Introduction to D-Space & C- Space

Nikhitha P B
F.D.P Substitute & Department of Mathematics & University of Calicut
P.M.Government College, Chalakudy
Kerala, India

Abstract
To reference in my paper, at first, I would like a text of 'Introduction to General Topology'that addressed convergence of a sequence in topological spaces. The concept seems plausible .Quite interested to consider the convergence of real sequence (1/n)with respect to different topology onreal space$\mathbb{R}$ *. All of us knew that (1/n) converges to 0with respect to usual topology on* $\mathbb{R}$*. Here I pursue my work on discussing the space on which (1/n) divergent and stronger than usual topology on* $\mathbb{R}$*. In consideration of this I am trying to define a new concept of D- Space and C- Space.*

Keywords
(R,τ_1) : Indiscrete Topology
(R,τ_2) : Discrete Topology
(R,τ_3) : Co-FiniteTopology
(R,τ_4) : Co-Countable Topology
(R,τ_5) : Usual Topology
(R,τ_6) : Upper-Limit Topology
(R,τ_7) : Lower -Limit Topology
(R,τ_8) : Scattering Topology
(R,τ_9) : Ray Topology
$\emptyset$: *Null Set*
$\in$: *Belongs To*
$\forall$: *For All*
$\exists$: *There Exists*

I. INTRODUCTION

As we all knew, sequences can have different behavior with respect to different topological spaces. When we give our attention to Convergence and Divergence, arrives at a new concept of D-Space and C-Space.Consider a sequence $\{x_n\}$ which is convergent with respect to some topologyτ, it is well known that$\{x_n\}$ convergent with respect to weaker topology $\tau' \subset \tau$.So we are interested to find out a topology strongerthan τ and on which the sequence $\{x_n\}$is divergent. Here I am discussing some ideas by introducing D- Space and C- Space.

II. TOPOLOGICAL SPACE

Let X be any set and τ be a family of its subset. Then τ is said to be a topology if it satisfies the following conditions
1.$\emptyset$,X $\in \tau$
2.$\tau$is closed under arbitrary union(ie,if $A_\propto \in \tau \quad \forall\propto$ then $\bigcup_\propto A_\propto \in \tau$)
3 . $\tau$is closed under finite intersection(ie, if $F_i \in \tau$ for $i = 1,2,3, \dots, n$ then $\bigcap_{i=1}^{n} F_i \in \tau$)
ifτ is a topology on X then (X, τ) is said to be a topological space and members of τ are called open sets in (X, τ).
Then 1, 2, 3 can be restated as
1. $\emptyset$,X areopen sets in (X, τ).
2. Arbitrary union of open sets is open.
3. Finite intersection of open set is open.

A. *Convergence and Divergence*

A sequence $\{x_n\}$ in a topological space (X,τ) is said be convergent and converges to a point $x \in$ X if for every open set U in X with $x \in$ U,$\exists$ a natural number N such that$x_n \in$ U$\forall$ n $\geq$ N. Otherwise we say that $\{x_n\}$ diverges to x.

B. *Remarks*

1. Eventually constant sequences are convergent with respect to any topological space. And converges to repeating term.
2. Any sequence is convergent with respect to indiscrete topology.
3. Eventually constant sequence is the only convergent sequence with respect to discrete topological space.
4. A sequence in co-finite topological spaces is convergent if and only if there is at most one term which repeats infinitely many often.
5. Eventually constant sequence is the only convergent sequence with respect to co-countable topological space.

III. D- SPACE AND C- SPACE

1. Let (X,τ) be any topological space, Let $\{x_n\}$ be any sequence (other than eventually constant sequence) converges to x in (X,τ).Then **D- Space** is a topological space (X, τ') on which $\{x_n\}$ is diverges to x and$\tau \subset \tau'$. And is denotedby D(x_n, τ, x).

2. Let (X,τ) be any topological space, Let $\{x_n\}$ be any sequence(other than eventually constant sequence) diverges to x in (X,τ).Then **C- Space** is a topological space (X, τ') on which $\{x_n\}$ is converges to x and $\tau' \subset \tau$. And is denoted byC(x_n, τ, x).

*Throughout this paper we will proscribe Eventually Constant Sequence.

A. *Remarks*

1. Indiscrete topological space is a C-space for any sequence.
2. Discrete topological space is a D-space for any sequence.

B. *Convergence with respect to Base*

A sequence $\{x_n\}$ is converges to x with respect to Base $\mathcal{B}$ of (X,τ) if for every base point **B** in $\mathcal{B}$with $x \in$ **B** , ∃ a natural number N such that $x_n \in$ **B**∀ n ≥ N.

1) Theorem(a):

A sequence $\{x_n\}$ is convergent with respect to (X,τ) iff the sequence $\{x_n\}$ is convergent with respect to Base $\mathcal{B}$ of (X,τ). .

Proof-

Necessary part

Suppose $\{x_n\}$ be any sequence converges to x∈ ***X*** with respect to (X,τ)
Let x∈ ***B*** ;**B** ∈ $\mathcal{B}$
Since **B**is a base point, it is open with respect to (X,τ).
Hence by definition of convergence with respect to (X,τ), ∃a natural number N such that$x_n \in$ **B**∀ n ≥ N.

Sufficient part

Suppose $\{x_n\}$ is converges to x with respect to base $\mathcal{B}$of (X,τ).
Let **U** be any open set containing x
By definition of base there exists a base point **B** in $\mathcal{B}$ such that x ∈ **B** ⊂ **U.**
But then we can find a natural number N such that $x_n \in$ **B**∀ n ≥ N (hypothesis)
ie,∃a natural number N such that $x_n \in$ **B** ⊂ **U**∀ n ≥ N
ie,∃a natural number N such that $x_n \in$ **U**∀ n ≥ N
Since **U** is an arbitrary open set,$\{x_n\}$ converges to x with respect to (X,τ).

C. *Convergence with respect to Sub-Base*

A sequence $\{x_n\}$ is converges to x with respect to Sub-Base$\mathcal{S}$ of (X,τ) if for every Sub-Base point **S** in $\mathcal{S}$with $x \in$ **S** , ∃ a natural number N such that $x_n \in$ **S** ∀ n ≥ N.

1) Theorem (b):

A sequence $\{x_n\}$ isconvergent with respect to (X,τ) iff the sequence $\{x_n\}$ is convergent with respect to sub base$\boldsymbol{\mathcal{S}}$ of (X,τ).

Proof-

Necessary part

Suppose $\{x_n\}$ be any sequence converges to x with respect to (X,τ)
Let x∈ **S** ;**S** ∈ $\boldsymbol{\mathcal{S}}$
Since **S**is a sub-base point, it is open with respect to (X,τ).
Hence by definition of convergence with respect to (X,τ), ∃ a natural number N such that x_n∈ **S**∀ n ≥ N.

Sufficient part

Suppose $\{x_n\}$ is converges to x with respect to sub-base$\boldsymbol{\mathcal{S}}$ of (X,τ).
Let **U** be any open set containing x
By definition of base there exists a base point **B** such that x ∈ **B** ⊂ **U.**
We know that $\mathbf{B} = \mathbf{S_1} \cap \mathbf{S_2} \cap \mathbf{S_3} \ldots \cap \mathbf{S}_{\boldsymbol{n}}$ **.**
Hencex ∈ $\mathbf{S}_{\boldsymbol{n}}$ ∀ n **.**
But then we can find,
Anatural number N_1 such that x_n∈ $\mathbf{S_1}$ ∀ n ≥ N_1
A natural number N_2 such thatx_n∈ $\mathbf{S_2}$ ∀ n ≥ N_2
.
.
.
A natural number $N_n$such that x_n∈ $\mathbf{S}_{\boldsymbol{n}}$ ∀ n ≥ N_n
Let N = Max{ $N_1, N_2, \ldots, N_n$ }
Then we can say that x_n∈ $\mathbf{B} = \mathbf{S_1} \cap \mathbf{S_2} \cap \mathbf{S_3} \ldots \cap \mathbf{S}_{\boldsymbol{n}}$ ∀ n ≥ N
ie,∃a natural number N such that x_n∈ **B** ⊂ **U** ∀ n ≥ N
ie,∃a natural number N such that x_n∈ **U**∀ n ≥ N
Since **U** is an arbitrary open set ,$\{x_n\}$ converges to x with respect to (X,τ).

D. Remarks

1. We can always find a **C- Space** corresponding to any sequence.
Proof-

We knew that any collection $\boldsymbol{\mathcal{S}}$ generates a topology.
And $\boldsymbol{\mathcal{S}}$ will be a sub base for that topology.
Let us take any collection,on which the sequence is convergent,
T hen by Theorem (b)we can find a C-Space

2. We can always find a **D- Space** corresponding to any sequence
Proof-
Consider a sequence $\{x_n\}$ converges to x .
We knew that any collection $\boldsymbol{\mathcal{S}}$ generates a topology.
And $\boldsymbol{\mathcal{S}}$ will be a sub base for that topology.
Let $\boldsymbol{\mathcal{S}}$= τ ∪ $\{x\}$,and τ′ be the topology generated by $\boldsymbol{\mathcal{S}}$.
Clearly,τ ⊂ τ′…………………………(1)
Since$\{x\}$ is a sub base point,$\{x_n\}$ diverges to x with respect to sub base$\boldsymbol{\mathcal{S}}$ ofτ′.
∴ By Theorem (b)
$\{x_n\}$ diverges to xwith respect to τ′……..(2)
(1) And (2) implies that τ′ is a D-Space.

E. Examples

1. Consider the sequence {1/n},Then

- D($1/n$, τ_1, 0) = (R,τ_6)
- D($1/n$, τ_2, 0) = (R,τ_8)
- D($1/n$, τ_3, 0) = (R,τ_4)

- $D(1/n, \tau_{4,}\ 0) = (\mathbb{R}, \tau_3)$
- $D(1/n, \tau_{5,}\ 0) = (\mathbb{R}, \tau_6)$
- $D(1/n, \tau_{6,}\ 0) = (\mathbb{R}, \tau_5)$
- $D(1/n, \tau_{7,}\ 0) = (\mathbb{R}, \tau_2)$
- $D(1/n, \tau_{8,}\ 0) = (\mathbb{R}, \tau_2)$
- $D(1/n, \tau_{9,}\ 0) = (\mathbb{R}, \tau_6)$

2. Consider the sequence {n},Then

- $D(n, \tau_{1,}\ 0) = (\mathbb{R}, \tau_5)$
- $C(n, \tau_{2,}\ 0) = (\mathbb{R}, \tau_3)$
- $D(n, \tau_{3,}\ 0) = (\mathbb{R}, \tau_4)$
- $C(n, \tau_{4,}\ 0) = (\mathbb{R}, \tau_3)$
- $C(n, \tau_{5,}\ 0) = (\mathbb{R}, \tau_3)$
- $C(n, \tau_{6,}\ 0) = (\mathbb{R}, \tau_3)$
- $C(n, \tau_{7,}\ 0) = (\mathbb{R}, \tau_3)$
- $C(\ n, \tau_{8,}\ 0) = (\mathbb{R}, \tau_3)$
- $D(n, \tau_{9,}\ 0) = (\mathbb{R}, \tau_5)$

IV. CONCLUSIONS

1. We can always find a**C- Space** corresponding to any sequence.
2. We can always find a**D- Space** corresponding to any sequence.

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