$(gsp)^*$-Closed Sets In Topological Spaces

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ABSTRACT

In this paper we have introduced a new class of sets called $(gsp)^*$-closed sets which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce two news paces namely, T^*_{gsp}-space, gT^*_{gsp}-space.Further, $(gsp)^*$-continuous, and $(gsp)^*$- irresolute mappings are also introduced and investigated.

Keywords:$(gsp)^*$-closed set,$(gsp)^*$-continuous map, $(gsp)^*$-irresolute map, T^*_{gsp}, gT^*_{gsp}-spaces.

1. Introduction

Levine **[10]** introduced the class of g-closed sets in 1970. Maki.et.al **[12]** defined αg-closed sets in 1994. Arya and Tour **[3]** defined gs-closed sets in 1990. Dontchev **[8],** Gnanambal **[9]** Palaniappan and Rao**[17]** introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively.Veerakumar **[18]** introduced g^* -closed sets in 1991.J.Dontchev **[8]** introduced gsp-closed sets in 1995. Levine **[10]** Devi.et.al**.[5,6**] introduced $T_{1/2}$- spaces, T_b spaces and $_{\alpha}T_b$ spaces respectively. Veerakumar **[18]** introduced $T_{1/2}$ *-spaces.The purpose of this paper is to introduce the concepts of $(gsp)^*$-closed sets, $(gsp)^*$-continuous map, $(gsp)^*$-irresolute maps. T^*_{gsp}-space, gT^*_{gsp}-space are introduced and investigated.

2. Preliminaries

Throughout this paper (X,τ),(Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ),cl(A)and int(A) denote the closure and the interior of A respectively.

The class of all closed subsets of a space (X,τ) is denoted by C(X,τ). The smallest semi-closed (resp.pre-closed and α-closed) set containing a subset A of (X,τ) is called the semi-closure (resp.pre-closure and α-closure) of A and is denoted by scl(A)(resp.pcl(A) and αcl(A))

Definition 2.1: A subset A of topological space (X,τ) is called

(1) a pre-open set**[14]** if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if $cl(\text{int}(A)) \subseteq A$

(2) a semi-open set **[11]** if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if $\text{int}(cl(A)) \subseteq A$

(3) a semi-preopen set**[1]** if $A \subseteq cl(\text{int}(cl(A))$ and a semi-preclosed set**[1]** if $\text{int}(cl(\text{int}(A))) \subseteq A$

(4) an α-open set **[15]** if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α-closed set **[15]** if $cl(\text{int}(cl(A))) \subseteq A$

(5) a $regular\text{–}open$ set [**14]** if int(cl(A)) = A and an $regular\text{–}closed$ set **[14]** if A= int(cl(A))

Definition 2.2: A subset A of topological space (X,τ) is called

(1) a generalized closed set (briefly g-closed) **[10]** if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

(2) generalized semi-closed set(briefly) gs-closed **[3**] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(3) an α- generalized closed set (briefly αg-closed) [**12]** if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

(4) a generalized semi pre-closed set (briefly gsp-closed) **[8]** if sp$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

(5) a regular generalized closed set (briefly rg-closed) [**17**] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ)

(6) a generalized pre-closed set (briefly gp-closed) **[13]** if p$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

(7) a generalized pre regular-closed set (briefly gpr-closed) **[9]** if p$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ)

(8) a g^*-closed set **[18**] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ)

(9) a wg-closed set **[16]** if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

Definition 2.3:A function *f:* (X,τ)→ (Y,σ) is called

(1) g-continuous **[4]** if f^{-1}(V) is a g-closed set of (X,τ) for every closed set V of (Y,σ)

(2) αg-continuous**[9]** if f^{-1}(V) is an αg-closed set of (X,τ) for every closed set V of (Y,σ)

(3) gs-continuous **[7]** if f^{-1}(V) is a gs-closed set of (X,τ) for every closed set V of (Y,σ)

(4) gsp-continuous **[8]** if f^{-1}(V) is a gsp-closed set of (X,τ) for every closed set V of (Y,σ)

(5) rg-continuous [**17]** if f^{-1}(V) is a rg-closed set of (X,τ) for every closed set V of (Y,σ)

(6) gp-continuous **[2]** if f^{-1}(V) is a gp-closed set of (X,τ) for every closed set V of (Y,σ)

(7) gpr-continuous **[9]** if f^{-1}(V) is a gpr-closed set of (X,τ) for every closed set V of (Y,σ)

(8) g^*-continuous **[18]** if f^{-1}(V) is a g^*-closed set of (X,τ) for every closed set V of (Y,σ)

(9) wg-continuous **[16]** if f^{-1}(V) is a wg-closed set of (X,τ) for every closed set V of (Y,σ).

Definition: 2.4: A topological space (X,τ) is said to be

(1) a $T_{1/2}$ space **[10]** if every g-closed set in it is closed.

(2) a T_b space **[6]** if every gs-closed set in it is closed.

(3) a αT_b space **[5]** if every αg-closed set in it is closed.

(4) a $T^*_{1/2}$ **[18]** space if every g^*-closed set in it is closed.

3.BASIC PROPERTIES OF $(gsp)^*$ - CLOSED SETS

We introduce the following definitions

Definition 3.1: A subset A of (X,τ) is said to be a $(gsp)^*$- closed set if cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is gsp-open in X.

Proposition 3.2: Every closed set is $(gsp)^*$-closed.

Proof follows from the definition.

Proposition 3 .3: Every $(gsp)^*$- closed set is g-closed.

Proof: Let A be a $(gsp)^*$- closed set. Let $A \subseteq U$ and U be open. Then $A \subseteq U$ and U is gsp-open and $cl(A) \subseteq U$ since A is $(gsp)^*$-closed.$\therefore$ A is g-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.4: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={a,c} is g-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.5: Every $(gsp)^*$-closed set is gs-closed.

Proof: Let A be a $(gsp)^*$- closed set.Let $A \subseteq U$ and U be open. Then $cl(A) \subseteq U$ since U is gsp-open and A is $(gsp)^*$-closed. $scl(A) \subseteq cl(A) \subseteq U$ .Hence A is gs-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.6: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={b} is gs-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.7: Every $(gsp)^*$-closed set is αg-closed, but not conversely.

Proof: Let A be a gsp^*- closed set. $cl(A) \subseteq U$ since U is gsp-open and A is $(gsp)^*$-closed.But

$\alpha\, cl(A) \subseteq cl(A) \subseteq U$ Hence A is αg-closed.

Example 3.8: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={b} is αg-closed but not $(gsp)^*$-closed in(X,τ).

Proposition3.9: Every $(gsp)^*$-closed set is gsp-closed but not conversely.

Proof: Let A be a $(gsp)^*$-closed set. Let $A \subseteq U$ and U be open. $cl(A) \subseteq U$ since U is gsp-open and A is$(gsp)^*$-closed. sp $cl(A) \subseteq cl(A) \subseteq U$ $\therefore$ A is gsp-closed.

Example 3.10: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={b} is gsp-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.11: Every $(gsp)^*$-closed set is rg-closed.

The converse of the above proposition is not true.

Example 3.12: Let X= {a, b, c}, τ = {φ, X,{b},{a, b}}.Then A={a} is rg-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.13: Every $(gsp)^*$-closed set is gp-closed but not conversely.

Example 3.14: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={b} is gp-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.15: Every $(gsp)^*$-closed set is gpr-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.16: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={a} is gpr-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.17: Every $(gsp)^*$-closed set is g^*-closed.

Proof: Let A be a $(gsp)^*$-closed set. Let $A \subseteq U$ and U be g-open. Then U is gsp-open.

and $cl(A) \subseteq U$ since A is$(gsp)^*$-closed.Hence A is g^*-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.18: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={a,c} is g^* -closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.19: Every $(gsp)^*$-closed set is wg-closed but not conversely.

Proof: Let A be a $(gsp)^*$-closed set.Let $A \subseteq U$ and U be open.Then U is gsp-open and $cl(A) \subseteq U$ since A is $(gsp)^*$-closed. cl(int(A))$\subseteq$cl(A)$\subseteq U$ $\therefore$ A is wg-closed.

Example 3.20: Let X= {a, b, c}, τ = {φ, X,{a},{a, b}}.Then A={b} is wg-closed but not $(gsp)^*$-closed in(X,τ).

Proposition 3.21: If A and B are $(gsp)^*$-closed sets then A∪B is also $(gsp)^*$-closed.

Proof: follows from the fact that cl(A∪B) = cl(A)∪cl(B)

Proposition 3.22: If A is $(gsp)^*$-closed set of (X,τ) such that $A \subseteq B \subseteq$cl(A), then B is also a $(gsp)^*$-closed set of (X,τ).

Proof: Let U be a gsp-open set of (X,τ) such that $B \subseteq U$ .Then A$\subseteq U$ where U is gsp-open.

Since A is $(gsp)^*$-closed,cl(A)$\subseteq U$ .Then cl(B) $\subseteq U$ Hence B is $(gsp)^*$-closed.

Proposition 3.23: If A is a $(gsp)^*$-closed set of (X,τ), then cl(A)\A does not contain any non-empty gsp-closed set.

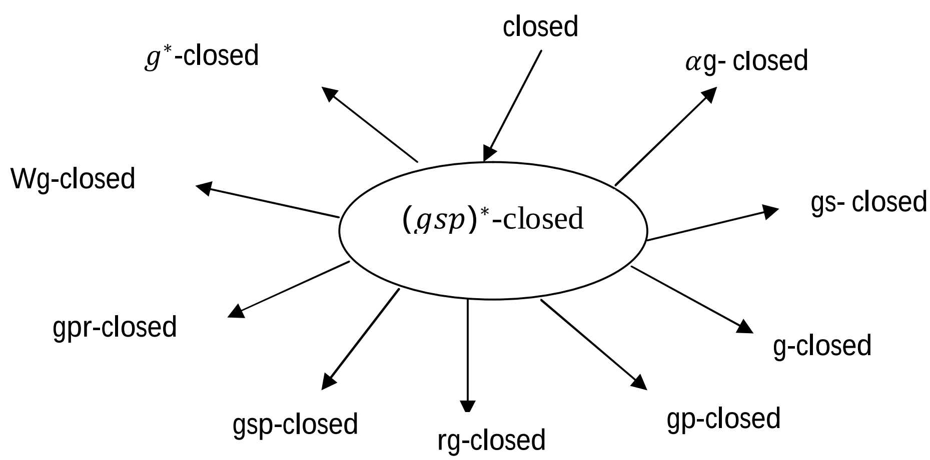
Proof: Let F be a gsp-closed set of (X,τ) such that $F \subseteq cl(A) \setminus A$ .Then $A \subseteq X \setminus F$ .Since A is gsp- closed cl(A)$\subseteq X \setminus F$ .This implies $F \subseteq X \setminus cl(A)$

Hence $F \subseteq (A \setminus cl(A)) \bigcap (cl(A) \setminus A) = \phi$

$\therefore$ F=φ. $\therefore$ cl(A)\A does not contain any non-empty gsp-closed set.

Proposition 3.24: If A is both gsp-open and$(gsp)^*$-closed then A is closed.

The above results can be represented in the following figure.



where A→B represents A implies B and B need not imply A.

4. $(gsp)^*$ -continuous and $(gsp)^*$ -irresolute maps.

We introduce the following definitions:

Definition 4.1: A function *f*: (X,τ) → (Y,σ) is called $(gsp)^*$ -continuous if f^{-1}(V) is a $(gsp)^*$-closed set in (X,τ) for every closed set V of (Y,σ)

Definition 4.2: A function *f*: (X,τ) →(Y,σ) is called $(gsp)^*$ irresolute if f^{-1}(V) is a $(gsp)^*$-closed set in (X,τ) for every $(gsp)^*$ closed set V of (Y,σ)

Theorem 4.3: Every continuous map is $(gsp)^*$-continuous

Proof: Let *f*: (X,τ) →(Y,σ) be a continuous map. Let F be a closed set in (Y,σ) Since f is continuous f^{-1}(F) is closed in (X,τ) and hence f^{-1}(F) is $(gsp)^*$ closed. Therefore f is $(gsp)^*$-continuous.

Theorem 4.4: Every $(gsp)^*$-continuous map is (1) g-continuous (2) gs-continuous (3) αg-continuous (4) gsp-continuous (5) rg-continuous (6) gp-continuous (7) gpr-continuous (8) g^*-continuous and (9) wg-continuous but not conversely.

Proof: Let *f*: (X,τ) →(Y,σ) be $(gsp)^*$-continuous and let F be a closed set of (Y,σ).Since *f* is $(gsp)^*$-continuous f^{-1}(F) is $(gsp)^*$ -closed in (X,τ) Then f^{-1}(F) is g-closed, gs-closed, αg-closed, gsp-closed, rg-closed, gp-closed, gpr-closed,g^*-closed and wg-closed. Hence f is g-continuous, gs-continuous,αg-continuous, gsp-continuous, rg-continuous, gp-continuous, gpr-continuous, g^*-continuous and wg-continuous.

Example 4.5: Let X= {a, b, c}=Y.τ = {φ ,X,{a},{a,b}},σ = {φ,Y,{b}}. Let *f*: (X,τ) →(Y,σ) be the identity map. The closed sets of Y are φ,Y and {a, c}.f^{-1}{a, c}={a ,c} is not $(gsp)^*$-closed in (X,τ).Hence *f* is not $(gsp)^*$-continuous . f^{-1}{a, c}={a ,c} is g-closed , αg-closed, gp-closed and g^*-closed. Hence *f* is g-continuous, αg-continuous, gp-continuous, and g^*-continuous.

Example 4.6: Let X= {a, b, c}= Y.τ = {φ ,X,{a},{a,b}},σ = {φ,Y,{a, c}}Let *f*: (X,τ) →(Y,σ) be the identity map .The closed sets of Y are φ,Y {b}. f^{-1}{b}={b} is not $(gsp)^*$-closed in (X,τ).Therefore f is not $(gsp)^*$-continuous .(1) f^{-1}{b}={b} is gs-closed and hence f is gs-continuous.(2) f^{-1}{b}={b} is gsp-closed and hence *f* is gsp-continuous.

Example 4.7: Let X= {a, b, c} = Y.τ = {φ , X,{b},{a, b}},σ = {φ,Y,{a}} Let *f*: (X,τ) →(Y,σ) be the identity map .The closed sets of Y are φ,Y and {b, c}. f^{-1}{b, c}={b, c} is rg-closed but not $(gsp)^*$-closed and hence *f* is rg-continuous but not $(gsp)^*$-continuous.

Example 4.8: let X= {a ,b, c}=Y,τ ={φ,X,{a},{a,b}},σ = {φ,Y,{a}},Let *f*: (X,τ) →(Y,σ) be defined as *f*(a) = c, *f*(b) = a, *f*(c) = b. f^{-1}{b, c} ={a,c} is gpr-closed in (X,τ),but not $(gsp)^*$-closed in (X,τ). ∴ f is gpr- continuous but not $(gsp)^*$ continuous.

Example 4.9: let X= {a ,b, c}=Y,τ ={φ,X,{a},{a,b}}, σ = {φ,Y,{c}},Let *f*: (X,τ) →(Y,σ) be defined as *f*(a) = a, *f*(b) =c, *f*(c) = b. f^{-1}{a, b}={a, c} is wg-closed in (X,τ),but not $(gsp)^*$-closed in (X,τ). Hence *f* is wg- continuous but not $(gsp)^*$ continuous.

Theorem 4.10: Every $(gsp)^*$-irresolute is $(gsp)^*$ continuous.

Proof: Let f: (X,τ) →(Y,σ) be a $(gsp)^*$-irresolute. Let V be a closed set of (Y,σ).Then V is $(gsp)^*$-closed and f^{-1}(V) is $(gsp)^*$-closed since f is a $(gsp)^*$-irresolute. Hence f is $(gsp)^*$- continuous.

Theorem 4.11: Let f: (X,τ) →(Y,σ) be a $(gsp)^*$ - irresolute then f is 1.αg- continuous 2.gs- continuous 3.g- continuous 4.gp- continuous 5.rg- continuous 6. g^*- continuous7.wg-continuous 8.gpr- continuous but not conversely.

Proof: Since every $(gsp)^*$ - irresolute is $(gsp)^*$ continuous, f is $(gsp)^*$ continuous.Then by theorem 4.4 the result follows.

Example 4.12: Let X= {a, b, c} = Y; τ = {φ,X,{a},{a,b},σ={φ,Y,{a,b}}.Let f: (X,τ) →(Y,σ) be the identity map.f^{-1}{ c}= {c} is αg-closed and hence f is αg-continuous. $(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{a, c}={a, c} is not $(gsp)^*$-closed in (X,τ).Hence f is not a $(gsp)^*$ - irresolute.

Example 4.13: Let X ={a, b, c} = Y,τ={φ,X,{a},{a,b}},σ = {φ,Y,{a,b}}.Let f: (X,τ) →(Y,σ) be defined by f(a) = a, f(b) = c, f(c) = b. f^{-1}{c}={b} is gs-closed in (X,τ) and hence f is gs-continuous. $(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{c} = {b} is not $(gsp)^*$-closed in (X,τ).Hence f is not a $(gsp)^*$-irresolute.

Example 4.14:Let X={a, b, c} = Y,τ={φ,X,{a},{a,b}},σ = {φ,Y,{a,b}}.Let f: (X,τ) →(Y,σ) be defined by f(a) = c, f(b) = a, f(c) = c. f^{-1}{c}={a,c} is g-closed in (X,τ) and hence f is g-continuous.$(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}. f^{-1}{c}={a,c} is not $(gsp)^*$-closed in (X,τ).Hence f is not a $(gsp)^*$-irresolute.

Example 4.15: Let X={a, b, c} = Y,τ={φ,X,{a},{a,b}},σ = {φ,Y,{a,b}}.Let f: (X,τ) →(Y,σ) be defined by f(a) = a, f(b) = c, f(c) = b. f^{-1}{c}={b} is gp-closed in (X,τ).∴ f is gp-continuous.$(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{c}={b} is not $(gsp)^*$-closed in (X,τ). Hence f is not a $(gsp)^*$-irresolute.

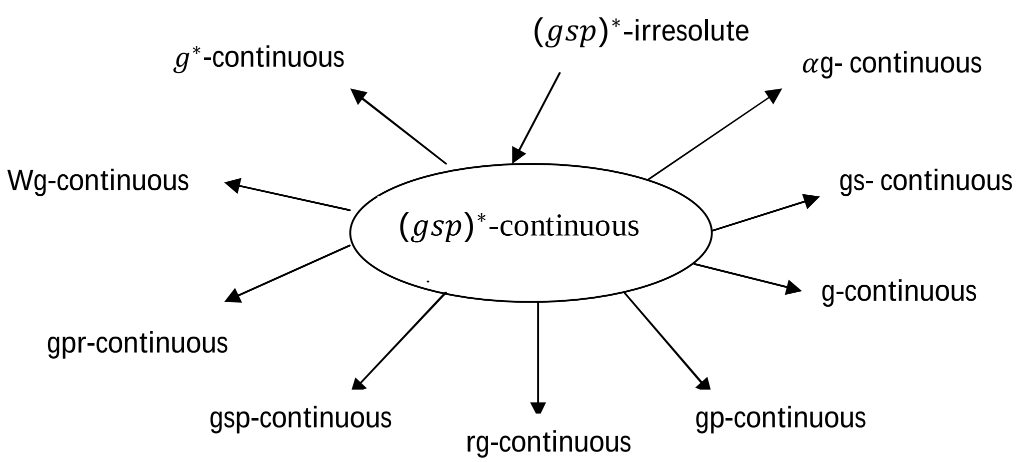
Example 4.16: Let X={a, b, c} = Y,τ={φ,X,{b},{a,b}},σ = {φ,Y,{a,b}}.Let f: (X,τ) →(Y,σ) be defined by f(a) = b, f(b) = c, f(c) = a. f^{-1}{c} ={b} is rg-closed in (X,τ).∴ f is rg-continuous.$(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{c}={b} is not $(gsp)^*$-closed in (X,τ).Hence f is not a $(gsp)^*$-irresolute.

Example 4.17: Let X={a, b, c} = Y,τ={φ,X,{a},{a,b}},σ = {φ,Y,{a,b}}.Let *f*: (X,τ) →(Y,σ) be defined by *f*(a) = c, *f*(b) = a, *f*(c) = c. f^{-1}{c}={a,c} is g^*-closed in (X,τ).∴ f is g^*-continuous. $(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{c} ={a,c} is not $(gsp)^*$-closed in (X,τ). Hence *f* is not a $(gsp)^*$-irresolute.

Example 4.18: Let X={a, b, c} = Y,τ={φ,X,{a},{a,b}},σ = {φ,Y,{a,b}}.Let *f*: (X,τ) →(Y,σ) be defined by *f*(a) = a, *f*(b) = c, *f*(c) = b. f^{-1}{c} ={b} is wg-closed in (X,τ).∴ f is wg-continuous. $(gsp)^*$-closed sets of (Y,σ) are φ,Y {c},{a, c} and {b, c}.f^{-1}{c}={b} is not $(gsp)^*$-closed in (X,τ).Hence *f* is not a $(gsp)^*$-irresolute.

Example 4.19: Let X={a, b, c} = Y,τ={φ,X,{a},{a, b}},σ = {φ,Y,{a,b}}.Let *f*: (X,τ) →(Y,σ) be defined by *f*(a) = b, *f*(b) = c, *f*(c) = a. f^{-1}{c} ={b} is gpr-closed in (X,τ).∴ f is gpr-continuous. $(gsp)^*$-closed sets of (Y,σ) are φ ,Y {c},{a, c} and {b, c}.f^{-1}{c}={b} is not $(gsp)^*$-closed in (X,τ). Hence *f* is not a $(gsp)^*$-irresolute.

The above results can be represented in the following figure.



where A→B represents A implies B and B need not imply A.

5. Applications of $(gsp)^*$-closed sets

Definition 5.1: A space (X,τ) is called a T^*_{gsp}- space if every $(gsp)^*$-closed set is closed.

Definition 5.2: A space (X,τ) is called a gT^*_{gsp}- space if every g-closed set is $(gsp)^*$ closed.

Theorem 5.3: Every $T_{1/2}$-space is T^*_{gsp}- space

Proof: Let (X, τ) be a $T_{1/2}$- space . Let A be a $(gsp)^*$-closed set.Since every $(gsp)^*$-closed set is g-closed, A is g-closed.Since (X,τ) is a $T_{1/2}$-space, A is closed. Hence (X,τ) is a T^*_{gsp}- space

The converse of the above proposition need not be true in general as seen in the following example.

Example 5.4: Let X = {a, b, c} and τ= {φ,X,{a},{a,b}}.(X,τ) is a T^*_{gsp}-space since all the $(gsp)^*$-closed sets of (X,τ) are closed. A= {a ,c} is g-closed but it is not closed and hence it is not a $T_{1/2}$- space.Hence a T^*_{gsp}- space need not be a $T_{1/2}$- space.

Theorem 5.5: Every αT_b-space is T^*_{gsp}- space but not conversely.

Proof: Let (X, τ) be a αT_b- space. Let A be a $(gsp)^*$-closed set.Every $(gsp)^*$-closed set is αg-closed and hence A is αg-closed.Since (X,τ) is a αT_b-space, A is closed.Hence (X,τ) is a T^*_{gsp}- space.

Example 5.6: Let X = {a, b, c} and τ= {φ,X,{a},{a,b}}.(X,τ) is a T^*_{gsp}-space since all the $(gsp)^*$-closed sets of (X,τ) are closed. A= {b} is αg-closed, but it is not closed and hence it is not a αT_b- space.Hence a T^*_{gsp}- space need not be a αT_b- space.

Theorem 5.7: Every $T^*_{1/2}$-space is a T^*_{gsp}- space.

The converse is not true as seen in the following example.

Example 5.8: Let X = {a, b, c} and τ= {φ, X,{a},{a,b}}.(X,τ) is a T^*_{gsp}-space. A= {a, c} is g^*-closed, but it is not closed and hence it is not a $T^*_{1/2}$- space.Hence a T^*_{gsp}- space need not be a $T^*_{1/2}$- space.

Theorem 5.9: Every T_b-space is T^*_{gsp}- space but not conversely.

Example 5.10: Let X = {a, b, c} and τ= {φ, X, {a},{a,b}}.(X,τ) is a T^*_{gsp}-space A= {b} is gs-closed, but it is not closed, and hence it is not a T_b- space.Hence a T^*_{gsp}- space need not be a T_b- space.

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