A New Property of the Roots of an Equation

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Abstract

If all roots of the "n" degree equation $a_0x^n+a_1x^{(n-1)}+a_2x^{(n-2)}+\ldots+a_n=0$*[Where* $n\in\mathbb{N}$ *and* $a_0, a_1, \ldots, a_n$ *are real numbers] are real numbers then,*

$$(n-1)a_1^2 \geq 2na_0a_2$$

Keywords - *Titu's Lemma, Tchebycheff's inequality,Roots,Equation*

I. INTRODUCTION

Apart from some well known properties of equations in this article I am proposing a new condition related to the roots of an "n" degree equation.

II. PROPOSAL

If all roots of the "n" degree equation $a_0x^n+a_1x^{(n-1)}+a_2x^{(n-2)}+\ldots+a_n=0$[Where n is an natural number and $a_0, a_1, \ldots, a_n$ are real numbers] are real numbers then,

$$(n-1)a_1^2 \geq 2na_0a_2$$

III. PROOF

Proof-I

Let, roots of the equation $a_0x^n+a_1x^{(n-1)}+a_2x^{(n-2)}+\ldots+a_n=0$ [Where n is a Natural number and $a_0, a_1, \ldots, a_n$ are real numbers] are b_i[i=1,2,…,n]

We know that,

$$\sum_{i=1}^{n} b_i = -\frac{a_1}{a_0}$$

We also know that,

$$\sum_{1\leq i<j\leq n} b_ib_j = \frac{a_2}{a_0}$$

So, from the two equation we get,

$$\sum_{i=1}^{n} b_i^2 = \left(\sum_{i=1}^{n} b_i\right)^2 - 2\sum_{1\leq i<j\leq n} b_ib_j = \frac{(a_1^2-2a_0a_2)}{a_0^2}$$

If all $b_1, b_2, \ldots, b_n$ are real numbers then,

$$\sum_{i=1}^{n}\left(\frac{b_i^2}{1}\right) \geq \frac{\left(\sum_{i=1}^{n} b_i\right)^2}{\sum_{i=1}^{n} 1}$$ [By using **Titu's Lemma**]

$$\Rightarrow \quad \sum_{i=1}^{n} b_i^2 \geq \frac{\left(\sum_{i=1}^{n} b_i\right)^2}{n}$$

$$\Longrightarrow \quad n\frac{(a_1^2-2a_0a_2)}{a_0^2} \geq \left(-\frac{a_1}{a_0}\right)^2$$

By simplifying this we get,

$$\Longrightarrow \quad (n-1)a_1^2 \geq 2na_0a_2$$

Proof-2

If all $b_1, b_2, \ldots, b_n$ are real numbers then,

$$\left(\sum_{i=1}^{n} b_i\right)^2 \leq n\sum_{i=1}^{n} b_i^2$$ [This is a corollary of **Tchebycheff's inequality.**]

Or, $\left(-\frac{a_1}{a_0}\right)^2 \leq n\frac{(a_1^2-2a_0a_2)}{a_0^2}$

Or, $a_1^2 \leq (na_1^2 - 2na_0a_2)$

Or, $(n-1)\, a_1^2 \geq 2na_0a_2$

IV. CONCLUSION

Thus apart from some well known properties of equations I have proposed a new property related to the roots of an “n” degree equation.

V. ACKNOWLEDGEMENTS

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REFERENCES

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