(i,j) Strongly a**-Closed Sets in Bitopological Spaces

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ABSTRACT

In this paper we introduce a new class of sets namely, (i,j) strongly α^{**}-closed sets. Properties of this set are investigated and we introduce new spaces namely, (i,j)$T_{s\alpha^{**}}$ spaces, (i,j)$g^*T_{s\alpha^{**}}$ spaces and (i,j)sg^* $T_{s\alpha^{**}}$ spaces.

Keywords: (i,j) strongly α^{**}-closed sets, (i,j)$T_{s\alpha^{**}}$ spaces, (i,j)$g^*T_{s\alpha^{**}}$ spaces, (i,j)sg^* $T_{s\alpha^{**}}$ spaces.

1.INTRODUCTION

A triple (X,τ_1,τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies on X is called bitopological space and Kelly[4] initiated the study of such spaces.Veera Kumar[15] introduced the concept of g^*-closed sets in 2000.Parimezlazhagan and Subramania Pillai[9] introduced the strongly g^*-closed sets in 2012.The purpose of this paper is to introduce the concept of strongly α^{**} closed sets in bitopological spaces. Three new spaces (i,j) $T_{s\alpha^{**}}$space,(i,j) $_{g^*}T_{s\alpha^{**}}$space,(i,j) $_{sg^*}T_{s\alpha^{**}}$space in bitopological spaces are introduced and some of their properties are investigated.

2.PRELIMINARIES

Throughout this paper (X,τ_1,τ_2) ,(Y,σ_1,σ_2) represent non-empty bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. If A is a subset of X with topology τ then cl(A) and int(A) denote the closure of A and the interior of A in X respectively. We recall the following definitions , which will be used often throughout this paper:

Definition 2.1:A subset A of a topological space (X,τ) is called

(1) a semi-open set[6] if $A\subseteq cl(int(A))$ and a semi-closed set if $int(cl(A))\subseteq A$

(2)a α-open set[8] if $A\subseteq int(cl(int(A))$ and a α-closed set if $cl(int(cl(A))\subseteq A$

(3)a g-closed set [7] if $cl(A) \subseteq U$, whenever $A\subseteq U$ and U is open in (X,τ).

(4)a g^*-closed set [13] if $cl(A) \subseteq U$, whenever $A\subseteq U$ and U is g open in (X,τ).

(5)a g^{**}-closed set [10] if $cl(A) \subseteq U$, whenever $A\subseteq U$ and U is g^*-open in (X,τ).

(6)a strongly g^*-closed set [9] if $cl(int(A)) \subseteq U$, whenever $A\subseteq U$ and U is g-open in (X,τ).

(7) a sg-closed set [3] if $scl(A) \subseteq U$, whenever $A\subseteq U$ and U is semi-open in (X,τ).

(8) a gs-closed set [2] if scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in (X,τ).

(9) a ψ-closed or sg^* closed set [14] if scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is sg-open in (X,τ).

(10) a α^*-closed set [15]if cl(A) $\subseteq$U ,whenever A $\subseteq$ U and U is α-open in (X,τ)

(11)a α^{**}-closed set [17] if cl(A) $\subseteq$U,whenever A $\subseteq$ U and U is α^*-open in (X,τ)

Definition 2.2:

A subset A of bitopological space (X,τ_1, τ_2) is called

(1) an (i,j) g^*-closed [11] if τ_j- cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is g open in $\tau_{i.}$

(2)an (i,j) α^{**}-closed [17] if τ_j- cl(A) $\subseteq$U,whenever A $\subseteq$ U and U is α^*-open in $\tau_{i.}$

(3)an (i,j) gs-closed [12] if τ_j - scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in $\tau_{i.}$

(4)an (i,j) sg-closed [4] if τ_j -scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is semi-open in $\tau_{i.}$

(5)an (i,j) ψ-closed or sg^* closed[1] if τ_j -scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is sg-open in $\tau_{i.}$

(6)an (i,j) strongly g^*-closed [16] if τ_j-cl(int(A)) $\subseteq$ U , whenever A $\subseteq$ U and U is g-open in $\tau_{i.}$

Definition 2.3:

A bitopological space (X,τ_1, τ_2) is called

(1) an (i,j)- $T_{1/2}^*$ space[11] if every (i,j)-g^*-closed set is τ_j-closed.

(2) an (i,j)- T_d space[12] if every (i,j)-gs-closed set in it is g-closed.

3.BASIC PROPERTIES OF (i,j)STRONGLY α^{**}-CLOSED SETS

We introduce the following definition:

Definition 3.1:A subset A of a bitopological space (X,τ_1, τ_2) is called (i,j)-strongly α^{**}closed if τ_j-cl(int(A)) $\subseteq$U whenever A $\subseteq$U and U is τ_i-α^*open.

Proposition 3.2: Every τ_j closed set A is (i,j) strongly α^{**}-closed

Proof follows from the definition.

The following example supports that a (i,j) strongly α^{**}-closed set need not be closed in general

Example 3.3: Let X={a,b,c}; τ_1={ϕ,{c},{a,c},X}; τ_2={ϕ,{a},X}. Then A={b} is (i,j) strongly α^{**}-closed but not τ_j-closed in (X, τ_1, τ_2).

Proposition 3.4: Every (i,j) α^{**}-closed is (i,j) strongly α^{**}-closed.

Proof: Let A be (i,j) α^{**}-closed. Let A $\subseteq$U and U be τ_i - open. Since A is (i,j) α^{**}-closed., τ_j -cl(A) $\subseteq$U, τ_j -cl(int(A)) $\subseteq \tau_j$ -cl(A) $\subseteq$U. Hence A is (i,j) strongly α^{**}-closed.

The converse of above proposition need not be true in general as seen in following example:

Example 3.5: Let X={a,b,c}; τ_1={ϕ,X,{b}}; τ_2={ϕ,X,{c}}. Then A={b} is (i,j) strongly α^{**}-closed but not a (i,j)-α^{**}closed in (X, τ_1, τ_2).

Proposition 3.6: Every (i,j) g^*-closed set is (i,j) strongly α^{**}-closed.

Proof: Let A be (i,j) g^*-closed.Let A⊆U and U be τ_i-α^*open. Then U is τ_i-g open and hence τ_j-cl(A)⊆U, since A is (i,j)-g^* closed. But τ_j-cl(int(A))⊆τ_j-cl(A)⊆U. Then A is (i,j) strongly α^{**}-closed.

The converse of above proposition need not be true in general as seen in following example:

Example 3.7: Let X={a,b,c}; τ_1={ϕ,X,{b}}; τ_2={ϕ,X,{c}}. Then A={a} is (i,j) strongly α^{**}-closed but not (i,j)-g^*closed in (X, τ_1, τ_2).

Proposition 3.8: Every (i,j) strongly g^*-closed set is (i,j) strongly α^{**}-closed.

The converse of above proposition need not be true in general as seen in following example:

Example 3.9: Let X={a,b,c}; τ_1={ϕ,X,{b}}; τ_2={ϕ,X,{c}}. Then A={c} is (i,j) strongly α^{**}-closed but not (i,j)-strongly g^*closed in (X, τ_1, τ_2).

Remark 3.10: (i,j) strongly α^{**}-closedness is independent of (i,j) sg-closedness.

Example 3.11: Let X={a,b,c}; τ_1={ϕ,X,{a},{b},{a,b}}; τ_2={ϕ,X,{a},{a,b}}. Then A={a,c} is (i,j) strongly α^{**}-closed but not (i,j) sg-closed in (X, τ_1, τ_2).

Example 3.12: Let X={a,b,c}; τ_1={ϕ,X,{a},{a,b}}; τ_2={ϕ,X,{a},{b},{a,b}}. Then A={a} is (i,j) sg-closed but not (i,j) strongly α^{**}-closed in (X, τ_1, τ_2).

Hence (i,j) strongly α^{**}-closedness is independent of (i,j) sg-closedness.

Remark 3.13: (i,j) strongly α^{**}-closedness is independent of (i,j) gs-closedness.

Example 3.14: Let X={a,b,c}; τ_1={ϕ,X,{a},{a,b}}; τ_2={ϕ,X,{a},{b,c}}. Then A={b} is (i,j) strongly α^{**}-closed but not (i,j) gs-closed in (X, τ_1, τ_2).

Example 3.15: Let X={a,b,c}; τ_1={ϕ,X,{a},{a,b}}; τ_2={ϕ,X,{a},{b},{a,b}}. Then A={b} is (i,j) gs-closed but not (i,j) strongly α^{**}-closed in (X, τ_1, τ_2).

Hence (i,j) strongly α^{**}-closedness is independent of (i,j) gs-closedness.

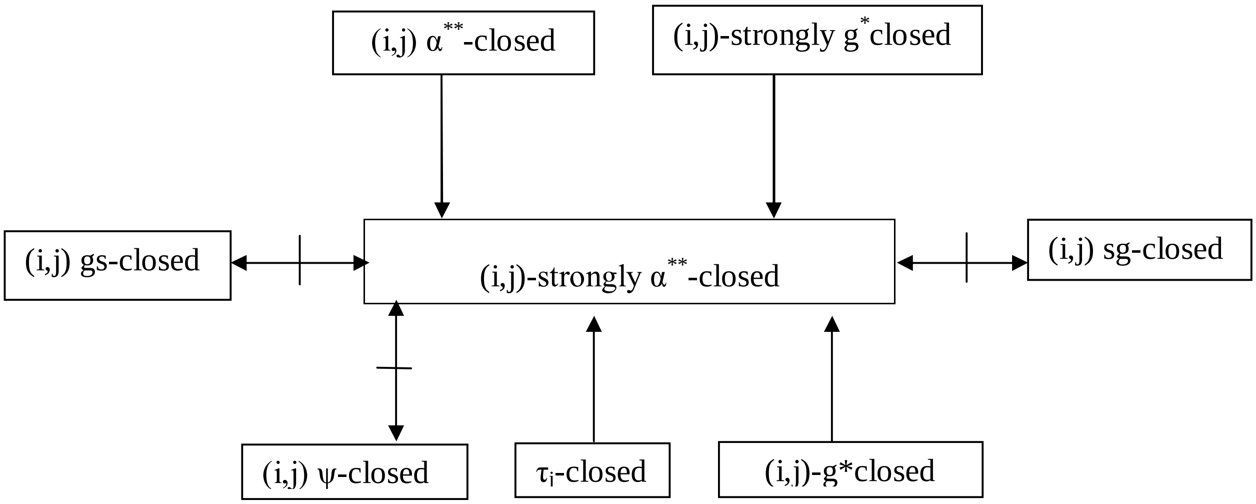
Remark 3.16: (i,j) strongly α^{**}-closedness is independent of (i,j) ψ-closedness.

Example 3.17: Let X={a,b,c}; τ_1={ϕ,X,{a},{b},{a,b}}; τ_2={ϕ,X,{a},{a,b}}. Then A={a,c} is (i,j) strongly α^{**}-closed set but not (i,j) ψ -closed in (X, τ_1, τ_2).

Example 3.18: Let X={a,b,c}; τ_1={ϕ,X,{a},{a,b}}; τ_2={ϕ,X,{a},{b},{a,b}}. Then A={a} is (i,j) ψ -closed but not (i,j) strongly α^{**}-closed in (X, τ_1, τ_2).

Hence (i,j) strongly α^{**}-closedness is independent of (i,j) ψ-closedness.

The above results can be represented in the following figure:



where A ⟶ B represents A implies B and B need not imply A

A ⟷ B represents A and B are independent.

4.APPLICATIONS OF (i,j) STRONGLY α^{**}-CLOSED SETS

As applications of (i,j) strongly α^{**}-closed sets, new spaces, namely (i,j)$T_{s\alpha^{**}}$ spaces, (i,j)g$^*T_{s\alpha^{**}}$ spaces, (i,j)sg* $T_{s\alpha^{**}}$ spaces are introduced.

We introduce the following definitions.

Definition 4.1:A space (X, τ_1, τ_2) is called a (i,j)$T_{s\alpha^{**}}$ space if every (i,j) strongly α^{**}-closed set is τ_j-closed.

Definition 4.2: A space (X, τ_1, τ_2) is called a (i,j) g$^*T_{s\alpha^{**}}$ space if every (i,j) strongly α^{**}-closed set is (i,j) g*-closed.

Definition 4.3: A space (X, τ_1, τ_2) is called a (i,j) sg$^*T_{s\alpha^{**}}$ space if every (i,j) strongly α^{**}-closed set is (i,j) strongly g*-closed.

Theorem 4.4: Every (i,j)$T_{s\alpha^{**}}$ space is a (i,j) $T_{1/2}{}^*$space

Proof: Let A be a (i.j)-g* closed set.Then A is (i,j) strongly α^{**}-closed .Since (X, τ_1, τ_2) is a (i,j)-$T_{s\alpha^{**}}$ space, A is τ_j-closed. ∴ (X, τ_1, τ_2) is a (i,j) $T_{1/2}{}^*$space.

The converse of above theorem need not be true in general as seen in following example

Example 4.5: Let X={a,b,c};τ_1={ϕ,X,{a}}; τ_2={ϕ,X,{a},{a,b}}.Then (X, τ_1, τ_2) is a $T_{1/2}{}^*$space.A={a,b} is (i,j) strongly α^{**}-closed but not τ_j-closed . ∴ The space (X, τ_1, τ_2) is not a (i,j) Tsα^{**} space.

Theorem 4.6: Every (i,j) $T_{s\alpha^{**}}$ space is a (i,j) sg$^*T_{s\alpha^{**}}$ space

Proof follows from the definitions

The converse of above theorem need not be true

Example 4.7: Let X={a,b,c};τ_1={ϕ,X,{a},{b},{a,b}}; τ_2={ϕ,X,{a},{a,b}}.Then (X, τ_1, τ_2) is a (i,j) $sg^*T_{s\alpha^{**}}$ space.A={b} is (i,j) strongly α^{**}-closed but not a τ_j-closed set. ∴The space (X, τ_1, τ_2) is not a (i,j) $Ts\alpha^{**}$ space.

Theorem 4.8: Every (i,j) $g^*T_{s\alpha^{**}}$ space is a (i,j) $sg^*T_{s\alpha^{**}}$ space

Proof: Let A be a (i,j) strongly α^{**}-closed set .Then A is (i,j)-g^* closed and hence ,A is (i,j) strongly g^* closed. ∴(X, τ_1, τ_2) is a (i,j) $sg^*T_{s\alpha^{**}}$ space

The converse of above theorem is not true.

Example 4.9: Let X={a,b,c};τ_1={ϕ,X,{a},{b},{a,b}}; τ_2={ϕ,X,{a},{a,b}}.Then (X, τ_1, τ_2) is a (i,j) $sg^*T_{s\alpha^{**}}$ space.A={b} is (i,j) strongly α^{**}-closed but not (i,j) g^*-closed. ∴The space (X, τ_1, τ_2) is not a (i,j) g^* $Ts\alpha^{**}$ space.

Theorem 4.10:A space (X, τ_1, τ_2) which is both (i,j) $T_{1/2}s^*$ space and (i,j) $sg^*T_{s\alpha^{**}}$ space is a (i,j) $g^*T_{s\alpha^{**}}$ space

Proof: Let A be a (i,j)-strongly α^{**}-closed set.Then A is (i,j)-strongly g^*-closed,since (X, τ_1, τ_2) is a $sg^*T_{s\alpha^{**}}$ space.Now A is τ_j-closed, since the space is a (i,j) $T_{1/2}s^*$ space.Hence A is (i,j) g^*-closed. ∴The space (X, τ_1, τ_2) is a (i,j) g^* $Ts\alpha^{**}$ space.

Remark 4.11:(i,j) $T_d$ness is independent from (i,j) -$Ts\alpha^{**}$ness,(i,j) $sg^*T_{s\alpha^{**}}$ness.

Example 4.12: Let X={a,b,c};τ_1={ϕ,X,{a}}; τ_2={ϕ,X,{a},{a,b}}. Then (X, τ_1, τ_2) is a (i,j) T_d space. A={b} is (i,j) - strongly α^{**}-closed but not a τ_2 closed set .Hence (X, τ_1, τ_2) is not a (i,j) -$Ts\alpha^{**}$ space.

Let X={a,b,c};τ_1={ϕ,X,{a},{a,b}}; τ_2={ϕ,X,{a},{b},{a,b}}. Then (X, τ_1, τ_2) is a (i,j)$T_{s\alpha}^{**}$ space. A={a} is (i,j) - gs closed but not a (i,j) g closed set .Hence (X, τ_1, τ_2) is not a (i,j) -T_d space.

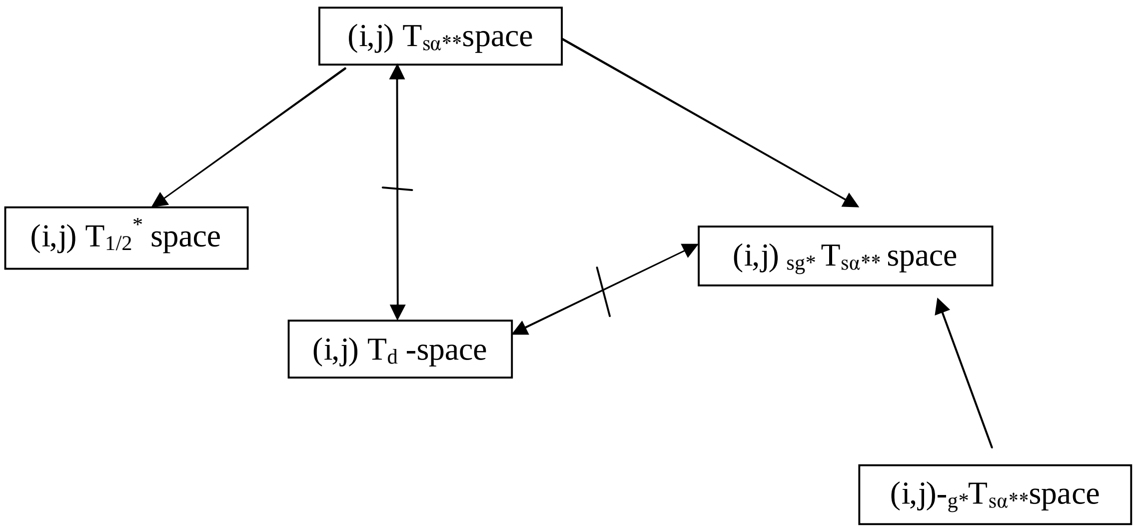
Hence (i,j) $T_d$ness is independent from (i,j) -$Ts\alpha^{**}$ness.

Example 4.13: Let X={a,b,c};τ_1={ϕ,X,{a}}; τ_2={ϕ,X,{a},{a,b}}. Then (X, τ_1, τ_2) is a (i,j) T_d space. A={a,b} is (i,j) - strongly α^{**}-closed but not (i,j) strongly g^*closed. Hence (X, τ_1, τ_2) is not a (i,j) - (i,j)$sg^*Ts\alpha^{**}$ space.

Let X={a,b,c};τ_1={ϕ,X,{a},{b},{a,b}}; τ_2={ϕ,X,{a},{a,b}}. Then (X, τ_1, τ_2) is a (i,j) $sg^*T_{s\alpha}^{**}$ space. A={b} is (i,j) - gs closed but not (i,j) g closed set .Hence (X, τ_1, τ_2) is not a (i,j) -T_d space.

Hence (i,j) $T_d$ness is independent from (i,j) $sg^*T_{s\alpha^{**}}$ness.

The above results can be represented in the following figure:



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