \cap N$.Then the characteristic complex intuitionistic flexible soft sets C_M and C_N are complex intuitionistic flexible soft right ideal and a complex intuitionistic flexible left ideal of S, respectively, by theorem 4.6.
At follows from the hypothesis.

$$\begin{aligned}
T_{CMN}(\alpha) &= (T_{CM} \bullet T_{CN})(\alpha) \\
&= (T_{CM} \cap T_{CN})(\alpha) \\
&= T_{CM \cap N}(\alpha) \\
&= 1.e^{i2\pi} \\
F_{CMN}(\alpha) &= (F_{CM} \bullet F_{CN})(\alpha) \\
&= (F_{CM} U F_{CN})(\alpha) \\
&= F_{CMN}(\alpha) \\
&= 0
\end{aligned}$$

So that $\alpha \in M_N$. Thus $\mathbf{M \cap N \subseteq MN.}$ Since the inclusion in the other direction always holds. We obtain that $R \cap L \subseteq RL$. It follows that S is regular.

Theorem 4.9: If C_1 and C_2 are two complex intuitionistic flexible soft sets of a semi group S, then

(i) $(C_1 \cap C_2)_x = C_{1x} \cap C_{2x}$

(ii) $(C_1 \cup C_2)_x = C_{1x} \cup C_{2x}$

(iii) $C_{1x} + C_{2x} \subseteq (C_1 + C_2)x$

Proof:

Let $f(x) = \sum_{i=0}^{n} a_i x^i$ be any element of S.Then

$$\begin{aligned}
\text{(i) } \delta(C_1 \cap C_2)_x (f(x)) &= \delta_{(C1 \cap C2)x} \\
&= (\inf)_i \{\delta_{(C1 \cap C2)}(ai)\} \\
&= (\inf)_i \{\delta_{C1}(ai), \inf\{\delta_{C2}(ai)\}\} \\
&= (\inf)_i \{\delta_{C1}(ai), \inf\{\delta_{C2}(ai)\}\} \\
&= (\inf) \{\delta_{C1x}(f(x)), \delta_{C2x}(f(x)\} \\
&= \delta_{(C1x \cap C2x)}(f(x)).
\end{aligned}$$

Similarly, we can shown that $\Delta\ (C_1 \cap C_2)_x = \Delta_{(C1 \cap C2)x}(f(x))$
Hence $(C_1 \cap C_2)_x = C_{1x} \cap C_{2x}$

(ii) $\delta\,(C_1 \bigcup C_2)_x\,(f(x)) = \delta_{(C1 \bigcup C2)x}$

$= (\inf)_i \{\delta_{(C1 \bigcup C2)}(ai)\}$

$= (\inf)_i \{\sup\{\delta_{C1}(ai), \delta_{C2}(ai)\}\}$

$\geq (\sup) \{(\inf)i\{\delta_{C1}(ai), \delta_{C2}(ai)\}\}$

$= (\sup)\{(\inf)i\,\delta_{C1}(ai), (\inf)i\,\delta_{C2}(ai)\}$

$= (\sup) \{\delta_{C1x}(f(x)), \delta_{C2x}(f(x))\}$

$= \delta_{(C1x \bigcup C2x)}(f(x))$.

Similarly, we can show that $\Delta\,(C_1 \bigcup C_2)_x = \Delta_{(C1 \bigcap C2)x}(f(x))$

Hence $(C_1 \bigcup C_2)_x \supseteq = C_{1x} \bigcup C_{2x}$

Now $\delta_{C1x + C2x}(f(x)) = \underset{f(x)= g(x)+h(x)}{\text{Sup}} \left\{ \inf\{\delta\,Ax(g(x)), \delta\,Bx\,(h(x))\}\, g(x) \right\}$

$$= \sum_{i=0}^{p} b_i x^i \ \ h(x)$$

$$= \sum_{i=0}^{p} C_i x^i$$

$= \underset{f(x)= g(x)+h(x)}{\sup}\left\{\inf\{\delta\,c1(bi), \delta\,c2\,(ci)\}\right\}$

$= \underset{a_i=b_i + c_i}{\sup}\left\{\inf\{(\delta\,c_1(bi), \delta\,c_2(ci)\}\right\}$

$\leq \underset{a_i=b_i+c_i}{(\inf)_i \max}\left\{\inf\{\delta\,c_1(bi), \delta\,c_2(ci)\}\right\}$

$= \delta\,(c_1+c_2)x\ (f(x))$.

Similarly, we can show that $\Delta_{c1x+c2x}\ (f(x)) = \Delta_{(c1+c2)x}\ (f(x))$.

CONCLUSION

In this paper, we introduce the notion of complex intuitionistic flexible fuzzy soft semi groups and itsCartesian product with the help of suitable example. We define direct product, ideals and prove some results.Also we investigate complex intuitionistic flexible fuzzy soft left (respectively right) ideal in semi group and study some results prove on its.

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