Composition of (α, β) – Derivations on BH-Algebras

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Abstract:

Motivated by some results on derivations on rings and the generalization of BCK and BCI algebra, in this paper, we define (α, β) derivations on BH-algebras and investigate some important results.

Keywords: *BH-Algebras, Derivations on BH-Algebras, (α, β)-derivations on BH-Algebras.*

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I. INTRODUCTION

BCK and BCI algebras are two new classes of algebras introduced by Imai and Isaki[2,3,4]. Jun Y. B, Roh E. M and Kim H. S [6] introduced the notion of BH-Algebras. In 2004, Y. B. Jun and X. L. Xin [5] introduced the notion of derivations on BCI-Algebras. Since then, many authors worked on the notion of derivations on several algebras such as BCI-Algebras[1], B-Algebras[7], BP-Algebras [8] and d-Algebras[9,10]. Motivated by this paper we introduce the notion of Composition of (α, β)-Derivations on BH-Algebras.

II. PRELIMINARIES

In this section we recall some basic definitions that are required in our work.

Definition 2.1[3] Let X be a set with a binary operation * and a constant 0. Then (X, *, 0) is called a BCk-algebra if it satisfies the following axioms:

1. $x * x=0$
2. $0 * x = x$
3. $((x * y) * (x * z)) * (z * y) = 0$
4. $(x * (x * y)) * y = 0$
5. $x * y = 0$ and $y * x = 0$ implies $x = y$ for all $x, y, z \in X$.

Definition 2.2 [4] Let X be a set with a binary operation * and a constant 0. Then (X, *, 0) is called a BCI-algebra if it satisfies the following axioms:

1. $((x * y) * (x * z)) * (z * y) = 0$
2. $(x * (x * y)) * y = 0$
3. $x * x = 0$
4. $x * y = 0$ and $y * x = 0$ implies $x = y$ for all $x, y, z \in X$.

Definition 2.3 [6] Let X be a set with a binary operation * and a constant 0. Then (X, *, 0) is called a BH-algebra if it satisfies the following axioms:

1. $x * x = 0$
2. $x * 0 = x$
3. $x * y = 0$ and $y * x = 0$ implies $x = y$ for all $x, y \in X$.

Example 2.4 [6] Let X = {0, 1, 2} be a set with the following Cayley table:

*	0	1	2
0	0	1	2

1	1	0	2
2	2	2	0

Definition 2.5 [6] Let S be a nonempty subset of a BH-algebra X, then S is called BH - subalgebra on X if $x * y \in S$ for all $x, y \in S$.

Example 2.6 [6] Let X = {0, 1, 2, 3} be a BH- Algebra in which the operation * is defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Definition 2.7[6] Let X be a BH- Algebra and $I (\neq 0) \subset X$. I is called an BH - ideal of X if it satisfies: for all $x, y \in X$

1. $0 \in I$
2. $x * y \in I$ and $y \in I$ imply $x \in I$

Example 2.8[6] Let X = {0, 1, 2} be a BH- Algebra with the following Cayley table:

*	0	1	2
0	0	0	1
1	1	0	2
2	2	2	0

Definition 2.9[6] Let X be a BH - Algebra. A map $g : X \to X$ is a left - right derivation on X. If g satisfies the identity,

$$g(xy) = g(x)y \wedge xg(y) \text{ for all } x, y \in X.$$

If g satisfies the identity,

$$g(xy) = xg(y) \wedge g(x)y \text{ for all } x, y \in X.$$

Then g is a right - left derivation on X.

Notation 2.10 For any x, y in a BH – Algebra (X, *, 0) we write x * y simply by xy. For elements x and y of a BH- Algebra X, denoted $x \wedge y = y (yx)$.

Definition 2.11 A mapping α of a BH – Algebra X into itself is called an endomorphism if

$$\alpha (x * y) = \alpha(x) * \alpha(y)$$

Definition 2.12 Let X be a BH-Algebra. A map $g_{(\alpha, \beta)} : X \to X$ is called a left - right (α, β) derivation on X. If it satisfies the identity,

$$g_{(\alpha, \beta)}(xy) = g_{(\alpha, \beta)}(x)\, \alpha(y) \wedge \beta(x)\, g_{(\alpha, \beta)}(y) \text{ for all } x, y \in X.$$

If $g_{(\alpha, \beta)}$ satisfies the identity

$$g_{(\alpha, \beta)}(xy) = \alpha(x)\, g_{(\alpha, \beta)}(y) \wedge g_{(\alpha, \beta)}(x)\, \beta(y) \text{ for all } x, y \in X.$$

Then $g_{(\alpha, \beta)}$ is called a right - left (α, β) derivation on X.

Definition 2.13 Let (X, *, 0) be a BH-Algebra and $x \in X$. Define $X * x = \{y * x / y \in X\}$. X is said to be an edge BH-Algebra if for any $x \in X$, $X * x = \{ 0, x \}$.

Example 2.14 Let X= { 0, 1, 2, 3} be a BH - Algebra with the operation * is defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	3	0

Define $g_{(\alpha,\beta)}: X \to X$ by

$$g_{(\alpha,\beta)}(x) = 0 \text{ if } x = 0, 1, 2$$
$$= 3 \text{ if } x = 3$$

Define an endomorphisms α and β on X as follows:

$$\alpha(x) = 0 \text{ if } x = 0, 1, 2$$
$$= 2 \text{ if } x = 3$$
$$\beta(x) = 0 \text{ if } x = 0, 1, 2$$
$$= 3 \text{ if } x = 3$$

$g_{(\alpha,\beta)}$ is a left - right (α, β) derivation . But in this case $g_{(\alpha,\beta)}$ is not a (r, l) - (α, β)- derivation because

$$g_{(\alpha,\beta)}(3,1) = g_{(\alpha,\beta)}(3) = 3$$

on the other hand

$$\alpha(3)\, g_{(\alpha,\beta)}(1) \wedge g_{(\alpha,\beta)}(3)\, \beta(1) = 0.$$

III. COMPOSITION OF (α, β)-DERIVATIONS ON BH- ALGEBRAS

In this section we define the notion of the composition of two self maps on BH-Algebras and we prove some results.

Definition 3.1 Let X be a BH - Algebra and $g_{(\alpha,\beta)}$, $g'_{(\alpha,\beta)}$ are two self map, we define $(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}) : X \to X$ by

$$g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}(x) = g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)) \text{ for all } x \in X.$$

Theorem 3.2 Let X be a BH - Algebra and $g_{(\alpha,\beta)}$, $g'_{(\alpha,\beta)}$ are two (l, r)- (α, β)- derivations on X such that $\alpha^2 = \alpha$. Then $g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}$ is a (l, r) - (α, β) derivation on X.

Proof: Let x, y ϵ X. Then

$$g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}(xy) = g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(xy))$$
$$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\,\alpha(y)) \wedge (\beta(x)\, g'_{(\alpha,\beta)}(y))$$
$$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\,\alpha(y))$$
$$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x))\,\alpha(\alpha(y)) \wedge \beta\,(g'_{(\alpha,\beta)}(x))\, g_{(\alpha,\beta)}(\alpha(y))$$
$$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x))(\alpha(\alpha(y))) \text{ because } x\,(xy) = y$$
$$= (g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x)\,\alpha^2(y)$$
$$= (g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x)\,\alpha(y),$$

we get

$$= \beta(x)(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(y)(\beta(x)(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(y)((g_{(\alpha,\beta)}\} \circ g'_{(\alpha,\beta)})(x)\,\alpha(y)))$$
$$= (g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})\,(x)\,\alpha(y) \wedge \beta(x)\,((g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})\,(y))$$

Theorem 3.3 Let X be a BH - Algebra. Let $g_{(\alpha,\beta)}$ and $g'_{(\alpha,\beta)}$ are (r, l) - (α, β)-derivations on X such that $\alpha^2 = \alpha$. Then $g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}$ is also a (r, l) derivation on X.

Proof:

$(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(xy) = g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(xy))$
$= g_{(\alpha,\beta)}(\alpha(x)\, g'_{(\alpha,\beta)}(y) \wedge g'_{(\alpha,\beta)}(x)\, \beta(y))$
$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\, \beta(y)(\, g'_{(\alpha,\beta)}(x)\, \beta(y)(\alpha(x)\, g'_{(\alpha,\beta)}(y))))$
$= g_{(\alpha,\beta)}(\alpha(x)\, g'_{(\alpha,\beta)}(y))$ because $(y(y(x)))=x)$
$= \alpha(\alpha(x))\, g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(y)) \wedge g_{(\alpha,\beta)}(\alpha(x))\, \beta(g'_{(\alpha,\beta)}(y))$
$= \alpha^2(x)\, g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(y)) \wedge g_{(\alpha,\beta)}(\alpha(x))\beta(g'_{(\alpha,\beta)}(y))$ because $(\alpha^2 = \alpha)$
$= \alpha(x)\, g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(y)) \wedge g_{(\alpha,\beta)}(\alpha(x))\, \beta(g'_{(\alpha,\beta)}(y))$
$= \alpha(x)\, g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(y))$ because $(y(yx) = x)$
$=\alpha(x)(\, g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(y)$
$= (g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x)\, \beta(y)((\, g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x)\, \beta(y)$
$(\alpha(x)(\, g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)})(y)))$
$= \alpha(x)(\, g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(y) \wedge (g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x)\, \beta(y)$

Hence $g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)}$ is a (r, l) - (α, β)- derivation on X.

Theorem 3.4 Let (X, * , 0) be a BH- Algebra and $g_{(\alpha,\beta)}$, $g'_{(\alpha,\beta)}$ are (α, β) derivations on X, such that $\alpha(x) = x$ for all $x \in X$. Then $g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)} = g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)}$.

Proof:

Let X be a BH- Algebra and $g_{(\alpha,\beta)}$, $g'_{(\alpha,\beta)}$ are the (α, β) derivations on X.
Since $g'_{(\alpha,\beta)}$ is a (l, r) - (α, β) derivation on X.
Then

$(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(xy) = g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(xy))$ for all $x, y \in X$
$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\, \alpha(y) \wedge \beta(x)\, g'_{(\alpha,\beta)}(y))$
$= g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\, \alpha(y))$ because $(y(y(x))) = x)$.

But $g'_{(\alpha,\beta)}$ is a (r, l)- (α, β) derivation on X.
Now

$g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x)\, \alpha(y)) = \alpha(g'_{(\alpha,\beta)}(x))\, g_{(\alpha,\beta)}(\alpha(\, y)) \wedge g_{(\alpha,\beta)}(g'_{(\alpha,\beta)}(x))\, \beta(\alpha(y))$
$= \alpha(g'_{(\alpha,\beta)}(x))\, g_{(\alpha,\beta)}(\alpha(y))$
$= g'_{(\alpha,\beta)}(x)\, g_{(\alpha,\beta)}(y)$ because $[\alpha(x) = x]$.

Hence

$(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(xy) = g'_{(\alpha,\beta)}(x)\, g_{(\alpha,\beta)}(y)$ for all $x, y \in X$ ....(1)
Also since $g_{(\alpha,\beta)}$ is a (r, l)- (α, β)- derivation on X.
Then

$(g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)})(xy) = g'_{(\alpha,\beta)}(g_{(\alpha,\beta)}\}(xy))$ for all $x, y \in X$.
$= g'_{(\alpha,\beta)}(\alpha(x)\, g_{(\alpha,\beta)}(y)) \wedge g_{(\alpha,\beta)}(x)\, \beta(y\,))$
$= g'_{(\alpha,\beta)}(\alpha(x)\, g_{(\alpha,\beta)}(y))$

Since $g'_{(\alpha,\beta)}$ is (l, r) - (α, β)- derivation on X,

$(g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)})(xy) = g_{(\alpha,\beta)}(\alpha(x)\, g_{(\alpha,\beta)}(y))$
$= g'_{(\alpha,\beta)}(\alpha(x))\, \alpha(g_{(\alpha,\beta)}(y)) \wedge \beta(\alpha(x))\, g'_{(\alpha,\beta)}(g_{(\alpha,\beta)}(y))$
$= g'_{(\alpha,\beta)}(\alpha(x))\, \alpha(g_{(\alpha,\beta)}(y))$
$= g'_{(\alpha,\beta)}(x)\, g_{(\alpha,\beta)}(y)$ because $(\alpha(x) = x)$........(2)

From (1) and (2), we have
$(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(xy) = (g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)}(xy))$ for all $x, y \in X$.
By putting $y = 0$ we get for all $x \in X$

$$(g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)})(x) = (g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)})(x)$$

Which implies $g_{(\alpha,\beta)} \circ g'_{(\alpha,\beta)} = g'_{(\alpha,\beta)} \circ g_{(\alpha,\beta)}$.

Definition 3.5 Let X be a BH - Algebra and $g_{(\alpha,\beta)}$, $g'_{(\alpha,\beta)}$ be two self maps on X. We define $g_{(\alpha,\beta)} \bullet g'_{(\alpha,\beta)} : X \rightarrow X$ as $(g_{(\alpha,\beta)} \bullet g'_{(\alpha,\beta)})(x) = g_{(\alpha,\beta)}(x)\, g'_{(\alpha,\beta)}(x)$ for all $x \in X$.

Notation 3.6 The set of all (α, β) left- right derivations on X, is denoted by Der(X).

Definition 3.7 Let $g_{(\alpha, \beta)}$, $g'_{(\alpha, \beta)}$ in Der(X). Define the binary operation $\wedge$ as
$(g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(x) = g_{(\alpha, \beta)}(x) \wedge g'_{(\alpha, \beta)}(x)$

Theorem 3.8 Let X be a BH - Algebra and $g_{(\alpha, \beta)}$, $g'_{(\alpha, \beta)}$ are (α, β) left -right derivations on X. Then $g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}$ is also a (α, β)- (l, r)- derivation on X.
Proof:

Let X be a BH - Algebra and $g_{(\alpha, \beta)}$, $g'_{(\alpha, \beta)}$ are (α, β) left -right derivation on X.
To Prove:

$(g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy) = (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(x)\ \alpha(y) \wedge \beta(x)((\ g_{(\alpha, \beta)} \wedge g'($

$$
\begin{aligned}
(g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy) &= g_{(\alpha, \beta)}(xy) \wedge g'_{(\alpha, \beta)}(xy) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y) \wedge \beta(x)\ g_{(\alpha, \beta)}(y)) \wedge (g'_{(\alpha, \beta)}(x)\ \alpha(y) \wedge \beta(x)\ g'_{(\alpha, \beta)}(y)) \\
&= ((\beta(x)\ g_{(\alpha, \beta)}(y)))((\beta(x)\ g_{(\alpha, \beta)}(y))(\ g_{(\alpha, \beta)}(x)\alpha(y))) \wedge\ (\beta(x)\ g'_{(\alpha, \beta)}(y) \\
&\qquad (\beta(x)\ g'_{(\alpha, \beta)}(y)\ (g'_{(\alpha, \beta)}(x)\ \alpha(y)))) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y)) \wedge (g'_{(\alpha, \beta)}(\alpha(y))) \text{ because } (x(xy) = y) \\
&= g'_{(\alpha, \beta)}(\alpha(y))(\ g'_{(\alpha, \beta)}(\alpha(y))(\ g_{(\alpha, \beta)}(x)\alpha(y))) \\
&= g_{(\alpha, \beta)}(x)\ \alpha(y)
\end{aligned}
$$

$$
\begin{aligned}
(g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy) &= (g'_{(\alpha, \beta)}(x)(\ g'_{(\alpha, \beta)}(x)(\ g_{(\alpha, \beta)}(x))))\ \alpha(y) \\
&= (g_{(\alpha, \beta)}(x) \wedge g'_{(\alpha, \beta)}(x))\ \alpha(y) \\
&= (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(x)\ \alpha(y) \\
&= \beta(x)(\ g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(y)\ ((\beta(x)(\ g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(y))\ (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(x)\ \alpha(y)) \\
&= (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(x)\ \alpha(y) \wedge \beta(x)(\ g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(y)
\end{aligned}
$$

This shows that $g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}$ is a (α, β)- (l, r) derivation on X.

oTheorem 3.9 Let X be a BH - Algebra. If $g_{(\alpha, \beta)}$, $g'_{(\alpha, \beta)}$, $g''_{(\alpha, \beta)}$ in Der(X).
Then $g_{(\alpha, \beta)} \wedge (g'_{(\alpha, \beta)} \wedge g''_{(\alpha, \beta)}) = (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}) \wedge g''_{(\alpha, \beta)}$.
Proof:

Let X be a BH - Algebra and $g_{(\alpha, \beta)}$, $g'_{(\alpha, \beta)}$, $g''_{(\alpha, \beta)}$ are (α, β) - (l, r)- derivation on X.

Now,
$$
\begin{aligned}
((g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}) \wedge g''_{(\alpha, \beta)})(xy) &= (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy) \wedge g''_{(\alpha, \beta)}(xy) \\
&= g''_{(\alpha, \beta)}(xy)(\ g''_{(\alpha, \beta)}(xy)(\ g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy)) \\
&= (g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)})(xy) \text{ because } x(xy) = y \\
&= g_{(\alpha, \beta)}(xy) \wedge g'_{(\alpha, \beta)}(xy) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y) \wedge \beta(x)\ g_{(\alpha, \beta)}(y)) \wedge (g'_{(\alpha, \beta)}(x)\ \alpha(y) \wedge \\
&\qquad \beta(x)\ g'_{(\alpha, \beta)}(y)) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y)) \wedge (g'_{(\alpha, \beta)}(x)\ \alpha(y)) \\
&= g'_{(\alpha, \beta)}(x)\ \alpha(y)\ (g'_{(\alpha, \beta)}(x)\ \alpha(y)(\ g_{(\alpha, \beta)}(x)\ \alpha(y))) \\
&= g_{(\alpha, \beta)}(x)\ \alpha(y) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)
\end{aligned}
$$

Also consider the following

$$
\begin{aligned}
(g_{(\alpha, \beta)} \wedge (g'_{(\alpha, \beta)} \wedge g''_{(\alpha, \beta)}))(xy) &= g_{(\alpha, \beta)}(xy) \wedge (g'_{(\alpha, \beta)} \wedge g''_{(\alpha, \beta)})(xy) \\
&= g_{(\alpha, \beta)}(xy) \wedge (g'_{(\alpha, \beta)}(xy) \wedge g''_{(\alpha, \beta)}(xy)) \\
&= g_{(\alpha, \beta)}(xy) \wedge (g''_{(\alpha, \beta)}(xy)\ \ (g''_{(\alpha, \beta)}(xy)\ g'_{(\alpha, \beta)}(xy))) \\
&= g_{(\alpha, \beta)}(xy) \wedge (g'_{(\alpha, \beta)}(xy)) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y) \wedge g_{(\alpha, \beta)}(y)) \wedge (g'_{(\alpha, \beta)}(x)\ \alpha(y) \wedge g'_{(\alpha, \beta)}(y)) \\
&= (g_{(\alpha, \beta)}(x)\ \alpha(y) \wedge g'_{(\alpha, \beta)}(x)\ \alpha(y)) \\
&= g'_{(\alpha, \beta)}(x)\ \alpha(y)\ (g'_{(\alpha, \beta)}(x)\ \alpha(y)(\ g_{(\alpha, \beta)}(x)\ \alpha(y))) \\
&= g_{(\alpha, \beta)}(x)\ \alpha(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)
\end{aligned}
$$

From (3) and (4) we get, $(g_{(\alpha, \beta)} \wedge (g'_{(\alpha, \beta)} \wedge g''_{(\alpha, \beta)}))(xy) = ((g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}) \wedge g''_{(\alpha, \beta)})(xy)$
put y = 0, we have
$(g_{(\alpha, \beta)} \wedge (g'_{(\alpha, \beta)} \wedge g''_{(\alpha, \beta)}))(x) = ((g_{(\alpha, \beta)} \wedge g'_{(\alpha, \beta)}) \wedge g''_{(\alpha, \beta)})(x)$
this implies that

$g_{(\alpha,\beta)} \wedge (g'_{(\alpha,\beta)} \wedge g''_{(\alpha,\beta)}) = (g_{(\alpha,\beta)} \wedge g'_{(\alpha,\beta)}) \wedge g''_{(\alpha,\beta)}$

Hence the theorem.

Theorem 3.10 Let (α, β) be two endomorphisms and $g_{(\alpha,\beta)}$ be a self map on BH - Algebra X such that $g_{(\alpha,\beta)}(x) = \alpha(x)$ for all x ϵ X. Then $g_{(\alpha,\beta)}$ is a (α, β) derivation on X.

Proof:

Let us take $g_{(\alpha,\beta)}(x) = \alpha(x)$ for all x ϵ X. Since x, y ϵ X, we have xy ϵ X.\

We have

$$\begin{aligned} g_{(\alpha,\beta)}(xy) &= \alpha(xy) \\ &= \alpha(x)\,\alpha(y) \\ &= g_{(\alpha,\beta)}(x)\,\alpha(y) \\ &= \beta(x)\,g_{(\alpha,\beta)}(y)\,(\beta(x)\,g_{(\alpha,\beta)}(y)(\,g_{(\alpha,\beta)}(x)\,\alpha(y))) \\ &= g_{(\alpha,\beta)}\,\alpha(y) \wedge \beta(x)\,g_{(\alpha,\beta)}(y) \end{aligned}$$

$g_{(\alpha,\beta)}$ is (l, r) - (α, β)- derivation.

Similarly, we take $g_{(\alpha,\beta)}(y) = \alpha(y)$ for all x ϵ X.

$$\begin{aligned} g_{(\alpha,\beta)}(x\,y) &= \alpha(xy) \\ &= \alpha(x)\,\beta(y) \\ &= \alpha(x)\,g_{(\alpha,\beta)}(y) \\ &= g_{(\alpha,\beta)}(x)\,\beta(y)(\,g_{(\alpha,\beta)}(x)\,\beta(y)\,(\alpha(x)\,g_{(\alpha,\beta)}(y))) \\ &= \alpha(x)\,g_{(\alpha,\beta)}(y) \wedge g_{(\alpha,\beta)}(x)\,\beta(y) \end{aligned}$$

$g_{(\alpha,\beta)}$ is (α, β)-(r, l)- derivation. Hence $g_{(\alpha,\beta)}$ is a (α, β)- derivation on X.

Theorem 3.11 Let $g_{(\alpha,\beta)}$ be a (α, β)- derivation on BH – Algebra X. Then the following result holds. i $g_{(\alpha,\beta)}(x)\,\alpha(x) = \alpha(x)\,g_{(\alpha,\beta)}(x)$

ii $g_{(\alpha,\beta)}(y)\,\alpha(y) = \alpha(y)\,g_{(\alpha,\beta)}(y)$

iii $g_{(\alpha,\beta)}(x)\,\alpha(y) = \alpha(x)\,g_{(\alpha,\beta)}(y)$

Proof:

i. Let $g_{(\alpha,\beta)}$ be a (α, β)- derivation on BH - Algebra X.

Since $g_{(\alpha,\beta)}$ is a (α, β)- derivation on X, because $g_{(\alpha,\beta)}$ is a (l, r)- (α, β)- derivation as well as (r, l)- (α, β) derivation.

Considering $g_{(\alpha,\beta)}$ as (l, r)- (α, β) derivation .

$$\begin{aligned} g_{(\alpha,\beta)}(0) &= g_{(\alpha,\beta)}(x\,x) \\ &= g_{(\alpha,\beta)}(x)\,\alpha(x) \wedge \beta(x)\,g_{(\alpha,\beta)}(x) \\ &= g_{(\alpha,\beta)}(x)\,\alpha(x) \end{aligned}$$

$g_{(\alpha,\beta)}(0) = g_{(\alpha,\beta)}(x)\,\alpha(x)$..............(7)

Similarly consider $g_{(\alpha,\beta)}$ as a (r, l) - derivation

$$\begin{aligned} g_{(\alpha,\beta)}(0) &= g_{(\alpha,\beta)}(x\,x) \\ &= \alpha(x)\,g_{(\alpha,\beta)}(x) \wedge g_{(\alpha,\beta)}(x)\,\beta(x) \\ &= \alpha(x)\,g_{(\alpha,\beta)}(x) \end{aligned}$$

$g_{(\alpha,\beta)}(0) = \alpha(x)\,g_{(\alpha,\beta)}(x)$ . . . .(8)

From (7) and (8) it follows that $g_{(\alpha,\beta)}(x)\,\alpha(x) = \alpha(x)\,g_{(\alpha,\beta)}(x)$.

ii. Interchanging the role of x and y, we get

$g_{(\alpha,\beta)}(y)\ \alpha(y) = \alpha(y)\,g_{(\alpha,\beta)}(y)$.

iii. When $g_{(\alpha,\beta)}$ is a (l, r) - (α, β) derivation, then

$$\begin{aligned} g_{(\alpha,\beta)}(xy) &= g_{(\alpha,\beta)}(x)\,\alpha(y) \wedge \beta(x)\,g_{(\alpha,\beta)}(y) \\ &= \beta(x)\,g_{(\alpha,\beta)}(y)\,(\beta(x)\,g_{(\alpha,\beta)}(y)(\,g_{(\alpha,\beta)}(x\,)\,\alpha(y) \\ &= g_{(\alpha,\beta)}(x)\,\alpha(y) \quad . . . . (9) \end{aligned}$$

Also using (r, l)- (α, β) derivation, we have

$$\begin{aligned} g_{(\alpha,\beta)}(xy) &= \alpha(x)\,g_{(\alpha,\beta)}(y) \wedge g_{(\alpha,\beta)}(x)\,\alpha(y) \\ &= g_{(\alpha,\beta)}(x)\,\alpha(y)\,(g_{(\alpha,\beta)}(x)\,\alpha(y)\,(\alpha(x)\,g_{(\alpha,\beta)}(y))) \\ &= \alpha(x)\,g_{(\alpha,\beta)}(y). . . . .(10) \end{aligned}$$

From (9) and (10) we get, $g_{(\alpha, \beta)}(x)\, \alpha(y) = \alpha(x)\, g_{(\alpha, \beta)}(y)$

IV. CONCLUSION

In this work we have studied BH - Algebra, an algebraic structure that arises from the study of algebraic formulations of propositional logic.Taking different theorems or statements of propositional logic, different algebraic structures could obtained. The BH - Algebra is one such algebra. We find its applications in processing algebras. In order to study the infinitesimal changes in different processes of those applications, we applied the notion of derivation on a BH - Algebra. Further study is required to investigate how these concepts could be applied to the field of computers for processing informations.

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