# New Paranormed Sequence Spaces $l_{\alpha}(p, \lambda), c(p, \lambda)$ and $c_{0}(p, \lambda)$ Generated by an Infinite Matrix 

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## Abstract

In this paper we introduce a set of new paranormed sequence spaces $l_{\infty}(p, \lambda), c(p, \lambda)$ and $c_{0}(p, \lambda)$ which are generated by an infinite lower uni triangular matrix $\lambda=S^{n}$ where $S=\left(s_{n k}\right)$ is an infinite matrix given by,
$S=\left(s_{n k}\right)=\left\{\begin{array}{lr}1 ; & 0 \leq k \leq n \\ 0 ; & k>n\end{array}\right.$
as defined in [6] . We also compute the basis for the spaces $c(p, \lambda)$ and $c_{0}(p, \lambda)$, obtain $\boldsymbol{\beta}$ - dual for all these spaces and characterize the matrix classes $\left(l_{\infty}(p, \lambda), l_{\infty}\right),\left(l_{\infty}(p, \lambda), c\right)$ and $\left(l_{\infty}(p, \lambda), c_{0}\right)$.
Keywords: Paranormed sequence spaces, $\boldsymbol{\beta}$ - dual, matrix transformation

## I. INTRODUCTION

By $\omega$ we mean the space of all complex valued sequences. A vector subspace of $\omega$ is called a sequence space. We shall write $l_{\infty}, c$ and $c_{0}$ for the spaces of all bounded, convergent and null sequence respectively. A linear topological space $X$ over the field R is said to be a paramormed space if there is a subadditive function $g: X \rightarrow \mathrm{R}$ such that $g(\theta)=0, g(x)=g(-x)$ and scalar multiplication is continuous i.e. $\left|\alpha_{n}-\alpha\right| \rightarrow 0$ and $g\left(x_{n}-x\right) \rightarrow 0$ imply $g\left(\alpha_{n} x_{n}-\alpha x\right) \rightarrow 0$, for all $\alpha$ 's in R and all x's in X , where $\theta$ is the zero vector in the linear space X . If $p=\left(p_{k}\right)$ be a bounded sequence of strictly positive real numbers, I.J. Maddox [1,2] defined the sequence spaces $l_{\infty}(p), c(p)$ and $c_{0}(p)$ as follows:

$$
\begin{aligned}
& l_{\infty}(p)=\left\{x=\left(x_{k}\right) \in \omega: \sup _{k}\left|x_{k}\right|^{p_{k}}<\infty\right\} \\
& c(p)=\left\{x=\left(x_{k}\right) \in \omega: \lim _{k \rightarrow \infty}\left|x_{k}-l\right|^{p_{k}}=0, \text { for some } l \in \square\right\} \\
& c_{0}(p)=\left\{x=\left(x_{k}\right) \in \omega: \lim _{k \rightarrow \infty}\left|x_{k}\right|^{p_{k}}=0\right\} .
\end{aligned}
$$

The space $c_{0}(p)$ is a complete paranormed space paranormed by $h(x)=\sup _{k}\left|x_{k}\right|^{\frac{p_{k}}{M}}$ and the spaces $l_{\infty}(p)$ and $c(p)$ are complete paranormed spaces paranormed by $h(x)$ if and only if inf $p_{k}>0$.

Let $X$ and $Y$ be any two sequence spaces and $A=\left(a_{n k}\right) ; n, k \in \mathrm{~N}$ be infinite matrix of complex numbers $a_{n k}$. Then we say that $A$ defines a matrix mapping $X$ into $Y$; and it is denoted by writing $A: X \rightarrow Y$ if for every sequence $x=\left(x_{k}\right) \in X$, the sequence $\left((A x)_{n}\right)$ is in $Y$, where
$(A x)_{n}=\sum_{k=1}^{\infty} a_{n k} x_{k} ; n \in \square$
By $(X, Y)$ we denote the class of all matrices $A$ such that $A: X \rightarrow Y$. Thus, $A \in(X, Y)$ if and only if the series on right side of (1.1) converges for each $n \in \square$ and every $x \in X$; and we write, $A x=\left\{(A x)_{n}\right\}_{n \in \square} \in Y$ for all $x \in X$.

The matrix domain $X_{A}$ of an infinite matrix $A$ in a sequence space $X$ is defined by
$X_{A}=\left\{x=\left(x_{k}\right) \in \omega: A x \in X\right\}$
which is a sequence space.
We now introduce new sequence spaces $X(p, \lambda)$
for $X \in\left\{l_{\infty}, c, c_{0}\right\}$ as,
$X(p, \lambda)=\left\{x=\left(x_{k}\right): \lambda x \in X(p)\right\}$ where
$\lambda=\left(\lambda_{n k}\right)=S^{n}=\left\{\begin{array}{lc}n-k+1 ; & n \geq k \\ 0 ; & \text { otherwise }\end{array}\right.$
and $S=\left(s_{n k}\right)=\left\{\begin{array}{lr}1 ; & 0 \leq k \leq n \\ 0 ; & k>n\end{array}\right.$.
$X(p, \lambda)$ is now the set of all sequences $\left\{u_{k}\right\}$ whose $\lambda=S^{n}$ - transforms are in the sequence space $X \in\left\{l_{\infty}, c, c_{0}\right\}$ where the sequence
$\left\{u_{k}\right\}=\left\{\sum_{i=1}^{k}(k-i+1) x_{i}\right\}$
Using the notation as in (1.2) we can represent $X(p, \lambda)$ as
$X(p, \lambda)=[X(p)]_{\lambda}$.
We shall now establish some propositions.
Proposition 1: Sequence space $c_{0}(p, \lambda)$ is linear metric space paranormed by g , defined by

$$
\begin{aligned}
g(x) & =\sup _{k}|\lambda x|^{\frac{p_{k}}{M}}, \text { where } M=\max \left(1, \sup _{k} p_{k}\right) \\
& =\sup _{k}\left|u_{k}\right|^{\frac{p_{k}}{M}}
\end{aligned}
$$

Proof: From the definition of $g$ it is clear that $g(x)=0 \Leftrightarrow x=0$ and $g(-x)=g(x)$ for all $x \in c_{0}(p, \lambda)$. To show linearity of $c_{0}(p, \lambda)$ with respect to coordinate-wise addition and scalar multiplication, let us take any two sequences $x, y \in c_{0}(p, \lambda)$ and scalars $\alpha, \beta \in \square$. Since $\lambda$ is linear operator by [8], we note that

$$
\begin{aligned}
& g(\alpha x+\beta y)=\sup _{k}|\lambda(\alpha x+\beta y)|^{\frac{p_{k}}{M}} \\
& \leq \max \{1,|\alpha|\} \sup _{k}|\lambda x|^{\frac{p_{k}}{M}}+\max \{1,|\beta|\} \sup _{k}|\lambda y|^{\frac{p_{k}}{M}} \\
& \quad=\max ^{\max }\{1,|\alpha|\} g(x)+\max \{1,|\beta|\} g(y)
\end{aligned}
$$

This follows the sub additivity of g i.e.
$g(x+y) \leq g(x)+g(y)$
Now it remained to show the continuity of scalar multiplication in $c_{0}(p, \lambda)$. For it, let $\left\{x^{n}\right\}$ be any sequence of the points in $c_{0}(p, \lambda)$ such that
$g\left(x^{n}-x\right) \rightarrow 0$ and $\left\{\alpha_{n}\right\}$ be sequence of real scalars such that $\alpha_{n} \rightarrow \alpha$. Now by using (1.5), we have
$g\left(x^{n}\right) \leq g(x)+g\left(x^{n}-x\right)$
Further,
$g\left(\alpha_{n} x^{n}-\alpha x\right)=\sup _{k}\left|\lambda\left(\alpha_{n} x^{n}-\alpha x\right)\right|^{\frac{p_{k}}{M}}$
$\leq\left(\left|\alpha_{n}-\alpha\right|^{\frac{p_{k}}{M}} g\left(x^{n}\right)+\left|\alpha_{n}-\alpha\right|^{\frac{p_{k}}{M}} g\left(x^{n}-x\right)\right)<\infty$
( for all n)
(1.6)

Since $\left\{g\left(x^{n}\right)\right\}$ is bounded, we find from (1.6) that
$g\left(\alpha_{n} x^{n}-\alpha x\right)<\infty$ for all $n \in \square$.
That is, the scalar multiplication for g is continuous and therefore g is a paranorm on the sequence space $c_{0}(p, \lambda)$.
It can easily be verified that g is the paranorm for the spaces $l_{\infty}(p, \lambda)$ and $c(p, \lambda)$ if and only if inf $p_{k}>0$.
Proposition 2 :The sequence spaces $X(p, \lambda)$ for $X \in\left\{l_{\infty}, c, c_{0}\right\}$ are complete metric spaces paranormed by g , defined as in proposition 1.
Proof: We prove this proposition for $c_{0}(p, \lambda)$. Take a Cauchy sequence $\left\{x^{n}\right\}$ in the space $c_{0}(p, \lambda)$, where
$x^{n}=\left\{x_{0}^{(n)}, x_{1}^{(n)}, x_{2}^{(n)}, \ldots\right\}$.
Now for given $\varepsilon>0$, there exists a positive integer $n_{0}(\varepsilon)$ such that,
$g\left(x^{n}-x^{m}\right)<\varepsilon$ for all $m, n \geq n_{o}(\varepsilon)$.
Also, from the definition of g for each fixed $k \in \square$, we have
$\left.\mid\left\{\lambda x^{n}\right\}_{k}-\left\{\lambda x^{m}\right\}_{k}\right\}^{\frac{p_{k}}{M}}$
$\leq \sup _{k}\left|\left\{\lambda x^{n}\right\}_{k}-\left\{\lambda x^{m}\right\}_{k}\right|^{\frac{p_{k}}{M}}$
$<\varepsilon$
for all $m, n \geq n_{o}(\varepsilon)$.
Now , this implies that, $\left\{\left(\lambda x^{0}\right)_{k},\left(\lambda x^{1}\right)_{k},\left(\lambda x^{2}\right)_{k}, \ldots\right\}$ is a
Cauchy sequence in $\square$ for each fixed $k \in \square$. Since $\square$ is complete, the sequence $\left\{\lambda x^{n}\right\}_{k}$ converges and let
$\left\{\lambda x^{n}\right\}_{k} \rightarrow\{\lambda x\}_{k}$ as $n \rightarrow \infty$.
For each fixed $k \in \square, m \rightarrow \infty$ and $n \geq n_{o}(\varepsilon)$, it is clear that
$\left|\left\{\lambda x^{n}\right\}_{k}-\{\lambda x\}_{k}\right|^{\frac{p_{k}}{M}} \leq \frac{\varepsilon}{2}$
Since $x^{n}=\left\{x_{k}^{(n)}\right\} \in c_{0}(p, \lambda)$, we have
$\left|\left\{\lambda x^{n}\right\}_{k}\right|^{\frac{p_{k}}{M}} \leq \frac{\varepsilon}{2}$
for each fixed $k \in \square$.
Combining (1.7) and (1.8), we obtain that

$$
\begin{aligned}
& \left|\{\lambda x\}_{k}\right|^{\frac{p_{k}}{M}} \\
& \leq\left\{\left\{\lambda x^{n}\right\}_{k}-\left.\{\lambda x\}_{k}\right|^{\frac{p_{k}}{M}}+\left|\left\{\lambda x^{n}\right\}_{k}\right|^{\frac{p_{k}}{M}}\right. \\
& \leq \varepsilon
\end{aligned}
$$

for all $n \geq n_{o}(\varepsilon)$.
Hence, the sequence $\{\lambda x\} \in c_{0}(p)$. Since $\left\{x^{n}\right\}$ was an arbitrary Cauchy sequence in $c_{0}(p, \lambda)$, we conclude that the space $c_{0}(p, \lambda)$ is complete. It completes the proof.
Proposition 3: The sequence spaces $X(p, \lambda)$ for $X \in\left\{l_{\infty}, c, c_{0}\right\}$ are linearly isomorphic to the respective spaces X.
Proof: For each $x \in X(p, \lambda)$, we have $\lambda x \in X(p)$, where $\lambda$ as defined in (1.3). It is easy to verify that $\lambda$ is linear and bijective. Also the matrix $\lambda$ has an inverse given by,
$\mu=\left(\mu_{n k}\right)=\left\{\begin{array}{lr}1 ; & k=n, n \geq 3 \text { and } k \leq n-2 \\ 0 ; & k>n, n \geq 4 \text { and } k \leq n-3 \\ -2 ; & n \geq 2 \text { and } k \leq n-1\end{array}\right.$
Thus, the sequence spaces $X(p, \lambda)$ is linearly isomorphic to $X(p)$.
Proposition 4: Let $\zeta_{k}=(\lambda x)_{k}$ and $0<p_{k} \leq \sup _{k} p_{k}<\infty$ for all $k \in N$. We define the sequence $\mu^{k}=\left\{\mu_{n}^{(k)}\right\}_{n \in \square}$ for every fixed $k \in N$ as in (1.9). Then,
(i) the sequence $\left\{\mu_{n}{ }^{(k)}\right\}_{n \in \square}$ is the basis for the sequence space $c_{0}(p, \lambda)$ and any $x \in c_{0}(p, \lambda)$ has a unique representation $x=\sum_{k=1}^{\infty} \zeta_{k} \mu^{(k)}$ and
(ii) the set $\left\{v, \mu^{(k)}\right\}$ is a basis for the space $c(p, \lambda)$ and any $x \in c(p, \lambda)$ has a unique representation in the form
$x=l v+\sum_{k=1}^{\infty}\left(\zeta_{k}-l\right) \mu^{(k)}$
where $l=\lim _{k \rightarrow \infty}(\lambda x)_{k}$ and $v^{T}=(1,3,0,0, \ldots)$

## II. DUALS

In this section we find the $\beta$-dual of the sequence space $l_{\infty}(p, \lambda)$. If $X$ be a sequence space, we define $\beta$-dual of $X$ as:
$X^{\beta}=\left\{a=\left(a_{k}\right): \sum_{k=1}^{\infty} a_{k} x_{k}\right.$ is convergent for each $\left.x \in X\right\}$.
Theorem 2.1: Let $p_{k}>0$ for every $k \in N$. Then $l_{\infty}^{\beta}(p, \lambda)=M_{\infty}(p, \lambda)$ where
$M_{\infty}(p, \lambda)=\bigcap_{N=2}^{\infty}\left\{a=\left(a_{k}\right): \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{\frac{1}{p_{k}}}<\infty\right\}$ and $\Delta^{2} a_{k}=\Delta a_{k}-\Delta a_{k+1}$.
Proof: Let $a \in M_{\infty}(p, \lambda)$ and $x \in l_{\infty}(p, \lambda)$. We choose an integer $N>\max \left(1, \sup _{k}\left|u_{k}\right|^{p_{k}}\right)$. Then we have,

$$
\begin{aligned}
& \left|\sum_{k=1}^{m} a_{k} x_{k}\right|\left|=\left|\sum_{k=1}^{m}\left(\Delta a_{k}-\Delta a_{k+1}\right) u_{k}\right| ; \text { where } u_{k}=\sum_{i=1}^{k}(k-i+1) x_{i}\right. \\
& =\left|\sum_{k=1}^{m} \Delta^{2} a_{k} u_{k}\right| \\
& \leq \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right|\left|u_{k}\right| \\
& \leq \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{\frac{1}{p_{k}}} \\
& <\infty .
\end{aligned}
$$

Hence, $M_{\infty}(p, \lambda) \subseteq l_{\infty}^{\beta}(p, \lambda)$.
On the other hand, let $a \in l_{\infty}^{\beta}(p, \lambda)$ but $a \notin M_{\infty}(p, \lambda)$. Then there exists an integer $\mathrm{N}>1$ such that
$\sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{\frac{1}{p_{k}}}=\infty$.
Then, $\left\{\Delta^{2} a_{k}\right\} \notin l_{\infty}^{\beta}(p)=M_{\infty}(p)$.
Hence, there exists a sequence $y=\left\{y_{k}\right\} \in l_{\infty}(p)$ such that $\sum_{k=1}^{\infty} \Delta^{2} a_{k} y_{k}$ does not converge. Although if we define the sequence $\mu=\left\{\mu_{k}\right\}$ by
$\mu_{k}=y_{k-1}-2 y_{k}+y_{k+1}$ with $y_{j}=0$ for $j \leq 0$,
then $\mu \in l_{\infty}(p, \lambda)$ and ,therefore,$\sum_{k=1}^{\infty} a_{k} \mu_{k}=\sum_{k=1}^{\infty} \Delta^{2} a_{k} y_{k}$. Hence it follows that the series $\sum_{k=1}^{\infty} a_{k} \mu_{k}$ does not converge ; which is contradiction to our assumption that $a \in l_{\infty}{ }^{\beta}(p, \lambda)$.Hence we must have,
$\sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{\frac{1}{p_{k}}}<\infty$
This shows that $l_{\infty}{ }^{\beta}(p, \lambda) \subseteq M_{\infty}(p, \lambda)$. It completes the proof.
Theorem 2.2: Let $p_{k}>0$ for every $k \in N$. Then $c_{0}{ }^{\beta}(p, \lambda)=M_{0}(p, \lambda)$ where
$M_{0}(p, \lambda)=\bigcup_{N>1}^{\infty}\left\{a=\left(a_{k}\right): \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{-\frac{1}{p_{k}}}<\infty\right\}$ 。
Proof: Let $a \in M_{0}(p, \lambda)$ and $x \in c_{0}(p, \lambda)$. Then
$\sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{-\frac{1}{p_{k}}}<\infty$ for some $\mathrm{N}>1$ and
$\left|u_{k}\right|^{p_{k}}<\frac{1}{N}$ for all sufficiently large k ; whence for such k ,
$\sum_{k=1}^{\infty}\left|a_{k} x_{k}\right|=\sum_{k=1}^{\infty}\left|\Delta^{2} a_{k} u_{k}\right|$
$\leq \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right|\left|u_{k}\right|$
$\leq \sum_{k=1}^{\infty}\left|\Delta^{2} a_{k}\right| N^{-\frac{1}{p_{k}}}$
< $\infty$
Hence, $M_{0}(p, \lambda) \subseteq c_{0}{ }^{\beta}(p, \lambda)$.
On the other hand, let $a \in c_{0}{ }^{\beta}(p, \lambda)$ but $a \notin M_{0}(p, \lambda)$. Then convergence of $\sum_{k=1}^{\infty} a_{k} x_{k}$ for all $x \in c_{0}(p, \lambda)$ implies that $a \in M_{0}(p, \lambda)$. For otherwise, as in the proof of theorem 2.1, we can easily construct a sequence $\mu \in c_{0}(p, \lambda)$ such that $\sum_{k=1}^{\infty} a_{k} \mu_{k}$ does not converge; which becomes contradiction.
Hence, $c_{0}{ }^{\beta}(p, \lambda) \subseteq M_{0}(p, \lambda)$.
This completes the proof.
Corollary: Let $p_{k}>0$ for every $k \in \square$. Then $c^{\beta}(p, \lambda)=M_{0}(p, \lambda) \cap c s$

## III. MATRIX TRANSFORMATIONS

In this section we characterize the classes $\left(l_{\infty}(p, \lambda), l_{\infty}\right),\left(l_{\infty}(p, \lambda), c\right)$ and $\left(l_{\infty}(p, \lambda), c_{0}\right)$.
Theorem 3.1 :Let $p_{k}>0$ for every $k \in \square$. Then $A \in\left(l_{\infty}(p, \lambda), l_{\infty}\right)$ if and only if
$\sup _{n} \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}<\infty$ for every integer $\mathrm{N}>1$.
Proof:Let the condition holds. Then we have,
$\sup _{n} \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}<\infty$.
Take $x \in l_{\infty}(p, \lambda)$. Then $\lambda x \in l_{\infty}(p)$ and hence $\sup _{k}|\lambda x|^{p_{k}}<\infty$. So there exists an integer $N \geq 1$ such that
$|\lambda x| \leq N^{\frac{1}{p_{k}}}$.
Then, $\left|\sum_{k=1}^{\infty} a_{n k} x_{k}\right|=\left|\sum_{k=1}^{\infty} \Delta^{2} a_{n k} u_{k}\right|$ where $u_{k}=\sum_{i=1}^{k}(k-i+1) x$
$\leq \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right|\left|u_{k}\right|$
$\leq \sup _{n} \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{-\frac{1}{p_{k}}}$
$<\infty$.
Hence it follows that $\sum_{k=1}^{\infty} a_{n k} x_{k}$ converges for each $n \in \square$ and $A x \in l_{\infty}$.
On the other hand, let $A \in\left(l_{\infty}(p, \lambda), l_{\infty}\right)$. As a contrary let us assume that there exists an integer $N>1$ such that
$\sup _{n} \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}=\infty$.
Then the matrix $\left(\Delta^{2} a_{n k}\right) \notin\left(l_{\infty}(p), l_{\infty}\right)$, as in theorem 3 [3]and so there exists a $y=\left(y_{k}\right) \in l_{\infty}(p)$ with $\sup _{k}\left|y_{k}\right|=1$ such that
$\sum_{k} \Delta^{2} a_{n k} y_{k} \neq O(1)$
Although if we define the sequence $\mu=\left\{\mu_{k}\right\}$ by
$\mu_{k}=y_{k-2}-2 y_{k-1}+y_{k}$; with $y_{j}=0$ for $j \leq 0$, then
$\mu \in l_{\infty}(p, \lambda)$ and therefore
$\sum_{k=1}^{\infty} a_{n k} \mu_{k}=\sum_{k=1}^{\infty} \Delta^{2} a_{n k} y_{k}$.
It follows that the sequence $\left\{A_{n}(\mu)\right\} \notin l_{\infty}$; which is contradiction to our assumption.
Thus, $\sup _{n} \sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}<\infty$ and it completes the proof.
Theorem 3.2 : Let $p_{k}>0$ for every $k \in N$. Then $A \in\left(l_{\infty}(p, \lambda), c\right)$ if and only if
(i) $\sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}$ converges uniformly in n for all integers $\mathrm{N}>1$.
(ii) $\lim _{n \rightarrow \infty} \Delta^{2} a_{n k}=\Delta^{2} \alpha_{k}$ for some fixed k.

Proof: Let the conditions (i) and (ii) hold. Then from corollary of theorem 3 [3] the matrix
$\left(\Delta^{2} a_{n k}\right) \in\left(l_{\infty}(p), c\right)$.
By using $\sum_{k=1}^{\infty} a_{n k} x_{k}=\sum_{k=1}^{\infty} \Delta^{2} a_{n k} u_{k}$,
we have, $\left(A_{n}(x)\right) \in\left(l_{\infty}(p, \lambda), c\right)$ for every $n \in \square$.
Hence, $A \in\left(l_{\infty}(p, \lambda), c\right)$.
On the other hand let $A \in\left(l_{\infty}(p, \lambda), c\right)$. Thenfrom (3.1) it follows that
$\left(\Delta^{2} a_{n k}\right) \in\left(l_{\infty}(p), c\right)$.
Hence from corollary of theorem 3 [3], we arrive at the result that the conditions (i) and (ii) hold.
Using the same arguments as in the theorems (3.1) and (3.2), it is straight forward matter to prove the theorem:

Theorem 3.3: Let $p_{k}>0$ for every $k \in N$. Then $A \in\left(l_{\infty}(p, \lambda), c_{0}\right)$ if and only if
(i) $\sum_{k=1}^{\infty}\left|\Delta^{2} a_{n k}\right| N^{\frac{1}{p_{k}}}$ converges uniformly in n for all integers $\mathrm{N}>1$
(ii) $\lim _{n \rightarrow \infty} \Delta^{2} a_{n k}=\Delta^{2} \alpha_{k}$ with $\alpha_{k}=0$ for all $k \in \square$.

## IV. CONCLUSIONS

We conclude that, the new sequence spaces $l_{\infty}(p, \lambda), c(p, \lambda)$ and $c_{0}(p, \lambda)$ have been constructed and studied as a generalization of the sequence spaces $l_{\infty}(p), c(p)$ and $c_{0}(p)$ respectively by infinite matrix $S^{n}=\lambda$ as the generator. The sequence $b=\left(b_{k}\right)=\{(1,0,0, \ldots),(-2,1,0,0, \ldots),(1,-2,1,0,0, \ldots), \ldots\} \notin l_{\infty}(p)$ but $\lambda b \in l_{\infty}(p)$.

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