(g*p)*-Closed Sets In Topological Spaces

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Abstract: In this paper, we have introduced a new class of sets called (g*p)*-closed sets which is properly placed in between the class of closed sets and the class of g-closed sets. As an application, we introduce two new spaces namely, ${}_gT_p$ and ${}_gT_p$* spaces. We have also introduced (g*p)*-continuous and (g*p)*-irresolute maps and their properties are investigated.

Keywords: (g*p)*-closed sets, (g*p)*-continuous maps, (g*p)*-irresolute maps and ${}_gT_p$ and ${}_gT^*_p$ spaces

1 INTRODUCTION

Levine [9]introduced the class of g-closed sets in 1970.Maki.et.al[11]defined αg -closed sets and $g\alpha$ -closed sets in 1994..Arya and Tour[3] defined gs-closed sets in 1990.Dontchev[7], Gnanambal[8] and Palaniappan and Roa[16]introduced gsp-closed sets gpr-closed sets and rg-closed sets respectively. Veerakumar[17] introduced and studied the concepts of g*-preclosed sets and g*-precontinuity in topological spaces in 1991.

The purpose of the paper is to introduce the concept of (g*p)*-closed sets, ${}_gT_p$ spaces and ${}_gT^*_p$ spaces. Further we have introduced (g*p)*-continuous and (g*p)*-irresolute maps.

2. PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a (X,τ) space, cl(A) and int(A) denote the closure and the interior of A respectively.

The class of all closed subsets of a space of a space (X,τ) is denoted by C(X,τ) .

Definition 2.1: A subset A of a topological space (X,τ) is called

(1) $pre-open$ [13] if $A \subseteq \text{int}(cl(A))$ and a $pre-closed$ set if $cl(\text{int}(A)) \subseteq A$.

(2) $semi-open$ [10] if $A \subseteq cl(\text{int}(A))$ and a $semi-closed$ if $\text{int}(cl(A)) \subseteq A$.

(3) $semi-preopen$ [1] if $A \subseteq cl(\text{int}(cl(A)))$ and a $semi-preclosed$ if $\text{int}(cl(\text{int}(A))) \subseteq A$.

(4) $\alpha-open$ [14] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and $\alpha-closed$ if $cl(\text{int}(cl(A))) \subseteq A$.

(5) $regular-open$ [18] if $A = \text{int}(cl(A))$

Definition 2.2: A subset A of a topological space (X,τ) is called

(1) $generalized\ closed$ (briefly $g-closed$) [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(2) $regular\ generalized$ closed (briefly rg-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ).

(3) $generalized\ semi-closed$ (briefly $gs-closed$) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(4) $\alpha-generalized\ closed$ (briefly $\alpha g-closed$) [11] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(5) $wg-closed$ [17] if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(6) $generalized\ preregular\ closed$ (briefly gpr-closed) [8] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ).

(7) $generalized\ semi-preclosed$ (briefly $gsp-closed$) [7] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(8) $generalized\ preclosed$ (briefly $gp-closed$) [12] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(9) $g*-preclosed$ (briefly $g*p$-closed)[16] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g- open in (X,τ)

Definition 2.3: A function $f:(X,\tau) \to (Y,\sigma)$ is called

(1) $g-continuous$[4] if $f^{-1}(V)$ is a $g-$ closed set of (X,τ) for every closed set V of (Y,σ).

(2) $\alpha g-continuous$ [8] if $f^{-1}(V)$ is an $\alpha g-$ closed set of (X,τ) for every closed set V of (Y,σ)

(3) $gs-continuous$[6] if $f^{-1}(V)$ is a $gs-$ closed set of (X,τ) for every closed set V of (Y,σ).

(4) $rg-continuous$[15] if $f^{-1}(V)$ is a $rg-$ closed set of (X,τ) for every closed set V of (Y,σ)

(5) $gp-continuous$[2] if $f^{-1}(V)$ is a $gp-$ closed set of (X,τ) for every closed set V of (Y,σ)

(6) $wg-continuous$ [17] if $f^{-1}(V)$ is a $wg-$ closed set of (X,τ) for every closed set V of (Y,σ).

(7) $gsp-continuous$[7] if $f^{-1}(V)$ is a $gsp-$ closed set of (X,τ) for every closed set V of (Y,σ).

(8) $gpr-continuous$ [8] if $f^{-1}(V)$ is a $gpr-$ closed set of (X,τ) for every closed set V of (Y,σ).

Definition: 2.4: A topological space (X,τ) is said to be

(1) a $T_{1/2}$ $space$ [9] if every $g-$closed set in it is closed.

(2) a T_b $space$ [5] if every $gs-$closed set in it is closed

(3) a ${}_{\alpha}T_b$ $space$ [4] if every $\alpha g-$closed set in it is closed.

3. BASIC PROPERTIES OF (g*p)*-CLOSED SETS

We now introduce the following definition.

Definition 3.1: A subset A of a topological space (X,τ) is called a (g*p)*-closed set, if $cl(A)\subseteq U$.

whenever $A\subseteq U$ and U is $g*p-open$.

Proposition 3.2: Every closed set is (g*p)*-closed.

Proof follows from the definitions.

Proposition 3.3: Every (g*p)*-closed set is (1) g-closed (2) αg -closed (3) gs-closed (4) gp -closed (5) wg -closed (6) gsp-closed(7) rg -closed and (8) gpr -closed but not conversely.

Proof: Let A be a (g*p)*-closed set. Let $A\subseteq U$ and U be open .Then U is $g*p$–open.

Since A is (g*p)*-closed,

(1) $cl\subseteq U$ and hence A is g–closed.

(2) $\alpha cl(A) \subseteq cl(A) \subseteq U$ and hence A is αg –closed.

(3) $scl\,(A) \subseteq cl\,(A) \subseteq U$ and hence A is gs –closed.

(4) $pcl\,(A) \subseteq cl\,(A) \subseteq U$ and hence A is gp –closed.

(5) $cl \subseteq U$ and which implies $cl(\text{int}(A) \subseteq cl\,(A) \subseteq U)$ hence A is wg –closed.

(6) $cl(A) \subseteq U$ and hence $spcl\,(A) \subseteq U$ therefore A is gsp–closed.

Proof for 7 & 8

(7) $cl(A) \subseteq U$ and hence A is rg –closed.

(8) $pcl\,(A) \subseteq cl\,(A) \subseteq U$ and hence A is gpr –closed.

Example 3.4: Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a,b\}$. Then A is g–closed, αg -closed, gs-closed, rg-closed, gp-closed, wg-closed,gsp-closed, gpr-closed but it is not (g*p)*- closed.

Proposition 3.5: If A and B are (g*p)*- closed sets, then $A \cup B$ is also a (g*p)*- closed set.

Proof follows from the fact that $cl\,(A \cup B) = cl\,(A) \cup cl\,(B)$.

Proposition 3.6: If A is both $g * p$ –open and (g*p)*- closed, then A is closed.

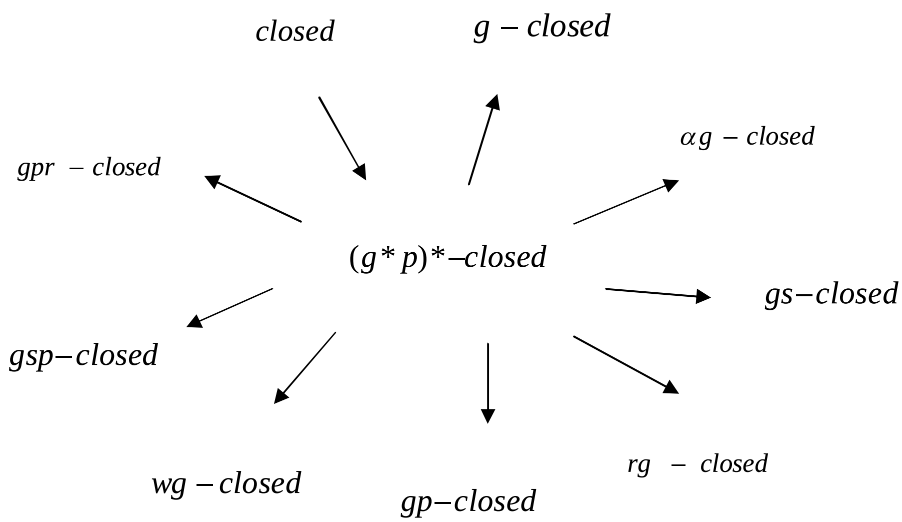
Proof follows from the definition of (g*p)*- closed sets.

Proposition 3.7: If A is (g*p)* closed set of (X,τ), such that $A \subseteq B \subseteq cl(A)$, then B is also a (g*p)*- closed set of (X,τ).

Proof: Let U be a $g * p -$open set of (X,τ) , such that $B \subseteq U$ .Then $A \subseteq U$ where U is g*p–open.

Since A is (g*p)*- closed, $cl(A) \subseteq U$ .Then $cl(B) \subseteq U$ .Hence B is (g*p)*- closed.

The above results can be represented in the following figure.



Where A ⟶ B represents A implies B and B need not imply A

4. (g*p)*-CONTINUOUS MAPS AND (g*p)*-IRRESOLUTE MAPS

We introduce the following definitions.

Definition: 4.1: A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called $(g*p)*$- continuous if the inverse image of every closed set in (Y,σ) is $(g*p)*$- closed in (X,τ).

Definition: 4.2: A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be a $(g*p)*$- irresolute map if $f^{-1}(V)$ is a $(g*p)*$ - closed set in (X,τ) for every (g*p)*- closed set V of (Y,σ).

Theorem 4.3: Every continuous map is (g*p)* - continuous.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a continuous map and let F be a closed set in (Y,σ). Then $f^{-1}(F)$ is closed in (X,τ). Since every closed set is (g*p)*- closed, $f^{-1}(F)$ is (g*p)*- closed. Then f is (g*p)*-continuous.

Theorem 4.4: Every (g*p)* -continuous map is $g-$ continuous, αg –continuous, gs -continuous, rg-continuous, gp -continuous, wg -continuous, gsp –continuous and gpr-continuous but not conversely.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a $(g*p)*$ - continuous map. Let V be a closed set in (Y,σ). Since f is (g*p)* -continuous, $f^{-1}(V)$ is (g*p)* closed in (X,τ). Then $f^{-1}(V)$ is g –closed, αg -closed, gs -closed ,rg-closed, gp -closed, wg -closed, gsp –closed and gpr-closed set of (X,τ).

Example 4.5: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}$ $, \sigma = \{\phi, Y, \{c\}\}$ $f : (X,\tau) \to (Y,\sigma)$ be the identity map. Then $f^{-1}(\{a,b\}) = \{a,b\}$ is not (g*p)* -closed in (X,τ).But $\{a,b\}$ is g –closed set, αg -closed set, gs-closed set. Then f is g -continuous, αg - continuous, gs - continuous but not (g*p)*-continuous.

Example 4.6: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}$ $, \sigma = \{\phi, Y, \{c\}\}$. $f : (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = b, f(b) = c, f(c) = a$. Then $f^{-1}(\{a,b\}) = \{a,c\}$ is not (g*p)*- closed in (X,τ).But $\{a,c\}$ is rg -closed. Hence f is rg - continuous but not (g*p)* -continuous.

Example 4.7: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}$ $, \sigma = \{\phi, Y, \{c\}\}$. $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = b, f(b) = c, f(c) = a$. Then $f^{-1}(\{a,b\}) = \{a,c\}$ is gp -closed but not (g*p)*-closed. Then f is gp -continuous but not (g*p)*-continuous.

Example 4.8: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}$ $, \sigma = \{\phi, Y, \{b,c\}\}$. $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = c, f(b) = a, f(c) = b$. Then $f^{-1}(\{a\}) = \{b\}$ is wg -closed but not (g*p)*-closed in (X,τ). Hence f is wg - continuous but not (g*p)* -continuous.

Example 4.9: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}, \sigma = \{\phi, Y, \{a\}, \{a,b\}\}$.

$f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = b, f(b) = a, f(c) = c$.Then $f^{-1}(\{b,c\}) = \{a,c\}$ is not

(g*p)*-closed in (X,τ),but it is gsp -closed .Hence f is gsp - continuous but not (g*p)*-continuous.

Example 4.10: Let $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}$ $, \sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}\}$.

$f : (X,\tau) \to (Y,\sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$. Then $f^{-1}(\{b,c\}) = \{a,c\}$ is not (g*p)*-closed in (X,τ), but it is gpr-closed .Hence f is gpr-continuous but not (g*p)* -continuous.

Theorem 4.11: Every (g*p)*- irresolute map is (g*p)* -continuous**.**

Proof follows from the definition.

Theorem 4.12: Every (g*p)* -irresolute map is g – continuous**,** αg - continuous, gs -continuous, rg-continuous, gp - continuous, wg - continuous, gsp - continuous and gpr-continuous

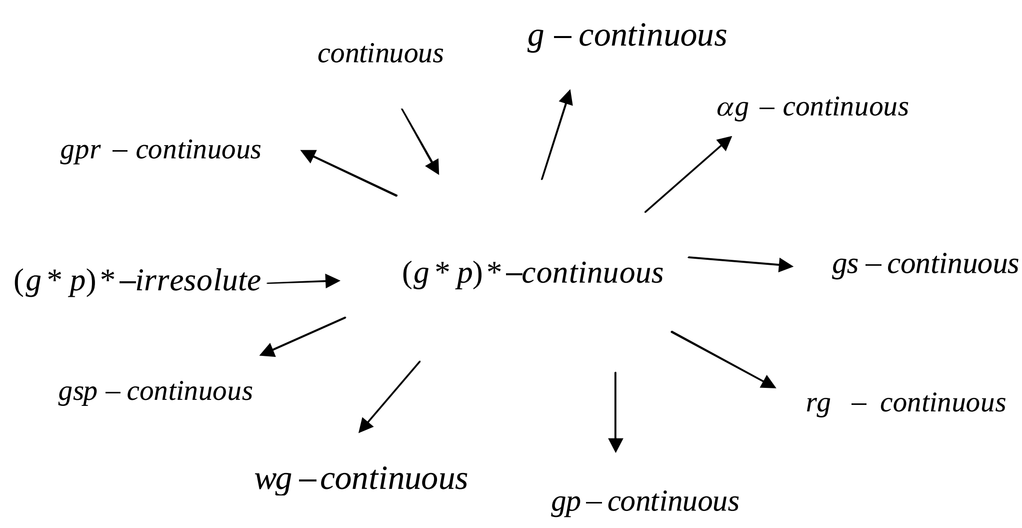
Proof follows from theorems (4.4) and (4.11).

The converse of the above theorem need not be true in general as seen in the following examples.

Example 4.13: Let $X = Y = \{a,b,c\}$ $\tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{c\}, \{a,c\}\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity map. ϕ, Y, $\{b\}$, $\{a,b\}$, $\{b,c\}$ are closed sets of Y. $f^{-1}(\{a,b\}) = \{a,b\}$, $f^{-1}(\{b,c\}) = \{b,c\}$ and $f^{-1}\{b\} = \{b\}$ are g-closed, gs-closed, rg-closed set, gp-closed, wg-closed, gsp-closed, gpr-closed. Hence f is $g-$continuous, gs -continuous, rg-continuous, gp - continuous, wg - continuous, gsp - continuous and gpr-continuous.(g*p)*-closed sets of Y are ϕ, Y, $\{b\}$, $\{a,b\}$, $\{b,c\}$. $f^{-1}(\{a,b\}) = \{a,b\}$ is not (g*p)*-closed in (X,τ). Hence f is not a (g*p)*-irresolute.

Example 4.14: Let $X = Y = \{a,b,c\}$ $\tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{c\}, \{a,c\}\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then $f^{-1}(\{b\}) = \{c\}$, $f^{-1}(\{a,b\}) = \{a,c\}$ and $f^{-1}(\{b,c\}) = \{b,c\}$ are αg -closed, and hence f is αg -continuous. (g*p)*-closed sets are ϕ, Y, $\{b\}$, $\{a,b\}$, $\{b,c\}$. $f^{-1}(\{a,b\}) = \{a,c\}$ is not (g*p)*-closed in (X,τ). Hence f is αg -continuous but not a (g*p)*-irresolute.

The above results can be represented in the following figure.



where A ⟶ B represents A implies B and B need not imply A.

Theorem 4.15: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be two functions. Then

[1] $g \circ f$ is (g*p)*-continuous if g is continuous and f is (g*p)*-continuous.

[2] $g \circ f$ is (g*p)* -irresolute if both f and g are (g*p)* -irresolutes.

[3] $g \circ f$ is (g*p)*-continuous if g is (g*p)*-continuous and f is (g*p)*-irresolute.

Proof [1]: Let $f:(X,\tau)\to(Y,\sigma)$ be (g*p)*-continuous and $g:(Y,\sigma)\to(Z,\eta)$ be continuous. Let F be a closed set in (Z,η). Since g is continuous, $g^{-1}(F)$ is closed in (Y,σ). Since f is (g*p)*-continuous, $f^{-1}(g^{-1}(F))$ is (g*p)* -closed in (X,τ). Hence $(g\circ f)^{-1}(F)$ is (g*p)*-closed. $\therefore$ $g\circ f$ is (g*p)*-continuous.

Proof [2]: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be (g*p)*- irresolutes. Let F be a (g*p)*-closed set in (Z,η). $(g\circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$. Since g is (g*p)*-irresolute, $g^{-1}(F)$ is (g*p)*-closed in (Y,σ). Since f is (g*p)*-irresolute, $f^{-1}(g^{-1}(F))$ is (g*p)*-closed in (X,τ). $\therefore$ $g\circ f$ is (g*p)*-irresolute.

Proof [3]: Let $f:(X,\tau)\to(Y,\sigma)$ be (g*p)*-irresolute and $g:(Y,\sigma)\to(Z,\eta)$ be (g*p)*-continuous. Let F be closed in (Z,η). Since g is (g*p)*-continuous, $g^{-1}(F)$ is (g*p)*- closed in (Y,σ). Since f is (g*p)*-irresolute, $f^{-1}(g^{-1}(F))$ is (g*p)*-closed in (X,τ). $\therefore$ $g\circ f$ is (g*p)*-continuous

5. APPLICATION OF (g*p)*-CLOSED SETS

We introduce the following definitions.

Definition: 5.1: A space (X,τ) is called a ${}_gT_p - space$ if every set (g*p)*-closed set is closed.

Definition: 5.2: A space (X,τ) is called a ${}_gT^*_p$ if every $g-closed$ set is (g*p)*-closed.

Theorem 5.3: Every $T_{1/2} - space$ is a ${}_gT_p - space$.

Proof: Let (X,τ) be a $T_{1/2} - space$. Let A be a (g*p)*-closed set. Since every (g*p)*-closed set is $g-closed$, A is g-closed. Since (X,τ) is $T_{1/2} - space$, A is closed. $\therefore$ (X,τ) is a ${}_gT_p - space$.

The converse of the above theorem need not be true in general as seen in the following example.

Example 5.4: Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\}\}$. $(g*p)*$-closed sets of X are $\phi,X,\{b,c\}$ and the g-closed sets are $\phi,X,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}$. Every (g*p)*-closed set is closed. Hence the space (X,τ) is ${}_gT_p$- space. $A=\{b\}$ is g-closed but it is not closed. Hence the space (X,τ) is not $T_{\frac{1}{2}}$- space.

Theorem 5.5: Every ${}_{\alpha}T_b - space$ is a ${}_{g}T_p - space$ but not conversely.

Proof follows from the definitions.

Example 5.6: Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ Here $(g*p)*$-closed sets are $\phi, X, \{b,c\}$ and the αg - closed sets are $\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$. Since every (g*p)*-closed set is closed, the space (X,τ) is a ${}_{g}T_p$ - space. A = {c} is αg -closed but not closed. Therefore the space (X,τ) is not a ${}_{\alpha}T_b$ - space.

Theorem 5.7: Every T_b space is a ${}_{g}T_p$ –space

Proof follows from the definitions. The converse is not true.

Example 5.8: Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ $(g*p)*$-closed sets are $\phi, X, \{b,c\}$ and gs- closed sets are $\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$. Since every (g*p)*-closed set is closed, the space (X,τ) is a ${}_{g}T_p$ - space. A = {c} is gs-closed but not closed. Therefore the space (X,τ) is not a

T_b - space.

Theorem 5.9: Every $T_{\frac{1}{2}}$ - space is a ${}_{g}T^*{}_{p}$ - space.

Proof follows from the definitions.

Theorem 5.10: Every ${}_{\alpha}T_b$ -space is a ${}_{g}T^*{}_{p}$ - space.

Proof: Let (X,τ) be a ${}_{\alpha}T_b$ – space. Let A be g -closed. Then A is αg –closed. Since the space is

${}_{\alpha}T_b$ -space, A is closed and hence A is $(g*p)*$–closed. Therefore the space (X,τ) is a ${}_{g}T^*{}_{p}$ –space.

Theorem 5.11: Every T_b - space is a ${}_{g}T^*{}_{p}$ - space but not conversely.

Proof follows from the definitions.

Example 5.12: Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$. Here (g*p)*-closed sets are $\phi, X, \{b,c\}, \{a,b\}, \{b\}$, g- closed sets are $\phi, X, \{b\}, \{a,b\}, \{b,c\}$ and the gs-closed sets are $\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}$ .Since every (g*p)*-closed set is g-closed, the space (X,τ) is a

${}_{g}T^*{}_{p}$ - space. A = {c} is gs-closed but not closed. Therefore the space (X,τ) is not a T_b - space.

Theorem 5.13: Let f : (X, τ) $\rightarrow$(Y, σ) be a $(g*p)*$- continuous map and let (X, τ) be a ${}_gT_p$- space then f is continuous.

Proof: Let f : (X, τ) $\rightarrow$(Y, σ) be a $(g*p)*$- continuous map. Let F be a closed set of (Y, σ). Since f is $(g*p)*$-continuous, f^{-1}(F) is $(g*p)*$- closed in (X, τ). Since (X, τ) is a ${}_gT_p$- space, f^{-1}(F) is closed in (X, τ). Therefore f is continuous.

Theorem 5.14: Let f : (X, τ) $\rightarrow$(Y, σ) be a g - continuous map where (X, τ) is a ${}_gT*_p$-space. Then f is $(g*p)*$ -continuous.

Proof: Let f : (X, τ) $\rightarrow$(Y, σ) be a g - continuous map. Let F be a closed set in (Y, σ).Since f is g - continuous, therefore f^{-1}(F) is g - closed in (X, τ). Since (X, τ) is a ${}_gT*_p$ space, f^{-1}(F) is (g*p)*-closed. Therefore f is $(g*p)*$ -continuous.

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