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Edge Trimagic Total Labeling for Disconnected Graphs

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***Abstract* - An edge trimagic total labeling of a graph G(V, E) with p vertices and q edges is a bijection f from the set of vertices and edges to 1, 2, … , p+q such that for every edge uv in E, f(u)+f(uv)+f(v) is either λ_1 or λ_2 or λ_3. An edge trimagic total graph is called a super edge trimagic total if f(V) = {1, 2, . . ., p}. An edge trimagic total graph is called a superior edge trimagic total if f(E) = {1, 2, . . ., q}. In this paper we prove the disconnected graphs nP_3, $(K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r})$, $nP_2 \cup K_{1,\,n+1}$, t copies of the sun graph S_n, nC_4 and nC_6 admits edge trimagic total labeling.**

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I. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. All graphs considered here are finite, simple and undirected. The useful survey on graph labeling by J.A. Gallian(2012) can be found in [4].

A walk of a graph G is an alternating sequence of vertices and edges beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. A walk which begins and ends at the same vertex is called a closed walk. If the terminal vertices are distinct, the walk is known as open walk. An open walk in which no vertex appears more than once is called a path. A graph is said to be connected if there is at least one path between every pair of vertices otherwise, it is disconnected. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is another graph $G_3 = G_1 \cup G_2$ whose vertex set $V_3 = V_1 \cup V_2$ and the edge set $E_3 = E_1 \cup E_2$. A simple graph in which there exists an edge between every pair of vertices is called a complete graph; the complete graph with n vertices is denoted by K_n. A bigraph G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a point of V_1 to a point of V_2. If G contains every edge joining V_1 and V_2, then G is a complete bigraph. If V_1 and V_2 have m and n vertices, we write $G = K_{m,\,n}$.

A star is a complete bigraph $K_{1,\,n}$. A sun is a cycle C_n with an edge terminating in a vertex of degree one attached to each vertex, it is denoted by S_n. The t copies of the sun graph S_n is denoted by tS_n [8]

In 1970, Kotzing and Rosa introduced edge magic total labeling [7]. In 2004, J. Basker Bubujee introduced edge bimagic labeling of graphs [1]. In 2013, C. Jayasekaran, M. Regees and C. Davidraj introduced edge trimagic total labeling of graphs[5]. We proved that some families of graphs are edge trimagic total in [5, 6, 9, 10, 11]. In this paper, we prove that the disconnected graphs $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$, $nP_2 \cup K_{1,\,n+1}$, t copies of the sun graph tS_n, nC_4 and nC_6 are super edge trimagic total and nP_3 is superior edge trimagic total.

II. EDGE TRIMAGIC LABELING FOR DISCONNECTED GRAPHS

In this section we define the superior edge trimagic total labeling and prove that the disconnected graphs nP_3, $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$, $nP_2 \cup K_{1,\,n+1}$, t copies of the sun graph tS_n, nC_4 and nC_6 are edge trimagic total.

1) Definition: An edge trimagic total labeling of a (p, q) graph G is a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge

$xy \in E(G)$, the value of $f(x)+f(xy)+f(y)$ is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be an edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling of a graph is called a super edge trimagic if $f(V) = \{1, 2, \ldots, p\}$. An edge trimagic total labeling of graph is called a superior edge trimagic total labeling if $f(E) = \{1, 2, \ldots, q\}$.

2) Theorem: The graph nP_3 admits an edge trimagic total labeling for even n.

Proof: Let $V = \{u_i, v_i, w_i \,/\, 1 \le i \le n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i \,/\, 1 \le i \le n\}$ be the edge set of the disconnected graph nP_3. The graph nP_3 has 3n vertices and 2n edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 5n\}$ such that $f(u_i) = 3n+\frac{n}{2}+\frac{i+1}{2}$, $f(w_i) = 3n+\frac{i+1}{2}$ for $1 \le i \le n$, $i \equiv 1 \pmod 2$; $f(u_i) = 2n+\frac{i}{2}$, $f(w_i) = 3n-\frac{n}{2}+\frac{i}{2}$ for $1 \le i \le n$, $i \equiv 0 \pmod 2$; $f(v_i) = 5n - i + 1$, $1 \le i \le n$; $f(u_iv_i) = \frac{n}{2}+\frac{i+1}{2}$, $f(v_iw_i) = \frac{i+1}{2}$ for $1 \le i \le n$, $i \equiv 1 \pmod 2$; $f(u_iv_i) = n+\frac{n}{2}+\frac{i}{2}$, $f(v_iw_i) = n+\frac{i}{2}$ f or $1 \le i \le n$, $i \equiv 0 \pmod 2$.

Now we have to prove that the graph nP_3 has three distinct trimagic constants λ_1, λ_2 and λ_3.

Consider the edges u_iv_i;

For $1 \le i \le n$ and $i \equiv 1 \pmod 2$,

$$f(u_i)+f(u_iv_i)+f(v_i) = 3n+\frac{n}{2}+\frac{i+1}{2}+\frac{n}{2}+\frac{i+1}{2}+5n-i+1$$
$$= 9n+2 = \lambda_1\text{(say)}.$$

For $1 \le i \le n$ and $i \equiv 0 \pmod 2$,

$$f(u_i)+f(u_iv_i)+f(v_i) = 2n+\frac{i}{2}+n+\frac{n}{2}+\frac{i}{2}+5n-i+1$$
$$= \frac{17n+2}{2} = \lambda_2\text{(say)}.$$

Consider the edges v_iw_i;

For $1 \le i \le n$ and $i \equiv 1 \pmod 2$,

$$f(v_i)+f(v_iw_i)+f(w_i) = 5n-i+1+\frac{i+1}{2}+3n+\frac{i+1}{2} = 8n+2 = \lambda_3\text{(say)}.$$

For $1 \le i \le n$ and $i \equiv 0 \pmod 2$,

$$f(v_i)+f(v_iw_i)+f(w_i) = 5n-i+1+n+\frac{i}{2}+3n-\frac{n}{2}+\frac{i}{2}$$
$$= \frac{17n+2}{2} = \lambda_2.$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 9n+2$, $\lambda_2 = \frac{17n+2}{2}$ and $\lambda_3 = 8n+2$.

Hence the graph nP_3 admits an edge trimagic total labeling for even n.

3) Theorem: The graph nP_3 admits a superior edge trimagic total labeling for even n.

Proof: We have proved that the graph nP_3 has an edge trimagic total labeling when n is even. The labeling given in the proof of the Theorem 2, the edges get labels $f(u_iv_i) = \frac{n}{2}+\frac{i+1}{2}$, $f(v_iw_i) = \frac{i+1}{2}$ for $1 \le i \le n$ and $i \equiv 1 \pmod 2$; $f(u_iv_i) = n+\frac{n}{2}+\frac{i}{2}$, $f(v_iw_i) = n+\frac{i}{2}$ for $1 \le i \le n$ and $i \equiv 0 \pmod 2$. Clearly the graph nP_3 has 2n edges and the 2n edges get labels 1, 2, …, 2n. Hence the graph nP_3 is a superior edge trimagic total when n is even.

4) Example: A superior edge trimagic total labeling of the disconnected graph $4P_3$ is given in fig.1.

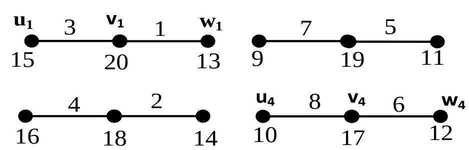

Fig.1 Graph $4P_3$ with $\lambda_1 = 38$, $\lambda_2 = 35$ and $\lambda_3 = 34$.

5) Theorem: The graph nP_3 admits an edge trimagic total labeling for odd n.

Proof: Let $V = \{u_i, v_i, w_i / 1 \leq i \leq n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i / 1 \leq i \leq n\}$ be the edge set of the disconnected graph nP_3. The graph nP_3 has 3n vertices and 2n edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 5n\}$ such that

$f(u_i) = 3n + \frac{n-1}{2} + \frac{i+1}{2}$, $f(w_i) = 3n + \frac{i+1}{2} - 1$ for $1 \leq i \leq n$ and $i \equiv 1 \pmod 2$; $f(u_i) = 2n + \frac{i}{2}$, $f(w_i) = 2n + \frac{n-1}{2} + \frac{i}{2}$ for $1 \leq i \leq n$ and $i \equiv 0 \pmod 2$; $f(v_i) = 5n - i + 1$, $1 \leq i \leq n$; $f(u_iv_i) = \frac{n+i+2}{2}$ and $f(v_iw_i) = \frac{i+1}{2}$ for $1 \leq i \leq n$ and $i \equiv 1 \pmod 2$; $f(u_iv_i) = n + \frac{n-1}{2} + \frac{i}{2} + 1$, $f(v_iw_i) = n + \frac{i}{2} + 1$ for $1 \leq i \leq n$ and $i \equiv 0 \pmod 2$.

Now we have to prove the graph nP_3 has three different trimagic constants λ_1, λ_2 and λ_3.

Consider the edges u_iv_i;

For $1 \leq i \leq n$ and $i \equiv 1 \pmod 2$,

$$f(u_i)+f(u_iv_i)+f(v_i) = 3n + \frac{n-1}{2} + \frac{i+1}{2} + \frac{n+i+2}{2} + 5n - i + 1$$
$$= 9n+2 = \lambda_1 \text{ (say)}.$$

For $1 \leq i \leq n$ and $i \equiv 0 \pmod 2$,

$$f(u_i)+f(u_iv_i)+f(v_i) = 2n + \frac{i}{2} + n + \frac{n-1}{2} + \frac{i}{2} + 1 + 5n - i + 1$$
$$= \frac{17n+3}{2} = \lambda_2 \text{(say)};$$

Consider the edges v_iw_i;

For $1 \leq i \leq n$ and $i \equiv 1 \pmod 2$,

$$f(v_i)+f(v_iw_i)+f(w_i) = 5n - i + 1 + \frac{i+1}{2} + 3n + \frac{i+1}{2} - 1$$
$$= 8n+1 = \lambda_3 \text{ (say)}.$$

For $1 \leq i \leq n$ and $i \equiv 0 \pmod 2$,

$$f(v_i)+f(v_iw_i)+f(w_i) = 5n - i + 1 + n + \frac{i}{2} + 1 + 2n + \frac{n-1}{2} + \frac{i}{2}$$
$$= \frac{17n+3}{2} = \lambda_3.$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 9n+2$, $\lambda_2 = 8n+1$ and $\lambda_3 = \frac{17n+3}{2}$. Therefore, the graph nP_3 admits an edge trimagic total labeling when n is odd.

6) Theorem: The graph nP_3 admits superior edge trimagic total labeling when n is odd.

Proof: We have proved that the graph nP_3 has edge trimagic total labeling when n is odd. The labeling given in the proof of the Theorem 5, the edges get labels for $i \equiv 1 \pmod 2$, $f(u_iv_i) = \frac{n+i+2}{2}$ and $f(v_iw_i) = \frac{i+1}{2}$ also for $i \equiv 0 \pmod 2$; $f(u_iv_i) = n + \frac{n-1}{2} + \frac{i}{2} + 1$, $f(v_iw_i) = n + \frac{i}{2} + 1$, $1 \leq j \leq n$. Clearly the graph nP_3 has 2n edges and the 2n edges get labels 1, 2, ..., 2n. Hence the graph nP_3 is a superior edge trimagic total.

7) Corollary: The disconnected graph nP_3 admits an edge trimagic labeling for all n.

8) Example: The disconnected graph $5P_3$ given in fig. 2 is a superior edge trimagic total graph.

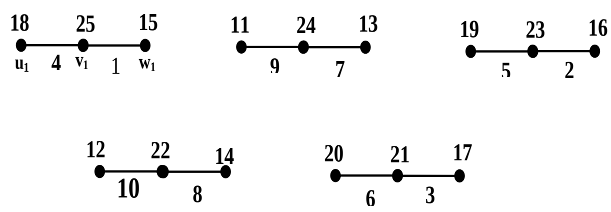

Fig. 2 Graph $5P_3$ with $\lambda_1 = 47$, $\lambda_2 = 44$ and $\lambda_3 = 41$.

9) Theorem: The graph $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$ admits an edge trimagic total labeling.

Proof: Let $V = \{ u_i, v_j, w_k \,/\, 1 \le i \le p+1;\ 1 \le j \le q+1;\ 1 \le k \le r+1\}$ be the vertex set and $E = \{u_1u_i, v_1v_j, w_1w_k \,/\, 2 \le i \le p+1;\ 2 \le j \le q+1;\ 2 \le k \le r+1\}$ be the edge set of the graph $(K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r})$.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 2p+2q+2r+3\}$ such that $f(u_1) = 1$; $f(v_1) = 2$; $f(w_1) = 3$;
$f(u_i) = p+q+r+5-i$, $2 \le i \le p+1$; $f(v_j) = q+r+5-j$, $2 \le j \le q+1$; $f(w_k) = r+5-k$, $2 \le k \le r+1$.
$f(u_1u_i) = p+q+r+2+i$, $2 \le i \le p+1$.
$f(v_1v_j) = 2p+q+r+2+j$, $2 \le j \le q+1$.
$f(w_1w_i) = 2p+2q+r+2+k$, $2 \le k \le r+1$.

Now we prove that the graph $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ admits an edge trimagic total labeling.
For the edges u_1u_i, $2 \le i \le p+1$;
$$f(u_1)+f(u_1u_i)+f(u_i) = 1+p+q+r+2+i+p+q+r+5-i$$
$$= 2(p+q+r)+8 = \lambda_1 \text{ (say)}.$$
For the edges v_1v_j, $2 \le j \le q+1$;
$$f(v_1)+f(v_1v_j)+f(v_j) = 2+2p+q+r+2+j+q+r+5-j$$
$$= 2(p+q+r)+9 = \lambda_2 \text{ (say)}.$$
For the edges w_1w_k, $2 \le k \le r+1$;
$$f(w_1)+f(w_1w_k)+f(w_k) = 3+2p+2q+r+2+k+r+5-k$$
$$= 2(p+q+r)+10 = \lambda_3 \text{ (say)}.$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constants $\lambda_1 = 2(p+q+r)+8$, $\lambda_2 = 2(p+q+r)+9$ and $\lambda_3 = 2(p+q+r)+10$.

Therefore, the disconnected graph $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$ admits an edge trimagic total labeling.

10) Theorem: The graph $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$ has a super edge trimagic total labeling.

Proof: We have proved that the graph $K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r}$ $(p \le q \le r)$ has an edge trimagic total labeling. The labeling given in the proof of the Theorem 9, the vertices get labels $f(u_1) = 1$; $f(v_1) = 2$; $f(w_1) = 3$; $f(u_i) = p+q+r+5-i$, $2 \le i \le p+1$; $f(v_j) = q+r+5-j$, $2 \le j \le q+1$; $f(w_k) = r+5-k$, $2 \le k \le r+1$.

The graph$(K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r})$ has $(p+q+r+3)$ vertices and the vertices get labels $1, 2, \ldots, (p+q+r+3)$.

Hence the graph $(K_{1,\,p} \cup K_{1,\,q} \cup K_{1,\,r})$ is a super edge trimagic total.

11) Example: The graph $(K_{1,\,6} \cup K_{1,\,8} \cup K_{1,\,10})$ given in fig. 3 is a super edge trimagic total.

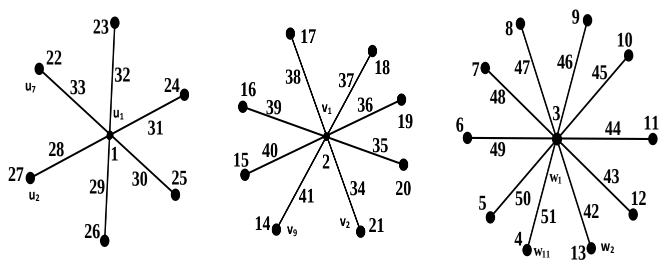

Fig. 3 $(K_{1,\,6} \cup K_{1,\,8} \cup K_{1,\,10})$ with $\lambda_1 = 56$, $\lambda_2 = 57$ and $\lambda_3 = 58$.

12) Theorem: The graph $nP_2 \cup K_{1,\,n+1}$ has an edge trimagic total labeling when n is odd.

Proof: Let $V = \{u_i, v_i \,/\, 1 \le i \le n\} \cup \{w, w_j \,/\, 1 \le j \le n+1\}$ be the vertex set and $E = \{u_iv_i/1 \le i \le n\} \cup \{ww_j \,/1 \le j \le n+1\}$ be the edge set of the disconnected graph $nP_2 \cup K_{1,\,n+1}$. The graph $nP_2 \cup K_{1,\,n+1}$ has $3n+2$ vertices and $2n+1$ edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 5n+3\}$ such that
$f(v_i) = n+\frac{i+1}{2}+1$, $f(u_i) = 2n+\frac{i+1}{2}+2$ for $1 \le i \le n$, $i \equiv 1 \pmod 2$, $f(v_i) = \frac{n+1}{2}+\frac{i}{2}$, $f(u_i) = 2n+\frac{i}{2}+\frac{n+1}{2}+2$ for $1 \le i \le n$, $i \equiv 0 \pmod 2$,

$f(v_iu_i) = 4n+3 - i$, $1 \le i \le n$,
$f(w_j) = 2(n+1) - \left(\frac{j-1}{2}\right)$, $f(ww_j) = 4n+\frac{j+1}{2} + 2$ for $1 \le j \le n+1$ and
$j \equiv 1 \pmod 2$,
$f(w_j) = \frac{n+1}{2} - \frac{j}{2} + 1$, $f(ww_i) = 4n+\frac{n+1}{2} + \frac{j}{2} +2$ for $1 \le j \le n+1$

and $j \equiv 0 \pmod 2$ and $f(w) = n+1$.

Now we prove that the graph $nP_2 \cup K_{1,n+1}$ have three trimagic constants λ_1, λ_2 and λ_3.
Consider the edges v_iu_i;
For $1 \le i \le n$ and $i \equiv 1 \pmod 2$;

$$f(v_i)+f(v_iu_i)+f(u_i) = n +\frac{i+1}{2} + 1 + 4n + 3 - i + 2n + \frac{i+1}{2} + 2$$
$$= 7n+7 = \lambda_1(\text{say}).$$

For $1 \le i \le n$ and $i \equiv 0 \pmod 2$;

$$f(v_i)+f(v_iu_i)+f(u_i) = \frac{n+1}{2} + \frac{i}{2} + 4n + 3 - i + 2n + \frac{n+1}{2} + \frac{i}{2} +2$$
$$= 7n+6 = \lambda_2(\text{say}).$$

Consider the edges ww_j;
For $1 \le j \le n + 1$ and $j \equiv 1 \pmod 2$;

$$f(w)+f(ww_j)+f(w_j) = n+1+4n + \left(\frac{j+1}{2}\right) + 2 + 2(n + 1) - \left(\frac{j-1}{2}\right)$$
$$= 7n+6 = \lambda_2.$$

For $1 \le j \le n + 1$ and $j \equiv 0 \pmod 2$;

$$f(w)+f(ww_j)+f(w_j) = n+1+4n+\frac{n+1}{2} + \frac{j}{2} + 2 + \frac{n+1}{2} - \frac{j}{2} + 1$$
$$= 6n+5 = \lambda_3(\text{say}).$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 7n+7$, $\lambda_2 = 7n+6$ and $\lambda_3 = 6n+5$.

Thus the graph $nP_2 \cup K_{1,\, n+1}$ has an edge trimagic total labeling when n is odd.

13) Theorem: The graph $nP_2 \cup K_{1,\, n+1}$ admits a super edge trimagic total labeling for odd n.

Proof: We have proved that the graph $nP_2 \cup K_{1,\, n+1}$ has edge trimagic total labeling when n is odd. The labeling given in the proof of the Theorem 12, the vertices get labels $f(v_i) = n+\frac{i+1}{2}+1$, $f(u_i) = 2n+\frac{i+1}{2} +2$ for $1 \le i \le n$ and $i \equiv 1 \pmod 2$ and $f(v_i) = \frac{n+1}{2} + \frac{i}{2}$, $f(u_i) = 2n+\frac{n+1}{2} + \frac{i}{2} + 2$ for $1\le i \le n$ and $i \equiv 0 \pmod 2$. Also $f(w) = n+1$, $f(w_j) = 2(n+1) - \left(\frac{j-1}{2}\right)$ for $1 \le j \le n+1$ and $j \equiv 1 \pmod 2$ and $f(w_j) = \frac{n+1}{2} - \frac{j}{2}+1$ for $1\le j\le n+1$, $j \equiv 0 \pmod 2$. Clearly the graph $nP_2 \cup K_{1,\, n+1}$ has $3n+2$ vertices and get labels 1, 2, …, $3n+2$.

Hence the graph $nP_2 \cup K_{1,\, n+1}$ is a super edge trimagic total.

14) Example: The disconnected graph $5P_2 \cup K_{1,\, 6}$ given in fig.4 is a super edge trimagic total.

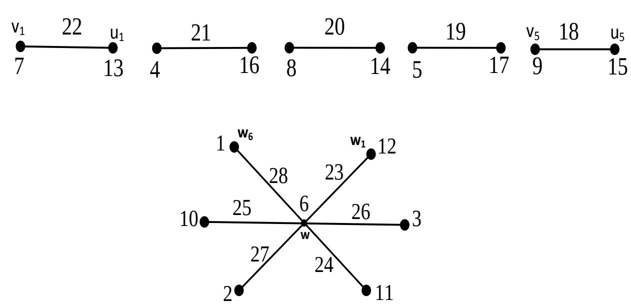

Fig. 4 Graph $5P_2 \cup K_{1,\, 6}$ with $\lambda_1 = 42$, $\lambda_2 = 41$ and $\lambda_3 = 35$.

15) Theorem: The graph $nP_2 \cup K_{1,\, n+1}$ has an edge trimagic total labeling when n is even.

Proof: Let V = $\{u_i, v_i / 1 \le i \le n\} \cup \{w, w_j / 1 \le j \le n + 1\}$ be the vertex set and E = $\{v_iu_i /1 \le i \le n\} \cup \{ww_j / 1 \le j \le n + 1\}$ be the edge set of the disconnected graph $nP_2 \cup K_{1,\, n+1}$. The graph $nP_2 \cup K_{1,\, n+1}$ has 3n+2 vertices and 2n+1 edges.

Define a bijection f: $V \cup E \to \{1, 2, \ldots, 5n + 3\}$ such that
$f(v_i) = n + \frac{i+1}{2} + 1$, $f(u_i) = 2n - \frac{n}{2} + \frac{i+1}{2} + 1$, $1 \le i \le n$, $i \equiv 1 \pmod 2$, $f(v_i) = \frac{n+i}{2} + 1$, $f(u_i) = 2n + \frac{n}{2} + \frac{i}{2} + 2$, $1 \le i \le n$, $i \equiv 0 \pmod 2$, $f(v_iu_i) = 4n+3- i$, $1 \le i \le n$;
$f(w_j) = 2(n+1) + \frac{n}{2} - \frac{j-1}{2}$, $f(ww_j) = 4n + \frac{j+1}{2} + 2$ for j = 1 to n+1 and $j \equiv 1 \pmod 2$,
$f(w_j) = \frac{n}{2} - \frac{j}{2} + 1$, $f(ww_j) = 4n + \frac{n}{2} + 3$, j = 1 to n+1, $j \equiv 0 \pmod 2$ and $f(w) = \frac{n}{2} + 1$.

Now we have to prove the graph $nP_2 \cup K_{1,\, n+1}$ has three distinct trimagic constants λ_1, λ_2 and λ_3.
Consider the edges v_iu_i;
For $1 \le i \le n$ and $i \equiv 1 \pmod 2$;
$f(v_i)+f(v_iu_i)+f(u_i)$
$= n + \frac{i+1}{2} + 1 + 4n + 3 - i + 2n + \frac{i+1}{2} - \frac{n}{2} + 1$
$= \frac{13n+12}{2} = \lambda_1$(say).
For $1 \le i \le n$ and $i \equiv 0 \pmod 2$;
$f(v_i)+f(v_iu_i)+f(u_i) = \frac{n+i}{2} + 1 + 4n + 3 - i + 2n + \frac{n}{2} + \frac{i}{2} + 2$
$= 7n+6 = \lambda_2$(say).
Consider the edges ww_j;
For $1 \le j \le n + 1$ and $j \equiv 1 \pmod 2$;
$f(w)+f(ww_j)+f(w_j) = \frac{n}{2} + 1 + 4n + \frac{j+1}{2} + 2 + 2(n+1) + \frac{n}{2} - \frac{j-1}{2}$
$= 7n+6 = \lambda_2$.
For $1 \le j \le n + 1$ and $j \equiv 0 \pmod 2$;
$f(w)+f(ww_j)+f(w_j) = \frac{n}{2} + 1 + 4n + \frac{n}{2} + \frac{j}{2} + 3 + \frac{n}{2} - \frac{j}{2} + 1$
$= \frac{11n+10}{2} = \lambda_3$(say).

Hence for each edge $uv \in E$, f(u)+f(uv)+f(v) yields any one of the trimagic constants $\lambda_1 = \frac{13n+12}{2}$, $\lambda_2 = 7n+6$ and $\lambda_3 = \frac{11n+10}{2}$. Thus the graph $nP_2 \cup K_{1,\, n+1}$ admits an edge trimagic total labeling when n is even.

16) Theorem: The graph $nP_2 \cup K_{1,n+1}$ has a super edge trimagic total labeling for even n.

Proof: We have proved that the graph $nP_2 \cup K_{1,\, n+1}$ has an edge trimagic total labeling when n is even. The labeling given in the proof of the theorem 15, the vertices get labels $f(v_i) = n + \frac{i+1}{2} + 1$, $f(u_i) = 2n - \frac{n}{2} + \frac{i+1}{2} + 1$ for $1 \le i \le n$ and $i \equiv 1 \pmod 2$; $f(v_i) = \frac{n+i}{2} + 1$, $f(u_i) = 2n + \frac{n}{2} + \frac{i}{2} + 2$ for $1 \le i \le n$ and $i \equiv 0 \pmod 2$; $f(w_j) = 2(n+1) + \frac{n}{2} - \frac{j-1}{2}$ for j = 1 to n+1 and $j \equiv 1 \pmod 2$, $f(w_j) = \frac{n}{2} - \frac{j}{2} + 1$ for j = 1 to n+1, $j \equiv 0 \pmod 2$ and $f(w) = \frac{n}{2} + 1$. Clearly the graph $nP_2 \cup K_{1,\, n+1}$ has 3n+2 vertices and get labels 1, 2, …, 3n+2. Hence the graph $nP_2 \cup K_{1,\, n+1}$ admits a super edge trimagic total labeling when n is even.

17) Corollary: The disconnected graph $nP_2 \cup K_{1,\, n+1}$ admits a super edge trimagic labeling for all n.

18) Example: The graph $6P_2 \cup K_{1,\, 7}$ given in fig. 5 is a super edge trimagic total.

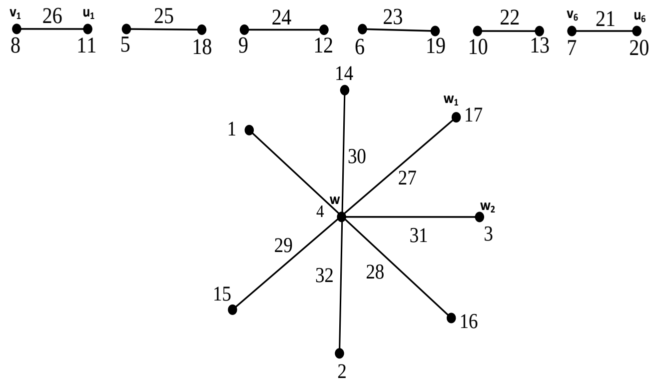

Fig. 5 Graph $6P_2 \cup K_{1,7}$ with $\lambda_1 = 45$, $\lambda_2 = 48$ and $\lambda_3 = 38$.

19) Theorem: For $n \geq 3$ and $t \geq 1$, the t copies the sun graph tS_n admits an edge trimagic total labeling.

Proof: Let tS_n be the t copies of the sun graph S_n with the vertex set $V = \{v_i^j, u_i^j / 1 \leq i \leq n; 1 \leq j \leq t\}$ and the edge set $E = \{v_i^j v_{i+1}^j, v_i^j u_i^j / 1 \leq i \leq n-1; 1 \leq j \leq t\} \cup \{v_1^j v_n^j / 1 \leq j \leq t\}$.

Define a bijecion f: $V(tS_n) \cup E(tS_n) \rightarrow \{1, 2, \ldots, 4nt\}$ such that $f(v_i^j) = n(j-1) + i$, $f(u_i^j) = nt + n(j-1) + i$, $f(v_i^j v_{i+1}^j) = 4nt - 2n(j-1) - 2(i-1)$, $f(v_i^j u_i^j) = 4nt - 2n(j-1) - 2(i-1) - 1$,
$1 \leq i \leq n-1$, $1 \leq j \leq t$ and $f(v_1^j v_n^j) = 4nt - 2n(j-1) - 2(n-1)$, $1 \leq j \leq t$.

Now we have to prove that the graph tS_n has three distinct trimagic constants λ_1, λ_2 and λ_3.

For the edges $v_1^j v_n^j$, $1 \leq j \leq t$;

$$f(v_1^j) + f(v_1^j v_n^j) + f(v_n^j)$$
$$= n(j-1)+1+ 4nt-2n(j-1) - 2(n-1)+ n(j-1)+n$$
$$= 4nt - n+3 = \lambda_1(\text{say}).$$

For the edges $v_i^j v_{i+1}^j$, $1 \leq i \leq n-1$, $1 \leq j \leq t$;

$$f(v_i^j) + f(v_i^j v_{i+1}^j) + f(v_{i+1}^j)$$
$$= n(j-1)+ i +4nt-2n(j-1)-2(i-1)+n(j-1)+i+1$$
$$= 4nt+3 = \lambda_2(\text{say}).$$

Also for the edges $v_i^j u_i^j$, $1 \leq i \leq n-1$, $1 \leq j \leq t$;

$$f(v_i^j) + f(v_i^j u_i^j) + f(u_i^j)$$
$$= n(j-1) + i+4nt -2n(j-1)-2(i-1)-1+nt+n(j-1)+i$$
$$= 5nt+1 = \lambda_3(\text{say}).$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 4nt-n+3$, $\lambda_2 = 4nt+3$ and $\lambda_3 = 5nt+1$.

Thus the t copies of a sun graph, tS_n admits an edge trimagic total labeling.

20) Example: A super edge trimagic total labeling of 3 copies of sun S_5 is given in fig. 6.

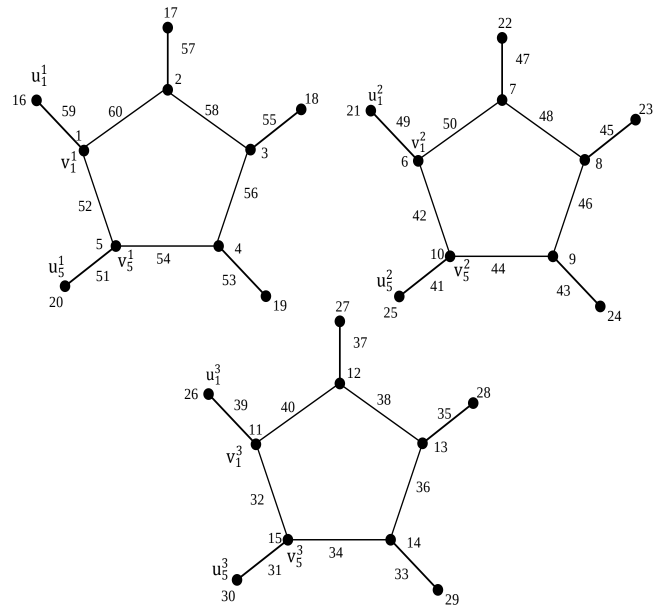

Fig. 6 Graph $3S_5$ with $\lambda_1 = 58$, $\lambda_2 = 63$ and $\lambda_3 = 76$.

21) Theorem: For $n \geq 3$ and $t \geq 1$, the t copies the sun graph tS_n is a super edge trimagic total.

Proof: We have proved that the graph tS_n admits an edge trimagic total labeling. The labeling given in the proof of the Theorem 19, the vertices get labels $f(v_i^j) = n(j-1)+i$, $f(u_i^j) = nt+n(j-1)+ i$, $1 \leq i \leq n-1$; $1 \leq j \leq t$. Clearly the graph tS_n has 2nt vertices and get labels 1, 2, …, 2nt.

Hence the graph tS_n is a super edge trimagic total graph

22) Theorem: The graph nC_4 admits an edge trimagic total labeling.

Proof: Let $V = \{u_i, v_i, w_i, x_i \,/ 1 \leq i \leq n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i, w_ix_i, x_iu_i \,/ 1 \leq i \leq n\}$ be the edge set of the disconnected graph nC_4. The graph nC_4 has 4n vertices and 4n edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 8n\}$ such that $f(u_i) = i$, $f(v_i) = n+i$, $f(w_i) = 3n+i$, $f(x_i) = 2n+i$ for all $1 \leq i \leq n$; $f(u_iv_i) = 8n-2i+2$, $f(v_iw_i) = 6n-2i+1$, $f(w_ix_i) = 6n-2i+2$ and $f(x_iu_i) = 8n-2i+1$ for all $1 \leq i \leq n$.

Now we prove that the graph nC_4 admits an edge trimagic total labeling.

For the edges u_iv_i, $1 \leq i \leq n$;

$f(u_i)+f(u_iv_i)+f(v_i) = i+8n-2i+2+n+i = 9n+2 = \lambda_1$(say).

For the edges v_iw_i, $1 \leq i \leq n$;

$f(v_i)+f(v_iw_i)+f(w_i) = n+i+6n-2i+1+3n+i = 10n+1 = \lambda_2$(say).

For the edges w_ix_i, $1 \leq i \leq n$;

$f(w_i)+f(w_ix_i)+f(x_i) = 3n+ i+6n-2i+2+2n+i = 11n+2 = \lambda_3$(say).

For the edges x_iu_i, $1\leq i\leq n$;

$f(x_i)+f(x_iu_i)+f(u_i) = 2n+i+8n-2i+1+i = 10n+1 = \lambda_2.$

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constant $\lambda_1 = 9n+2$, $\lambda_2 = 10n+1$ and $\lambda_3 = 11n+2$. Hence the disconnected graph nC_4 admits an edge trimagic total labeling.

23) Theorem: The graph nC_4 admits a super edge trimagic total labeling.

Proof: We have proved that the graph nC_4 has an edge trimagic total labeling. The labeling given in the proof of the theorem 22, the vertices get labels $f(u_i) = i$, $f(v_i) = n+ i$, $f(w_i) = 3n+i$, $f(x_i) = 2n+i$ for all $1\leq i\leq n$. Clearly the graph has 4n vertices and get labels 1, 2, …, 4n. Hence the graph nC_4 admits a super edge trimagic total labeling.

24) Example: A super edge trimagic total labeling of $4C_4$ is given in fig. 7

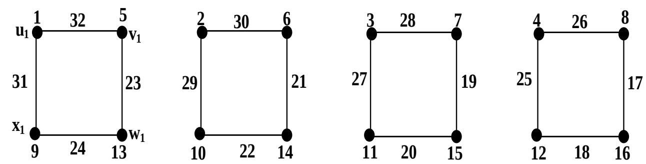

Fig. 7 Graph $4C_4$ with $\lambda_1 = 38$, $\lambda_2 = 41$ and $\lambda_3 = 46$.

25) Theorem: The graph nC_6 admits an edge trimagic total labeling.

Proof: Let $V = \{u_i, v_i, w_i, x_i, y_i, z_i / 1\leq i\leq n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i, w_ix_i, x_iy_i, y_iz_i, z_iu_i /1\leq i\leq n\}$ be the edge set of the disconnected graph nC_6. The graph nC_6 has 6n vertices and 6n edges.

Define a bijection $f: V\cup E\rightarrow\{1, 2, \ldots, 12n\}$ such that $f(u_i) = i$, $f(v_i) = 3n+ i$, $f(w_i) = n+i$, $f(x_i) = 4n+i$, $f(y_i) = 2n+i$, $f(z_i) = 5n+i$ for all $1\leq i\leq n$; $f(u_iv_i) = 12n-2i+2$, $f(v_iw_i) = 12n-2i+1$, $f(w_ix_i) = 8n-2i+1$, $f(x_iy_i) = 10n-2i+1$, $f(y_iz_i) = 8n-2i+2$ and $f(z_iu_i) = 10n-2i+2$ for all $1\leq i\leq n$.

Now we prove that the graph nC_6 admits an edge trimagic total labeling.

For the edges u_iv_i, $1\leq i\leq n$;

$f(u_i)+f(u_iv_i)+f(v_i) = i+12n-2i+2+3n+i = 15n+2 = \lambda_1$(say).

For the edges v_iw_i, $1\leq i\leq n$;

$f(v_i)+f(v_iw_i)+f(w_i) = 3n+ i+12n-2i+1+n+i = 16n+1 = \lambda_2$(say).

For the edges w_ix_i, $1\leq i\leq n$;

$f(w_i)+f(w_ix_i)+f(x_i) = n+ i+8n-2i+1+4n+i = 13n+1 = \lambda_3$(say).

For the edges x_iy_i, $1\leq i\leq n$;

$f(x_i)+f(x_iy_i)+f(y_i) = 4n+i+10n-2i+1+2n+i = 16n+1 = \lambda_2$.

For the edges y_iz_i, $1\leq i\leq n$;

$f(y_i)+f(y_iz_i)+f(z_i) = 2n+i+8n-2i+2+5n+i = 15n+2 = \lambda_1$.

For the edges z_iu_i, $1\leq i\leq n$;

$f(z_i)+f(z_iu_i)+f(u_i) = 5n+i+10n–2i+2+i = 15n+2 = \lambda_1$.

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constant $\lambda_1 = 15n+2$, $\lambda_2 = 16n+1$ and $\lambda_3 = 13n+1$. Hence the disconnected graph nC_6 admits an edge trimagic total labeling.

26) Theorem: The graph nC_6 admits a super edge trimagic total labeling.

Proof: We have proved that the graph nC_6 has an edge trimagic total labeling. The labeling given in the proof of the theorem 25, the vertices get labels $f(u_i) = i$, $f(v_i) = 3n+ i$, $f(w_i) = n+i$, $f(x_i) = 4n+i$, $f(y_i) = 2n+i$, $f(z_i) = 5n+i$ for all $1\leq i\leq n$. Clearly thegraph nC_6 has 6n vertices and get labels 1, 2, …, 6n. Hence the graph nC_6 admits a super edge trimagic total labeling.

27) Example: A super edge trimagic total labeling of $3C_6$ is given in fig. 8.

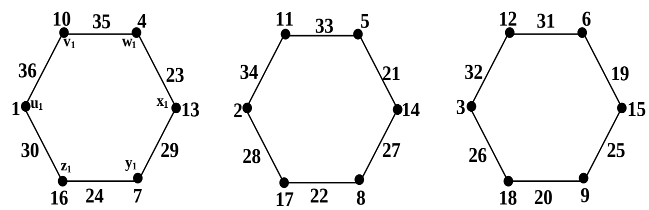

Fig. 8 Graph $3C_6$ with $\lambda_1 = 47$, $\lambda_2 = 49$ and $\lambda_3 = 40$.

III. CONCLUSIONS

In this paper, we proved that the disconnected graphs $K_{1,\,p}\cup K_{1,\,q}\cup K_{1,\,r}$ $(p \leq q \leq r)$, $nP_2\cup K_{1,\,n+1}$, t copies of the sun graph tS_n, nC_4 and nC_6 are super edge trimagic total and nP_3 is superior edge trimagic total. There may be many interesting trimagic graphs can be constructed also in future.

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