$\hat{g}*$-Closed Sets in Topological Spaces

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ABSTRACT

In this paper we have introduced a new class of sets called $\hat{g}*$-closed sets which is properly placed in between the class of closed sets and g -closed sets. As an application, we introduce three new spaces namely, $T_{\hat{g}}*$, ${}_{g}T_{\hat{g}}*$ and ${}_{g*}T_{\hat{g}}*$ spaces. Further, $\hat{g}*$-continuous and $\hat{g}*$-irresolute mappings are also introduced and investigated.

Keywords: $\hat{g}*$-closed sets, $\hat{g}*$-continuous map, $\hat{g}*$-irresolute map, $T_{\hat{g}}*$, ${}_{g}T_{\hat{g}}*$ and ${}_{g*}T_{\hat{g}}*$ spaces.

1. Introduction

Levine **[10]** introduced the class of g -closed sets in 1970. Maki.et.al **[12]** defined αg -closed sets in 1994. Arya and Tour **[3]** defined gs -closed sets in 1990. Dontchev **[8]**, Gnanamble **[9]** , Palaniappan and Rao**[17]** introduced gsp-closed sets, gpr-closed sets and rg -closed sets respectively. Veerakumar **[19]** introduced $\hat{g}$ -closed sets in 1991. Levine **[10]** Devi.et.al.**[5,6]** introduced $T_{1/2}$- spaces , T_b spaces and ${}_{\alpha}T_b$ spaces respectively. Veerakumar **[18]** introduced T_c, ${}_{\alpha}T_c$ and $T_{1/2}$ *-spaces. The purpose of this paper is to introduce the concepts of $\hat{g}*$-closed sets, $\hat{g}*$-continuous and $\hat{g}*$-irresolute mappings. we have also introduced $T_{\hat{g}}*$, ${}_{g}T_{\hat{g}}*$ and ${}_{g*}T_{\hat{g}}*$ spaces.

2. Preliminaries

Throughout this paper (X,τ),(Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ),cl(A)and int(A) denote the closure and the interior of A respectively.

The class of all closed subsets of a space (X,τ) is denoted by C(X,τ). The smallest semi-closed(resp.pre-closed and α-closed) set containing a subset A of (X,τ) is called the semi-closure(resp.pre-closure and α-closure) of A and is denoted by scl(A)(resp.pcl(A) and αcl(A))

Definition 2.1: A subset A of a topological space (X, τ) is called

(1) a $pre-open$ set**[14]**if A $\subseteq$ int(cl(A)) and a $pre-closed$ set if cl(int(A)) $\subseteq$ A

(2) a $semi-open$ set**[11]** if A $\subseteq$ cl(int(A)) and a $semi-closed$ set if int(cl(A)) $\subseteq$ A

(3) a $semi-preopen$ set **[1]** if A $\subseteq$ cl(int(cl(A)) and a $semi-preclosed$ set [1] if int(cl(int(A))) $\subseteq$ A

(4) an $\alpha-open$ set **[15]** if A $\subseteq$ int(cl(int(A))) and an $\alpha-closed$ set [15] if cl(int(cl(A))) $\subseteq$ A

(5) a $regular-open$ set**[14]** if int(cl(A))=A and an $regular-closed$ set[14] ifA=int(cl(A))

Definition 2.2: A subset A of a topological space (X, τ) is called

(1) a $generalized\ closed$ set (briefly $g-closed$) **[10]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in(X, τ).

(2) a $generalized\ semi-closed$ set (briefly $gs-closed$)**[3]** if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(3) an α - $generalized\ closed$ set (briefly $\alpha g-closed$)**[12]** if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(4) a $generalized\ semi-preclosed$ set (briefly $gsp-closed$)**[10]** if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(5) a $regular\ generalized\ closed$ set (briefly $rg-closed$)**[17]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).

(6) a $generalized\ preclosed$ set (briefly $gp-closed$)**[13]** if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(7) a $generalized\ preregular\ closed$ set (briefly $gpr-closed$)**[9]** if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).

(8) a $g^*-closed$ set**[18]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $g-open$ in (X, τ).

(9) a $wg-closed$ set**[16]** if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(10) a $\hat{g}-closed$ set**[19]**if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

Definition 2.3: A map f : (X, τ) $\rightarrow$ (Y, σ) is called

(1) $g-continuous$ **[4]** if f^{-1}(V) is a $g-closed$ set of (X, τ) for every closed set V of (Y, σ).

(2) $\alpha g-continuous$ **[5]** if f^{-1}(V) is a $\alpha g-closed$ set of (X, τ) for every closed set V of (Y, σ).

(3) gs-$continuous$ **[7]** if f^{-1}(V) is a $gs-closed$ set of (X, τ) for every closed set V of (Y, σ).

(4) gsp- $continuous$ **[8]** if f^{-1}(V) is a $gsp-closed$ set of (X, τ) for every closed set V of (Y, σ).

(5) rg-$continuous$ **[17]** if f^{-1}(V) is a $rg-closed$ set of (X, τ) for every closed set V of (Y, σ)

(6) gp-$continuous$ **[2]** if f^{-1}(V) is a $gp-closed$ set of (X, τ) for every closed set V of (Y, σ)

(7) gpr-$continuous$ **[9]** if f^{-1}(V) is a $gpr-closed$ set of (X, τ) for every closed set V of (Y, σ)

(8) g*- $continuous$ **[18]** if f^{-1}(V) is a $g^*-closed$ set of (X, τ) for every closed set V of (Y, σ)

(9) $wg-continuous$ **[16]** if f^{-1}(V) is a $wg-closed$ set of (X, τ) for every closed set V of (Y, σ)

Definition 2.4: A topological space (X, τ) is said to be

(1) a $T_{1/2}$ $space$ **[10]** if every $g-closed$ set in it is closed.

(2) a T_b $space$ **[6]** if every $gs-closed$ set in it is closed.

(3) an ${}_{\alpha}T_b$ $space$ **[5]** if every $\alpha g-closed$ set in it is closed.

(4) a $T_{1/2}*space$ **[18]** if every $g*-closed$ set in it is closed.

3. Basic properties of $\hat{g}$ *-closed sets

We now introduce the following definitions.

Definition 3.1: A subset A of a topological space (X, τ) is said to be a $\hat{g}$ * -closed set if cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\hat{g}$ -open in X.

Proposition 3.2: Every closed set is $\hat{g}$ * -closed.

Proof follows from the definition.

The following example supports that a $\hat{g}$ * -closed set need not be closed in general.

Example 3.3: Let X= {a, b, c}, τ={ϕ X, {a}, {a, b}}, A= {a,c} is $\hat{g}$ * - closed but not closed.

So, the class of $\hat{g}$ * -closed sets is properly contained in the class of closed sets.

Proposition 3.4: Every $\hat{g}$ *- $closed$ set is

(i) $g-closed$ (ii) $gs-closed$ (iii) $\alpha g-closed$ and (iv) $gsp-closed$

Proof: Let A be a $\hat{g}$ *- $closed$ set

Let A $\subseteq$U and U be open. Then U is $\hat{g}$ -open. Since A is $\hat{g}$ *- $closed$, $cl(A)\subseteq$U.

(i) Hence A is $g-closed$.

(ii) Then $scl(A)\subseteq cl(A)\subseteq$U. Hence A is $gs-closed$.

(iii) $\alpha cl(A)\subseteq cl(A)\subseteq$U and hence A is $\alpha g-closed$.

(iv) $spcl(A) \subseteq cl(A) \subseteq$ U. Hence A is $gsp-closed$.

The converse of the above proposition need not be true in general as seen in the following examples.

Example 3.5: Let X= {a, b, c}, τ = {ϕ, X, {b}, {a, c}} , A ={a} is $g-closed$ but not $\hat{g}$ *- $closed$.

Example 3.6: Let X= {a, b, c}, τ = {ϕ, X, {a}, {a, b}}and let A = {b}.Then A is $gs-closed$, $\alpha g-closed$ and $gsp-closed$ but not $\hat{g}$ *- $closed$.

Proposition 3.7: Every $\hat{g}$ *- $closed$ set is

(i) $gp-closed$ (ii) $wg-closed$ (iii) $rg-closed$ and (iv) $gpr-closed$

Proof: Let A be a $\hat{g}$ *- $closed$ set

(i)& (ii) Let $A \subseteq$ U and U be open. Then U is $\hat{g}$ -open. Since A is $\hat{g}$ *- $closed$,
$cl(A) \subseteq U$
$pcl(A) \subseteq cl(A) \subseteq$ U.HenceAis $gp-closed$.
$cl(\text{int}(A) \subseteq cl(A) \subseteq$ U. Hence A is $wg-closed$.

(iii) &(iv) Let $A \subseteq$ U an U be regular open .Then U is $\hat{g}$ -open. Since A is $\hat{g}$ *-$closed$,

$cl(A) \subseteq$ U. Hence A is $rg-closed$.

$pcl(A) \subseteq cl(A) \subseteq$ U. Then A is $gpr-closed$.

Example 3.8: Let X= {a, b, c}, τ = {ϕ, X, {a}, {b, c}}, A={b}is $rg-closed$ and $gpr-closed$ but not $\hat{g}$ *- $closed$.

Example 3.9: Let X= {a, b, c}, τ = {ϕ, X, {a}, {a, b}}, A= {b} is $gp-closed$ and $wg-closed$ but not $\hat{g}$ *- $closed$.

Proposition 3.10: Every g *-closed set is $\hat{g}$ *-closed.

Proof follows from the definition.

Example 3.11: Let X= {a, b, c}, τ = { ϕ , X, {a}}.Then A={b} is a $\hat{g}$ *-closed but not g *-closed.

Proposition 3.12: If A and B are $\hat{g}$ *-$closed$ sets, then $A\bigcup B$ is also a $\hat{g}$ *-$closed$ set.

Proof follows from the fact that $cl(A\bigcup B) = cl(A)\bigcup cl(B)$.

Remark 3.13: If A and B are $\hat{g}$ *-$closed$ sets, then $A\bigcap B$ need not be $\hat{g}$ *-$closed$ as seen in the following example.

Example 3.14: Let X= {a, b, c}, τ = { ϕ , X, {a}}.Then A={a, b} and B= {a, c}are $\hat{g}$ *-$closed$. But $A\bigcap B$ = {a} is not $\hat{g}$ *-$closed$.

Proposition 3.15: If A is a $\hat{g}*-closed$ set of (X, τ) such that $A\subseteq B\subseteq cl(A)$, then B is also a $\hat{g}*-closed$ set of (X, τ).

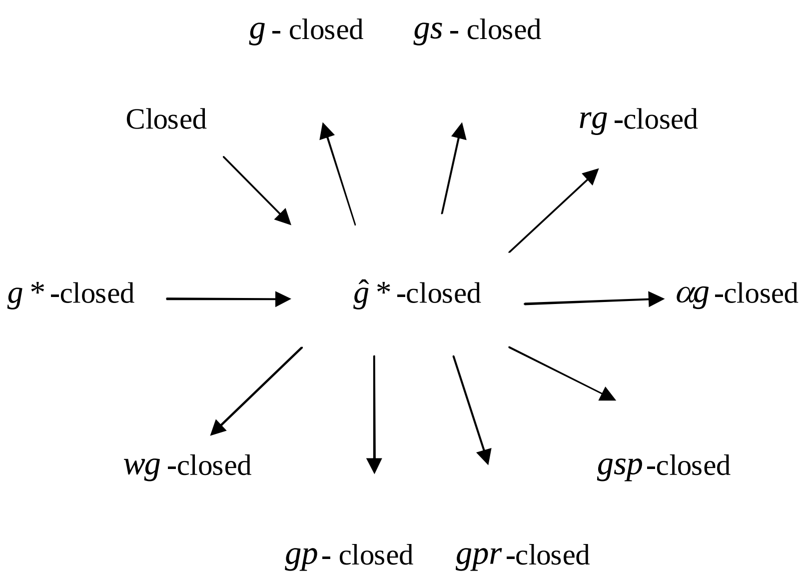
Proof: Let U be a $\hat{g}$ -$open$ set of (X, τ) such that B$\subseteq$U. Then A$\subseteq$U and since A is $\hat{g}$ *-closed, $cl(A)\subseteq$U. Now $cl(B)\subseteq cl(A)\subseteq$U. Then B is also a $\hat{g}*-closed$ set of (X, τ).

Proposition 3.16: If A is a $\hat{g}*-closed$ set of (X, τ) then $cl(A)\setminus A$ does not contain any non-empty $\hat{g}$ -closed set.

Proof: Let F be a $\hat{g}$ -closed set of (X, τ) such that F$\subseteq cl(A)\setminus A$.Then A$\subseteq$X\F. Since A is $\hat{g}*-closed$ and X\F is $\hat{g}$ -open, $cl(A)\subseteq$X\F. This implies F$\subseteq$X$\setminus cl(A)$.

So F$\subseteq$(X$\setminus cl(A)\bigcap(cl(A)\setminus A$)=$\phi$.Therefore F=$\phi$.

The above results can be represented in the following figure:



where A $\longrightarrow$ B represents A implies B and B need not imply A.

4. $\hat{g}$ *-continuous and $\hat{g}$ *-irresolute maps.

We introduce the following definitions.

Definition 4.1: A map *f:* $(X,\tau)\rightarrow(Y,\sigma)$ is said to be a $\hat{g}$ *-continuous map if $f^{-1}(V)$ is a $\hat{g}$ *- closed set of (X,τ) for every closed set V of (Y,σ).

Definition 4.2: A map *f:* $(X,\tau)\rightarrow(Y,\sigma)$ is said to be a $\hat{g}$ *-irresolute map if $f^{-1}(V)$is a $\hat{g}$ *- closed set of (X,τ) for every $\hat{g}$ *- closed set V of (Y,σ).

Theorem 4.3: Every *continuous* map is $\hat{g}*-continuous$.

Proof follows from the definitions.

The converse of the above theorem need not be true in general, as seen in the following example.

Example 4.4: Let X=Y= {a, b, c}, τ= {ϕ , X, {a}, {a, b}} and σ = {ϕ , Y, {a, c}}

Let $f : (X, \tau) \to (Y, \sigma)$ be defined by f (a)=b, f (b)=c, f (c)=b. The inverse image of all closed sets in (Y, σ) are $\hat{g}*-closed$ in(X, τ). Therefore f is $\hat{g}*-continuous$ f^{-1}({b}) = {a, c} is not closed in (X, τ) and hence f is not $continuous$

Theorem 4.5: Every $\hat{g}*-continuous$ map is $g-continuous$, αg-$continuous$, gs-$continuous$, gsp-$continuous$, gp-$continuous$, rg-$continuous$, gpr-$continuous$, and wg-$continuous$.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be a $\hat{g}*-continuous$ map. Let V be a closed set of (Y, σ).

Since f is $\hat{g}*-continuous$, f^{-1}(V) is a $\hat{g}*-closed$ set in (X, τ).

By the proposition (3.4) & (3.7) f^{-1}(V) is $g-closed$, $rg-closed$, $gpr-closed$, $wg-closed$, $\alpha g-closed$, $gs-closed$, $gsp-closed$, $gp-closed$. Hence f is $g-continuous$, αg-$continuous$, gs-$continuous$, gsp-$continuous$, gp-$continuous$, rg-$continuous$, gpr-$continuous$, and wg-$continuous$.

The converse of the above theorem need not be true in general, as seen in the following example.

Example 4.6: Let X=Y= {a, b, c}, τ= {ϕ, X, {b}, {a, c}} and σ = {ϕ, Y, {a, c}}

Let $f : (X, \tau) \to (Y, \sigma)$ be defined by f (a)=b, f (b)=c, f (c)=a. Then f^{-1}({b}) = {a} is $g-closed$ in (X, τ) but not $\hat{g}*-closed$ in(X, τ).Hence f is $g-continuous$ but not $\hat{g}*-continuous$.

Example 4.7: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a, c}}. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f^{-1}({b}) = {b} is not $\hat{g}$ *- $closed$ in X. But {b} is $gs-closed$, $\alpha g-closed$, $wg-closed$, $gsp-closed$ .Therefore f is gs-$continuous$,

gsp-$continuous$, wg-$continuous$, αg-$continuous$but f is not $\hat{g}*-continuous$

Example 4.8: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {b, c}}

Let f : (X, τ)$\to$(Y, σ) be defined by f(a)=c, f(b)=a, f(c)=b. Then f^{-1}({a}) = {b} is $gp-closed$ in (X, τ) but not $\hat{g}*-closed$ in(X, τ).Hence f is $gp-continuous$ but not $\hat{g}*-continuous$.

Example 4.9: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {b, c}} and σ = {ϕ, Y, {a, c}}. Let f : (X, τ)$\to$(Y, σ)be the identity map. Then f^{-1}({b}) = {b} is $rg-closed$, $gpr-closed$ but not $\hat{g}$ *- $closed$ in (X, τ).Therefore f is $rg-continuous$, $gpr-continuous$ but f is not $\hat{g}*-continuous$ .Thus the class of all $\hat{g}*-continuous$ maps is properly contained in the classes of $g-continuous$, αg-$continuous$, gs-$continuous$, gsp-$continuous$, gp-$continuous$, rg-$continuous$, gpr-$continuous$, and wg-$continuous$ maps.

Theorem 4.10: Every $g*$-$continuous$ map is $\hat{g}*-continuous$.

Proof: Let f : (X, τ)$\to$(Y, σ) be a $g*$-$continuous$and let V be a closed set of Y.

Then f^{-1}(V) is a $g*-closed$ set and hence it is $\hat{g}*-closed$ .Hence f is $\hat{g}*-continuous$.

Example 4.11: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}} and σ = {ϕ, Y, {a, b}}. Let f : (X, τ)$\to$(Y, σ) be the identity map. Then f^{-1}({c}) = {c} is $\hat{g}*-closed$ in(X, τ) but not $g*-closed$ in(X, τ).Hence f is $\hat{g}*-continuous$ but not $g*-continuous$.

Theorem 4.12: Every $\hat{g}$ *- $irresolute$ map is $\hat{g}$ *-$continuous$but not conversely.

Example 4.13: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a}}.

Let f : (X, τ)$\to$(Y, σ)be the identity map. Then f^{-1}({b, c}) = {b, c} is $\hat{g}*-closed$ in

(X, τ).Therefore f is $\hat{g}*-continuous$. {b} is $\hat{g}*-closed$ in (Y, σ) but f^{-1}{b} = {b} is not $\hat{g}*-closed$ in(X, τ). Therefore f is not $\hat{g}$ *- $irresolute$. Hence f is $\hat{g}$ *-$continuous$but not $\hat{g}$ *- $irresolute$.

Theorem 4.14: Every $\hat{g}$ *- $irresolute$ map is g -$continuous$, αg-$continuous$, gs-$continuous$, gsp-$continuous$, gp-$continuous$, rg-$continuous$, gpr-$continuous$, and wg-$continuous$ maps but not conversely.

Proof follows from the definitions.

Example 4.15: Let X=Y= {a, b, c}, τ= {ϕ, X, {b}, {a, c}} and σ = {ϕ, Y, {a}, {a, b}}.

Let f : (X, τ) →(Y, σ)be the identity map. All the subsets of X are $g-closed$ in(X, τ).Hence f^{-1}{b, c} and f^{-1}{c} are $g-closed$ in(X, τ) and hence f is g -$continuous$.{b, c} is $\hat{g}*-closed$ in Y but f^{-1}({b, c})= {b, c} is not $\hat{g}*-closed$ in (X, τ).Therefore f is not a $\hat{g}$ *- $irresolute$ map. Hence f is g -$continuous$ but not $\hat{g}$ *- $irresolute$.

Example 4.16: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a}}.

Closed sets of Y are ϕ, Y and {b, c}. Let f : (X, τ) →(Y, σ)be the identity map. Now f^{-1}({b, c}) = {b, c} is $gs-closed$, $\alpha g-closed$, $wg-closed$, $gsp-closed$ and $gp-closed$ in (X, τ). Hence f is gs-$continuous$, αg-$continuous$, wg-$continuous$, gsp-$continuous$ and gp-$continuous$. $\hat{g}*-closed$ sets of(X, τ) are ϕ, X, {c}, {a, c}, {b, c}and that of (Y, σ) are ϕ, Y, {b},{c},{a, b},{a, c},{b, c}.Now f^{-1}({ b}) = {b} is not $\hat{g}*-closed$ in(X, τ).Hence f is not a $\hat{g}$ *- $irresolute$.

Example 4.17: Let X=Y= {a, b, c}, τ= {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a}}.

Let f : (X, τ) →(Y, σ)be the identity map. f^{-1}({b, c}) = {b, c} is $rg-closed$ and $gpr-closed$ in (X, τ) and hence f is $rg-continuous$ and $gpr-continuous$. $\hat{g}*-closed$ sets of (X, τ) are ϕ, X, {a}, {b,c} and that of (Y, σ) are ϕ, Y, {b}, {c}, {a,b}, {a,c} and {b, c}. Then f^{-1}({b}) = {b}is not $\hat{g}*-closed$ in (X, τ) where{b} is $\hat{g}*-closed$ in(Y, σ). Therefore f is not $\hat{g}$ *- $irresolute$.

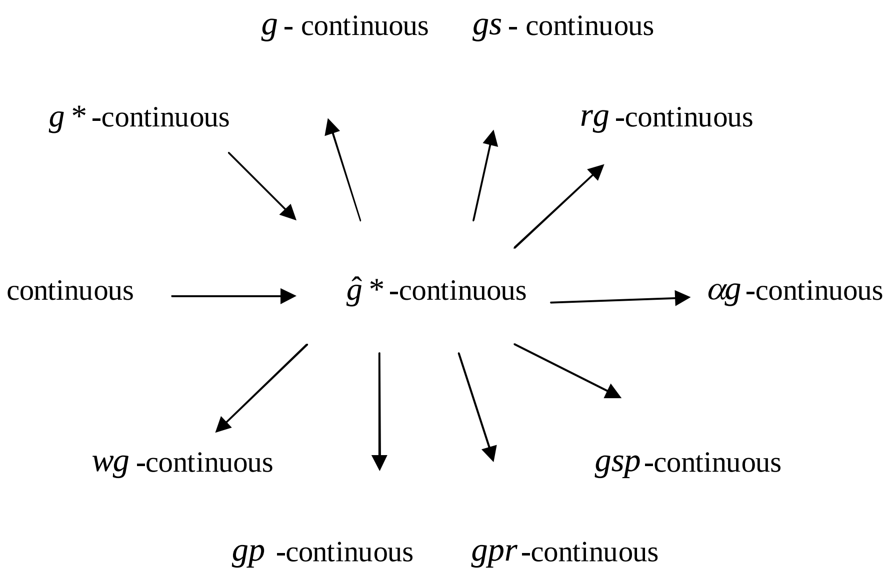
Theorem 4.18: Let f: (X, τ) →(Y, σ) and g: (Y, σ) →(Z, η) be two maps.

Then (i) gof is $\hat{g}$ *-$continuous$ if g is $continuous$ and f is $\hat{g}$ *-$continuous$.

(ii) gof is $\hat{g}$ *-$irresolute$ if both f and g are $\hat{g}$ *-$irresolute$ s .

(iii) gof is $\hat{g}$ *- *continuous* if g is $\hat{g}$ *- *continuous* and f is $\hat{g}$ *- *irresolute* .

The above results can be represented in the following figure:



where A $\longrightarrow$ B represents A implies B and B need not imply A.

5. APPLICATIONS OF $\hat{g}$ *-CLOSED SETS

As applications of $\hat{g}$ *- *closed* sets, new spaces namely, $T_{\hat{g}}$ *-space, ${}_{g}T_{\hat{g}}$ *-space and ${}_{g*}T_{\hat{g}}$ *-space are introduced.

Definition 5.1: A space (X, τ) is called a $T_{\hat{g}}$ *-space if every $\hat{g}$ *-closed set is closed.

Definition 5.2: A space (X, τ) is called a ${}_{g}T_{\hat{g}}$ *-space if every g -closed set is $\hat{g}$ *-closed.

Definition 5.3: A space (X, τ) is called a ${}_{g*}T_{\hat{g}}$ *-space if every $\hat{g}$ *-closed set is $g*$-closed.

Theorem 5.4: Every $T_{1/2}$ -space is a $T_{\hat{g}}$ *-space but not conversely.

Proof: Let (X, τ) be a $T_{1/2}$-space. Let A be a $\hat{g}$ *-closed set. By proposition (3.4), every $\hat{g}$ *-closed set is g -closed. Since (X, τ) is a $T_{1/2}$ -space, every g - closed set is closed. Hence (X, τ) is a $T_{\hat{g}}$ *-space.

Example 5.5: Let X= {a, b, c} and τ= {ϕ, X, {b}, {a, c}}.Since all the $\hat{g}$ *-closed sets is closed, (X, τ) is a $T_{\hat{g}}$ *-space. A = {a} is g -closed but not closed .Hence(X, τ) is not a $T_{1/2}$ -space.

Theorem 5.6: Every ${}_{\alpha}T_b$ -space is a $T_{\hat{g}}$ *-space but not conversely.

Proof: Let (X, τ) be a ${}_{\alpha}T_b$- space. Let A be a $\hat{g}$ *-closed set. Since every $\hat{g}$ *-closed set is αg -closed and (X, τ) is a ${}_{\alpha}T_b$ -space, A is closed. Therefore (X, τ) is a $T_{\hat{g}}$ *-space.

Example 5.7: Let X= {a, b, c} and τ= {ϕ, X, {b}, {a, c}}. Since all the $\hat{g}$ *-closed sets is closed, (X, τ) is a $T_{\hat{g}}$ *-space. A= {a} is αg - closed but not closed. Hence (X, τ) is not a ${}_{\alpha}T_b$ -space.

Theorem 5.8: Every $T_{\hat{g}}$ *-space is a $T_{1/2}$ *-space but not conversely.

Proof: Let (X, τ) be a $T_{\hat{g}}$ *- space. Let A be a g *-closed set .Then A is $\hat{g}$ *- closed. Since (X, τ) is a $T_{\hat{g}}$ *- space, A is closed. Then (X, τ) is a $T_{1/2}$ *-space.

Example 5.9: Let X= {a, b, c} and τ= {ϕ, X, {a}}. Since all the g *-closed sets is closed, (X, τ) is a $T_{1/2}$ *-space. A= {c} $\hat{g}$ *-closed but not closed .Hence (X, τ) is not a $T_{\hat{g}}$ *- space.

Theorem 5.10: Every T_b -space is a $T_{\hat{g}}$ *-space but not conversely.

Proof: Let (X, τ) be a T_b - space. Let A be a $\hat{g}$ *-closed set. Then A is gs -closed. Since (X, τ) is a T_b - space, A is closed and hence (X, τ) is a $T_{\hat{g}}$ *-space.

Example 5.11: Let X= {a, b, c} and τ= {ϕ, X, {b}, {a, c}}. Since all the $\hat{g}$ *-closed sets is closed, (X, τ) is a $T_{\hat{g}}$ *-space. A= {a} is gs-closed but not closed .Hence (X, τ) is not a T_b -space.

Theorem 5.12: Every $T_{\hat{g}}$ *-space is a $_{g*}T_{\hat{g}}$ *-space but not conversely.

Proof follows from the definitions.

Example 5.13: Let X= {a, b, c} and τ= {ϕ, X, {a}, {a, b}}. Since all the $\hat{g}$ *-closed sets is g*-closed, (X, τ) is a $_{g*}T_{\hat{g}}$ *-space. A= {a, c} is $\hat{g}$ *-closed but not closed .Hence (X, τ) is not a $T_{\hat{g}}$ *-space.

Theorem 5.14: A space which is both $T_{\hat{g}}$ *-space and $_{g}T_{\hat{g}}$ *-space is a $T_{1/2}$ -space.

Proof: Let A be a g - closed set. Then A is $\hat{g}$* -closed since(X, τ) is a $_{g}T_{\hat{g}}$ *-space and A is closed since (X, τ) is a $T_{\hat{g}}$ *-space. Hence (X, τ) is a $T_{1/2}$ -space.

Theorem 5.15: Let *f*: (X, τ) $\rightarrow$(Y, σ) be a $\hat{g}$ *- continuous map and let (X, τ) be a $T_{\hat{g}}$ *- space, then *f* is continuous.

Proof: Let *f*: (X, τ) $\rightarrow$(Y, σ) be a $\hat{g}$ *- continuous map. Let F be a closed set in (Y, σ).Then f^{-1}(F) is $\hat{g}$ *- closed in (X, τ).Since (X, τ) is a $T_{\hat{g}}$ *- space, f^{-1}(F) is closed in (X, τ). Therefore f is continuous.

Theorem 5.16: Let *f*: (X, τ) $\rightarrow$(Y, σ) be a $\hat{g}$ *-irresolute map and let (X, τ) be a $T_{\hat{g}}$ *- space. Then *f* is g- continuous.

Proof: Let *f*: (X, τ) $\rightarrow$(Y, σ) be a $\hat{g}$ *-irresolute map. Let F be a closed set in (Y, σ).Then F is

$\hat{g}$ *-closed in Y. f^{-1}(F) is $\hat{g}$ *-closed in (X, τ), since f is a $\hat{g}$ *-irresolute map. Since every $\hat{g}$ *-closed set is g - closed, f^{-1}(F) is g - closed. Hence f is g - continuous.

Theorem 5.17: Let *f*: $(X, \tau) \rightarrow (Y, \sigma)$ be a g - continuous map and let (X, τ) be a ${}_gT_{\hat{g}}$ *- space. Then *f* is $\hat{g}$ *-continuous.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a g - continuous map. Let F be a closed set in (Y, σ).

Then f^{-1}(F) is g - closed in (X, τ).Since (X, τ) is ${}_gT_{\hat{g}}$ *- space, f^{-1}(F) is $\hat{g}$ *-closed in (X, τ).

Hence f is $\hat{g}*-continuous$.

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