$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)}]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $n_2,\cdots,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2,\cdots,0\leqslant m_r,0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=m^{(k)}+1,\cdots,p^{(k)});(k=1,\cdots,r);$

$\mathrm{D}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=n^{(k)}+1,\cdots,q^{(k)});(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r).$

5) $c_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, n^{(k)}); (k = 1, \cdots, r); d_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, m^{(k)}); (k = 1, \cdots, r).$

$a_{kji_k} \in \mathbb{C}; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r).$

$b_{kji_k} \in \mathbb{C}; (j = 1, \cdots, q_{i_k}); (k = 2, \cdots, r).$

$d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r).$

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r).$

The contour $L_k$ is in the $s_k (k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where $\sigma$ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}} \left( 1 - a_{2j} + \sum_{k=1}^{2} \alpha_{2j}^{(k)} s_k \right) (j = 1, \cdots, n_2), \Gamma^{A_{3j}} \left( 1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k \right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}} \left( 1 - a_{rj} + \sum_{i=1}^{r} \alpha_{rj}^{(i)} \right) (j = 1, \cdots, n_r), \quad \Gamma^{C_j^{(k)}} \left( 1 - c_j^{(k)} + \gamma_j^{(k)} s_k \right) (j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$ to the right of the contour $L_k$ and the poles of $\Gamma^{D_j^{(k)}} \left( d_j^{(k)} - \delta_j^{(k)} s_k \right) (j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour $L_k$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi$ where

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)} \gamma_j^{(k)} - \tau_{i^{(k)}} \left( \sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)} \delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)} \gamma_{ji^{(k)}}^{(k)} \right)$$

$$-\tau_{i_2} \left( \sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha_{2ji_2}^{(k)} + \sum_{j=1}^{q_{i_2}} B_{2ji_2} \beta_{2ji_2}^{(k)} \right) - \cdots - \tau_{i_r} \left( \sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha_{rji_r}^{(k)} + \sum_{j=1}^{q_{i_r}} B_{rji_r} \beta_{rji_r}^{(k)} \right) \tag{1.4}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0( |z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r} ), max( |z_1|, \cdots, |z_r| ) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0( |z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r} ), min( |z_1|, \cdots, |z_r| ) \rightarrow \infty$ where $i = 1, \cdots, r :$

$\alpha_i = \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right]$ and $\beta_i = \max_{1 \leqslant j \leqslant n^{(i)}} Re \left[ C_j^{(i)} \left( \frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right]$

**Remark 1.**
If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

**Remark 2.**
If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [6].

**Remark 3.**
If $A_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=A_{rji_r}=B_{rji_r}=1$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}$ $=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [5].

**Remark 4.**
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [7,8].

In your investigation, we shall use the following notations.

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U=0,n_2;0,n_3;\cdots;0,n_{r-1};V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)} \tag{1.11}$$

$$X=p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)} \tag{1.12}$$

## 2. Required results.

In this section, we give two finite integrals. These results will utilized in the following section.

**Lemma 1.** ([3], Erdelyi, p. 450-452)

$$\int_0^{\frac{\pi}{2}} e^{\omega(a+b)\theta}(\sin\theta)^{a-1}(\cos\theta)^{b-1}\mathrm{d}\theta = e^{\frac{1}{2}\omega\pi a}\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \tag{2.1}$$

provided $\min\{Re(a), Re(b)\} > 0$

**Lemma 3 .** ([4] MacRobert, 1960-1961)

$$\int_0^1 \frac{x^{p-1}(1-x)^{q-1}}{[a'x+b'(1-x)]^{p+q}}{}_2F_1\left(e,f;p;\frac{a'x}{a'x+b'(1-x)}\right)\mathrm{d}x = \frac{\Gamma(p)\Gamma(q)\Gamma(p+q-e-f)}{a'^p b'^q\Gamma(p+q-e)\Gamma(p+q-f)} \tag{2.3}$$

provided $Re(p) > 0, Re(q) > 0, Re(p+q-e-f) > 0, |a'x+b'(1-x)] \neq 0, 0 \leqslant x \leqslant 1.$

## 3. Main integral.

In this section we evaluate a general double finite integrals.

**Theorem.**

$$\int_0^1\int_0^{\frac{\pi}{2}} e^{\omega(a+b)\theta)}(\sin\theta)^{a-1}(\cos\theta)^{b-1}x^{p-1}(1-x)^{q-1}(a'x+b'(1-x))^{-p-q}{}_2F_1\left(c,d;a;\frac{a'x}{a'x+b'(1-x)}\right)$$

$$\gimel\left(z_1 e^{\omega(a_1+b_1)\theta}(\sin\theta)^{a_1}(\cos\theta)^{b_1}\frac{(a'x)^{c_1}[b'(1-x)]^{d_1}}{[a'x+b'(1-x)]^{-c_1-d_1}},\cdots,z_r e^{\omega(a_r+b_r)\theta}(\sin\theta)^{a_r}(\cos\theta)^{b_r}\frac{(a'x)^{c_r}[b'(1-x)]^{d_r}}{[a'x+b'(1-x)]^{-c_r-d_r}}\right)$$

$$\mathrm{d}x d\theta = \frac{e^{\omega\pi\frac{a'}{2}}}{a'^p b'^q}\gimel^{U;0,n_r+5:V}_{X;p_{i_r}+5,q_{i_r}+3,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1 e^{\omega\pi\frac{a_1}{2}} & \mathbb{A};(1-a;a_1,\cdots,a_r;1),(1-b;b_1,\cdots,b_r;1), \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r e^{\omega\pi\frac{a_r}{2}} & \mathbb{B};\mathbf{B},(1-a-b;a_1+b_1,\cdots,a_r+b_r;1), \end{array}\right.$$

$$\left.\begin{array}{c} \text{(1-p;c}_1,\cdots,c_r;1),(1-q;d_1,\cdots,d_r;1),(1-p-q+c+d;c_1+d_1,\cdots,c_r+d_r;1),\mathbf{A}:A \\ \cdot \\ \cdot \\ \text{(1-p-q+c;c}_1+d_1,\cdots,c_r+d_r;1),(1-p-q+d;c_1+d_1,\cdots,c_r+d_r;1):B \end{array}\right) \tag{3.1}$$

provided

$a_i, b_i, c_i, d_i > 0 (i=1,\cdots,r),\ \ a, b$ are no zero integers $a'x+b'(1-x) \neq 0, 0 \leqslant x \leqslant 1$

$$Re(a) + \sum_{i=1}^r a_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0.\ \ Re(b) + \sum_{i=1}^r b_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0.$$

$$R(p) + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0. Re(q) + \sum_{i=1}^r d_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0.$$ and

$$\left|arg\left(z_1 e^{\omega(a_i+b_i)\theta}(\sin\theta)^{a_i}(\cos\theta)^{b_i}\frac{(a'x)^{c_i}[b'(1-x)]^{d_i}}{[a'x+b'(1-x)]^{-c_i-d_i}}\right)\right| < \frac{1}{2}A_i^{(k)}\pi \text{ where } A_i^{(k)} \text{ is defined by (1.4).}$$

Proof

To establish the theorem 1, expressing the multivariable Gimel-function in the Mellin-Barnes multiple integrals contour with the help of (1.1) and interchanging the order of integrations, which is justified under the conditions mentioned above, we get (say I)

$$I = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$$

$$\left[\int_0^{\frac{\pi}{2}} e^{\omega(a+b+\sum_{i=1}^r(a_i+b_i)s_i)\theta)}(\sin\theta)^{a+\sum_{i=1}^r a_i s_i-1}(\cos\theta)^{b+\sum_{i=1}^r b_i s_i-1}\mathrm{d}\theta\right]$$

$$\left[\int_0^1 x^{p-1}(1-x)^{q-1}(a'x+b'(1-x))^{-p-q}{}_2F_1\left(c,d;a;\frac{a'x}{a'x+b'(1-x)}\right)\mathrm{d}x\right] \tag{3.2}$$

Now Evaluating the inner integrals with the help of lemmae 1and 2 repectively and interpreting the resulting expression in Mellin-Barnes multiple integrals contour with the help of (1.1), we obtain the desired theorem 1.

## 4. Special case.

In this section, taking $a_i = -b_i(i = 1,\cdots,r)$ in the above theorem, we get

**Corollary**.

$$\int_0^1\int_0^{\frac{\pi}{2}} e^{\omega(a+b)\theta)}(\sin\theta)^{a-1}(\cos\theta)^{b-1}x^{p-1}(1-x)^{q-1}(a'x+b'(1-x))^{-p-q}{}_2F_1\left(c,d;a;\frac{a'x}{a'x+b'(1-x)}\right)$$

$$\gimel\left(z_1 e^{\omega(a_1+b_1)\theta}(\tan\theta)^{a_1}\frac{(a'x)^{c_1}[b'(1-x)]^{d_1}}{[a'x+b'(1-x)]^{-c_1-d_1}},\cdots,z_r e^{\omega(a_r+b_r)\theta}(\tan\theta)^{a_r}\frac{(a'x)^{c_r}[b'(1-x)]^{d_r}}{[a'x+b'(1-x)]^{-c_r-d_r}}\right)$$

$$\mathrm{d}x d\theta = \frac{e^{\omega\pi\frac{a'}{2}}}{a'^p b'^q\Gamma(a+b)}\gimel^{U;0,n_r+4:V}_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}\left(\begin{array}{c} \mathrm{z}_1 e^{\omega\pi\frac{a_1}{2}} \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{\omega\pi\frac{a_r}{2}} \end{array}\middle|\begin{array}{c} \mathbb{A};(1-a;a_1,\cdots,a_r;1), \\ \cdot \\ \cdot \\ \mathbb{B};\mathbf{B}, \end{array}\right.$$

$$\left.\begin{array}{c}(\text{1-p;c}_1,\cdots,c_r;1),(1-q;d_1,\cdots,d_r;1),(1-p-q+c+d;c_1+d_1,\cdots,c_r+d_r;1),\mathbf{A}:A \\ \cdot \\ \cdot \\ (\text{1-p-q+c;c}_1+d_1,\cdots,c_r+d_r;1),(1-p-q+d;c_1+d_1,\cdots,c_r+d_r;1):B\end{array}\right) \tag{4.1}$$

provided

$a_i, c_i, d_i > 0(i = 1,\cdots,r)$, $a, b$ are no zero integers $a'x+b'(1-x)\neq 0, 0\leqslant x\leqslant 1$

$$Re(a) + \sum_{i=1}^{r} a_i \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0. \ Re(b) - \sum_{i=1}^{r} a_i \max_{1 \leqslant j \leqslant n^{(i)}} Re \left[ C_j^{(i)} \left( \frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right] > 0.$$

$$R(p) + \sum_{i=1}^{r} c_i \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0. Re(q) + \sum_{i=1}^{r} d_i \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[ D_j^{(i)} \left( \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0. \text{ and}$$

$$\left| arg \left( z_1 e^{\omega(a_i + b_i)\theta} (\tan\theta)^{a_i} \frac{(a'x)^{c_i} [b'(1-x)]^{d_i}}{[a'x + b'(1-x)]^{-c_i - d_i}} \right) \right| < \frac{1}{2} A_i^{(k)} \pi \text{ where } A_i^{(k)} \text{ is defined by (1.4).}$$

**Remark :**
We obtain the same integrals with the functions cited in the section 1.

## 5. Conclusion.

The importance of our all the results lies in their manifold generality. Firstly, in view of general arguments utilized in these double finite integrals, we can obtain a large simpler double or single finite integrals. Secondly, by specialising the various parameters as well as variables in the generalized multivariable Gimel-function, we get a several formulae involving remarkably wide variety of useful functions ( or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several intersting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

## REFERENCES.


[1] F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

[2] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.

[3] A. Erdelyi, W. Magnus, F. Oberhettinger transforms and F.G. Tricomi, Higher Trancendental functions, Vol I, McGraw-Hill, New York (1953).

[4] T.M. MacRobert, Beta function formulae and integrals involving E-function, Math. Ann., 142(5) (1961), 450-452.

[6] Y.N. Prasad, Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , 231-237.

[7] J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

[8] H.M. Srivastava, A contour integral involving Fox's H-function, Indian. J. Math. 14(1972), 1-6.

[9] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[10] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

[11] H.M. Srivastava and N.P. Singh, The integration of certains products of the multivariable H-function with a general class of polynomials, Rend. CirC. Mat. Palermo. 32(2)(1983), 157-187.