

Soft P-Connectedness Via Soft P-Open Sets

J.Subhashinin¹, Dr.C.Sekar²

¹ Department of Mathematics, VV College of Engineering, Tisayanvilai- INDIA.

² Department of Mathematics, Aditanar College of Arts and Science, Tirchendur- INDIA.

Abstract

This paper introduces soft pre-connectedness in soft topological spaces. The notations of interior and closure are generalized using these sets. In a soft topological space, a soft set F_A is said to soft pre-open if there exists a soft open set F_O such that $F_A \subseteq F_O \subseteq \overline{F_A}$. A detail study is carried out on the soft pre-neighborhood system and soft pre-connectedness via soft pre-open sets.

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1. Introduction

Soft set theory was first introduced by Molodtsov [4] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2013, Bin Chen [1] studied some local properties by soft semi open sets. For example, soft semi-neighborhood of the soft point, soft semi-connectedness etc. Mahanta and P.K.Data [3] introduced semi open, semi closed soft sets and studied semi interior and semi closure of a soft set in a soft topological space. As well known, connectivity occupies very important place in topology. Many authors have presented different kinds of connectivity in general, fuzzy, intuitionistic fuzzy and soft topological spaces. In 2012, Santhi and Jayanthi [6] studied intuitionistic fuzzy generalized semi-pre connected space. At this juncture, in this paper we introduce a notation of soft pre-neighborhood system, soft pre-connectedness by means of soft pre-open sets and a detailed study of some of its properties. The organization of the paper is as follows: Section 2 is the preliminary part where definitions and some properties of soft sets (in our form) and features of soft topologies are described. In section 3, we define soft pre-

neighborhood system and studied their properties. In section 4, we define soft pre-separated sets, soft pre- connectedness via soft pre-open sets in a soft topological space. Section 5, concludes the paper.

2. Preliminaries

For basic notations and definitions not given here, the reader can refer [1-7].

2.1. Definition. [4] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)): x \in E, f_A(x) \in P(U)\}$, where E is a set of parameters, $A \subseteq E$, $P(U)$ is the power set of U , and $f_A: A \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called an approximate function of the soft set F_A . The value of $f_A(x)$ may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over U is denoted by $S(U)$.

2.2. Example. [2] Suppose that there are five cars in the universe. Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ under consideration, and that $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ is a set of decision parameters. The x_i ($i = 1, 2, 3, 4, 5, 6, 7, 8$) stand for the parameters “expensive”, “beautiful”, “manual gear”, “cheap”, “automatic gear”, “in good repair”, “in bad repair” and “costly” respectively. In this case, to define a soft set means to point out expensive cars, beautiful cars and so on. It means that, Consider the mapping f_E given by “cars (.)”, where (.) is to be filled in by one of the parameters $x_i \in E$. For instance, $f_E(x_1)$ means “car (expensive)”, and its functional value is the set $\{c \in U: c \text{ is an expensive car}\}$ and so,

Let $A \subseteq E$, the soft set F_A that describes the “attractiveness of the cars” in the opinion of a buyer say Ram, may be defined like $A = \{x_2, x_3, x_4, x_5, x_7\}$, $f_A(x_2) = \{c_2, c_3, c_5\}$, $f_A(x_3) = \{c_2, c_4\}$, $f_A(x_4) = \{c_1\}$, $f_A(x_5) = \{U\}$ and $f_A(x_7) = \{c_3, c_5\}$. We can view this soft set F_A as consisting of the following collection of approximations:

$$F_A = \{(x_2, \{c_2, c_3, c_5\}), (x_3, \{c_2, c_4\}), (x_4, \{c_1\}), (x_5, \{U\}), (x_7, \{c_3, c_5\})\}.$$

2.3. Definition. [7] The soft set $F_A \tilde{\in} S(U)$ is called a soft point in F_E , denoted by (x_{f_A}) , if for the element $x \in A$ and $f_A(x) \neq \emptyset$ and $f_A(x') = \emptyset$ for all $x' \in A - \{x\}$.

The soft point (x_{f_A}) is said to be in the soft set F_B , denoted by $(x_{f_A}) \tilde{\in} F_B$ if for the element $x \in A$ and $f_A(x) \subseteq f_B(x)$.

2.4. Definition. [5] Let $F_A \in S(U)$. The soft power set of F_A is defined by $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \subseteq F_A, i \in I \subseteq N\}$ and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.

2.5. Example. [5] Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_2, e_3\}$ and $F_A = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1, u_2\})\}$. Then $F_{A_1} = F_A$, $F_{A_2} = F_\emptyset$, $F_{A_3} = \{(e_2, \{u_2, u_3\})\}$, $F_{A_4} = \{(e_2, \{u_2\})\}$, $F_{A_5} = \{(e_2, \{u_3\})\}$, $F_{A_6} = \{(e_2, \{u_2\}), (e_3, \{u_1, u_2\})\}$, $F_{A_7} = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\}$, $F_{A_8} = \{(e_2, \{u_3\}), (e_3, \{u_2\})\}$, $F_{A_9} = \{(e_2, \{u_2\}), (e_3, \{u_1\})\}$, $F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}$, $F_{A_{11}} = \{(e_3, \{u_1\})\}$, $F_{A_{12}} = \{(e_3, \{u_2\})\}$, $F_{A_{13}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1\})\}$, $F_{A_{14}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_2\})\}$, $F_{A_{15}} = \{(e_2, \{u_3\}), (e_3, \{u_1\})\}$, $F_{A_{16}} = \{(e_2, \{u_2\}), (e_3, \{u_2\})\}$ are all soft subset of F_A . So $|\tilde{P}(F_A)| = 2^4 = 16$.

2.6. Definition. [5] Let $F_E \in S(U)$. A soft topology on F_E denoted by $\tilde{\tau}$ is a collection of soft subsets of F_E having the following properties:

- (i). $F_\emptyset, F_E \in \tilde{\tau}$
- (ii). $\{F_{E_i} \subseteq F_E : i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \bigcup_{i \in I} F_{E_i} \in \tilde{\tau}$
- (iii). $\{F_{E_i} \subseteq F_E : 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \bigcap_{i=1}^n F_{E_i} \in \tilde{\tau}$.

The pair $(F_E, \tilde{\tau})$ is called a soft topological space.

2.7. Example. Let us consider the soft subsets of F_A that are given in Example 2.5. Then $\tilde{\tau}_1 = \{F_A, F_\emptyset, F_{A_6}, F_{A_9}, F_{A_{11}}\}$, $\tilde{\tau}_2 = \{F_\emptyset, F_A, F_{A_5}, F_{A_8}, F_{A_4}\}$, $\tilde{\tau}_3 = \{\tilde{P}(F_A)\}$ are soft topologies on F_A .

2.8. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \subseteq F_E$. Then the collection $\tilde{\tau}_{F_A} = \{F_{E_i} \subseteq F_A : F_{E_i} \in \tilde{\tau}, i \in I \subseteq N\}$ is called a soft sub space topology on F_A . Hence $(F_A, \tilde{\tau}_{F_A})$ is called a soft topological subspace of $(F_E, \tilde{\tau})$.

2.9. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly F_\emptyset and F_E are soft open sets. The collection of all soft open set is denoted by $G_s(F_E)$. Let $F_C \subseteq F_E$. Then F_C is said to be soft closed if the soft set F_C^c is soft open in F_E . The collection of all soft closed set is denoted by $F_s(F_E)$.

2.10. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space, a soft set F_A is said to be soft pre-open set (soft P-open) if there exists a soft open set F_O such that $F_A \subseteq F_O \subseteq \overline{F_A}$. The set of all soft P-open set of

F_E is denoted by $G_{sp}(F_E, \tilde{\tau})$ or $G_{sp}(F_E)$. Then F_A^c is said to be soft P-closed. The set of all soft P-closed set of F_E is denoted by $F_{sp}(F_E, \tilde{\tau})$ or $F_{sp}(F_E)$.

2.11. Remark. A soft set F_A which is both soft P-open and soft P-closed is known as soft P-clopen set. Clearly F_\emptyset and F_E are soft P-clopen sets.

2.12. Proposition.

- (i) Every soft open set is a soft pre-open set.
- (ii) Every soft closed set is a soft pre-closed set.

2.13. Theorem.

- (i) Arbitrary soft union of soft P-open sets is a soft P-open set.
- (ii) The soft intersection of two soft P-open set need not be a soft P-open set.
- (iii) Arbitrary soft intersection of soft P-closed sets is soft P-closed set.
- (iv) The soft union of two soft P-closed set need not be a soft P-closed set,.

2.14. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \subseteq F_E$. Then the soft pre- interior (soft P-interior) of F_A denoted by $p(F_A)^o$ is defined as the soft union of all soft P-open subsets of F_A . Note that $p(F_A)^o$ is the biggest soft P- open set that contained in F_A .

2.15. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \subseteq F_E$. Then the soft pre closure (soft P-closure) of F_A denoted by $p(\overline{F_A})$ is defined as the soft intersection of all soft P-closed supersets of F_A . Note that, $p(\overline{F_A})$ is the smallest soft P- closed set that containing F_A .

2.16. Proposition. A soft set is soft P-open iff $p(F_A)^o = F_A$.

A soft set is soft P-closed iff $p(\overline{F_A}) = F_A$.

3. Soft P-Neighborhood system

3.1. Definition. A soft set $F_A \in S(U)$ in a soft topological space $(F_E, \tilde{\tau})$ is called a soft P-neighborhood (soft P-neighborhood) of the soft point $(x_{f_A}) \in F_E$ if there exist a soft P-open set F_O such that $(x_{f_A}) \in F_O \subseteq F_A$. The soft pre-neighborhood system of a soft point (x_{f_A}) denoted by $PN_{\tilde{\tau}}(x_{f_A})$ is the family of all its soft pre-neighborhoods.

3.2. Example. Let us consider the soft subsets of F_A that are given in Example 2.5. Define $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space and $\tilde{\tau}^c = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}\}$. Then the collection of all soft P-open set is, $G_{Sp}(F_A) = \{F_A, F_\emptyset, F_{A_3}, F_{A_5}, F_{A_7}, F_{A_8}, F_{A_{13}}, F_{A_{14}}, F_{A_{15}}\}$. Consider the soft point $(e_3, \{u_1\})$. Then the family of soft pre-neighborhood of $(e_3, \{u_1\})$ is $PN_{\tilde{\tau}}((e_3, \{u_1\})) = \{F_A, F_{A_{13}}, F_{A_{15}}\}$.

3.3. Definition. A soft set F_B in a soft topological space $(F_E, \tilde{\tau})$ is called a soft pre -neighborhood (soft P-neighborhood) of the soft set F_A if there exist a soft P-open set F_O such that $F_A \subseteq F_O \subseteq F_B$.

3.4. Theorem. A soft set F_A is soft P-open if and only if for each soft set F_B contained in F_A such that F_A is a soft P-neighborhood of F_B .

Proof: (\Rightarrow) Obvious.

(\Leftarrow) Since $F_A \subseteq F_A$, there exists a soft P-open set F_O such that $F_A \subseteq F_O \subseteq F_A$. Hence $F_O = F_A$ and F_A is soft P-open.

3.5. Theorem. The soft pre-neighborhood system $PN_{\tilde{\tau}}(x_{f_A})$ at (x_{f_A}) is in a soft topological space $(F_E, \tilde{\tau})$ has the following properties:

- (a) If $F_B \subseteq PN_{\tilde{\tau}}(x_{f_A})$, then $(x_{f_A}) \subseteq F_B$.
- (b) If $F_B \subseteq PN_{\tilde{\tau}}(x_{f_A})$ and $F_B \subseteq F_C$, then $F_C \subseteq PN_{\tilde{\tau}}(x_{f_A})$.
- (c) If $F_B \subseteq F_A$ is soft P-open if and only if F_B contains a soft P-neighborhood of each of its points.

Proof:

(a) If $F_B \subseteq PN_{\tilde{\tau}}(x_{f_A})$ then there is a soft P-open set F_O such that $(x_{f_A}) \subseteq F_O \subseteq F_B$. Therefore, we have $(x_{f_A}) \subseteq F_B$.

(b) Let $F_B \subseteq PN_{\tilde{\tau}}(x_{f_A})$ and $F_B \subseteq F_C$. Since $F_B \subseteq PN_{\tilde{\tau}}(x_{f_A})$, then there is a soft P-open set F_O such that $(x_{f_A}) \subseteq F_O \subseteq F_B$. Therefore, we have $(x_{f_A}) \subseteq F_O \subseteq F_B \subseteq F_C$ and so $F_C \subseteq PN_{\tilde{\tau}}(x_{f_A})$.

(c) (\Rightarrow) Suppose F_B is a soft P-open in F_A , and $(x_{f_A}) \subseteq F_B \subseteq F_A$ then F_B is a soft P-neighborhood of each $(x_{f_A}) \subseteq F_B$ in F_A .

(\Leftarrow) If each $(x_{f_A}) \subseteq F_B$ has a soft P-neighborhood $F_O \subseteq F_B$, then

$F_B = \{x_{f_A} : x_{f_A} \tilde{\in} F_B\} \cong \tilde{U}_{(x_{f_A}) \in F_B} F_O \cong F_B$ or $F_B = \tilde{U}_{(x_{f_A}) \in F_B} F_O$. This implies that F_B is soft P-open in F_A .

3.6. Remark. If $F_B, F_C \tilde{\in} PN_{\tilde{\tau}}(x_{f_A})$, then $F_B \tilde{\cap} F_C \tilde{\notin} PN_{\tilde{\tau}}(x_{f_A})$.

The following example shows the result.

3.7. Example. Let us consider the Example 3.2, Here $F_{A_{13}}, F_{A_{15}} \tilde{\in} PN_{\tilde{\tau}}((e_3, \{u_1\}))$. But $F_{A_{13}} \tilde{\cap} F_{A_{15}} = \{(e_3, \{u_1\})\} \tilde{\notin} PN_{\tilde{\tau}}((e_3, \{u_1\}))$.

4. Soft pre-connected space

In this section we introduce the concept of soft pre-disjoint sets, soft pre-separated sets and soft pre-connected space in a soft topological space. Also we discuss some of the main results based on the above with examples mentioned definitions.

4.1. Definition. Two soft pre-open sets F_A and F_B are said to be soft pre-disjoint if $A \cap B = \emptyset$ and $F_A \tilde{\cap} F_B = F_\emptyset$.

4.2. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. Two non-empty soft sub sets F_A and F_B of F_E are called soft pre-separated iff $p(\overline{F_A}) \tilde{\cap} F_B = F_A \tilde{\cap} p(\overline{F_B}) = F_\emptyset$. That means that a soft pre-separation of a soft topological space $(F_E, \tilde{\tau})$ is a pair F_A, F_B of soft pre-disjoint non-null soft pre-open sets whose union is F_E .

4.3. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space. Then the following statements are equivalent:

- (1): F_\emptyset and F_E are the only soft pre-clopen sets in $(F_E, \tilde{\tau})$.
- (2): $(F_E, \tilde{\tau})$ is not the soft union of two soft disjoint non-empty soft pre-open sets.
- (3): $(F_E, \tilde{\tau})$ is not the soft union of two soft disjoint non-empty soft pre-Close sets.
- (4): $(F_E, \tilde{\tau})$ is not the soft union of two non-empty soft pre-separated sets.

Proof

(1) \Rightarrow (2)

Suppose (2) is false and that $F_E = F_A \tilde{\cup} F_B$, where F_A and F_B are soft disjoint non-empty soft pre-open sets. Since $F_E - F_A = F_B$ is soft pre-open and non-empty. We have that F_A is a non-empty proper soft pre-clopen set in F_E . Which shows that (1) is false.

Therefore (1) \Rightarrow (2).

(2) \Leftrightarrow (3) This is clear.

(3) \Rightarrow (4)

If (4) is false, then $F_E = F_A \tilde{\cup} F_B$, where F_A and F_B are non-empty and soft pre-separated sets.

Since $F_A \tilde{\cap} p(\overline{F_B}) = F_\emptyset$. We conclude that $p(\overline{F_B}) \cong F_B$, so F_B is soft pre-closed. Similarly, F_A must be soft pre-closed. Therefore (3) is fails. Which shows (3) \Rightarrow (4).

(4) \Rightarrow (1).

Suppose (1) is false and that F_A is a non-empty proper soft pre-clopen subset of F_E . Then $F_B = F_E - F_A$ is non-empty and soft pre-clopen. So F_A and F_B are soft-separated sets. Since $F_E = F_A \tilde{\cup} F_B$.

Which shows that (4) is false. Therefore (4) \Rightarrow (1).

4.4. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. If there doesn't exist a soft pre-separation of F_E , then it is said to be soft pre-connected (soft P-connected) otherwise it is soft pre-disconnected (soft P-disconnected).

4.5. Example.

Let $(F_A, \tilde{\tau})$ be a soft topological space, where F_A and its soft subsets are considered as in Example 2.5.

Let $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}\}$ then $\tilde{\tau}^c = \{F_A, F_\emptyset, F_{A_6}, F_{A_9}, F_{A_{11}}\}$.

$G_{SP}(F_A) = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}, F_{A_3}, F_{A_7}, F_{A_{13}}, F_{A_{15}}\}$

$F_{SP}(F_A) = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}, F_{A_{10}}, F_{A_4}, F_{A_{12}}, F_{A_{16}}\}$

Here we show that F_A is soft P-connected. We first choose

$F_{A_4} = \{(e_2, \{u_2\})\}$ and $F_{A_7} = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\}$.

Then $p(\overline{F_{A_4}}) = \{(e_2, \{u_2\}), (e_3, \{u_1\})\}$ and so $p(\overline{F_{A_4}}) \tilde{\cap} F_{A_7} = \{(e_3, \{u_1\})\} = F_{A_{11}} \neq F_\emptyset$. We next

choose $F_{A_5} = \{(e_2, \{u_3\})\}$ and $F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}$. Then $p(\overline{F_{A_5}}) = F_A$ and so $p(\overline{F_{A_5}}) \tilde{\cap} F_{A_{10}} = \{(e_3, \{u_1, u_2\})\} = F_{A_{10}} \neq F_\emptyset$. We next choose, $F_{A_3} = \{(e_2, \{u_2, u_3\})\}$ and $F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}$.

Then $p(\overline{F_{A_3}}) = F_A$ and so $p(\overline{F_{A_3}}) \tilde{\cap} F_{A_{10}} = \{(e_3, \{u_1, u_2\})\} = F_{A_{10}} \neq F_\emptyset$. We next choose $F_{A_{11}} =$

$\{(e_3, \{u_1\})\}$ and $F_{A_{14}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_2\})\}$. Then $p(\overline{F_{A_{11}}}) = \{(e_3, \{u_1\})\} = p(\overline{F_{A_{14}}}) = F_A$.

But $p(\overline{F_{A_{11}}}) \tilde{\cap} F_{A_{14}} = F_\emptyset$ and $F_{A_{11}} \tilde{\cap} p(\overline{F_{A_{14}}}) = \{(e_3, \{u_1\})\} = F_{A_{11}} \neq F_\emptyset$. Finally we choose $F_{A_{12}} = \{(e_3, \{u_2\})\}$ and $F_{A_{13}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1\})\}$. Then $p(\overline{F_{A_{12}}}) = \{(e_2, \{u_2\}), (e_3, \{u_1, u_2\})\}$ and so $p(\overline{F_{A_{12}}}) \tilde{\cap} F_{A_{13}} = \{(e_2, \{u_2\}), (e_3, \{u_1\})\} = F_{A_9} \neq F_\emptyset$.

Thus we see that F_A can't be expressed as the soft union of two soft P-separated sets and hence F_A is soft P-connected.

4.6. Remark. Since every soft pre-connected space is soft connected space. But the converse is not true as shown by the following example.

4.7. Example. Soft connectedness does not imply soft P-connectedness.

Let $(F_A, \tilde{\tau})$ be the soft topological space. Where F_A and its soft subsets are considered as in Example 2.5. Consider $\tau = \{F_A, F_\emptyset, F_{A_3}, F_{A_{11}}, F_{A_{13}}\}$.

Here $\tilde{\tau}^c = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_{14}}, F_{A_{12}}\}$.

$G_{Sp}(F_A) = \{F_A, F_\emptyset, F_{A_3}, F_{A_{11}}, F_{A_{13}}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_9}, F_{A_{13}}, F_{A_{15}}\}$.

$F_{Sp}(F_A) = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_{14}}, F_{A_7}, F_{A_6}, F_{A_5}, F_{A_4}, F_{A_8}, F_{A_{12}}, F_{A_{16}}\}$. It is clear that $(F_A, \tilde{\tau})$ is soft connected.

Since the only soft clopen sets are F_A and F_\emptyset . But we show that it is not soft P-connected. Let $F_A = F_{A_4} \tilde{\cup} F_{A_7}$, then $p(\overline{F_{A_4}}) = \{(e_2, \{u_2\})\} = F_{A_4}$, $p(\overline{F_{A_7}}) = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\} = F_{A_7}$. We have $p(\overline{F_{A_4}}) \tilde{\cap} F_{A_7} = F_\emptyset$ and $F_{A_4} \tilde{\cap} p(\overline{F_{A_7}}) = F_\emptyset$. Hence F_A can be expressed as a soft union of two soft P-separated sets F_{A_4} and F_{A_7} . Hence F_A is not soft pre-connected.

4.8. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space. Then the following statements are equivalent:

- (1): $(F_E, \tilde{\tau})$ is a soft P-connected space.
- (2): $(F_E, \tilde{\tau})$ is not the soft union of any two soft P-separated sets.

Proof:

(1) \Rightarrow (2).

Assume (1), suppose (2) is false, then let F_A, F_B are two soft P-separated sets such that $F_E = F_A \tilde{\cup} F_B$. Since F_E is soft P-connected $p(\overline{F_A}) \tilde{\cap} F_B = F_A \tilde{\cap} p(\overline{F_B}) = F_\emptyset$. Since $F_A \tilde{\subseteq} p(\overline{F_A})$ and $F_B \tilde{\subseteq} p(\overline{F_B})$, then $F_A \tilde{\cap} F_B = F_\emptyset$. Now $p(\overline{F_A}) \tilde{\subseteq} F_E - F_B = F_A$. Hence $p(\overline{F_A}) = F_A$. Therefore F_A is soft P-closed set. By the same way we can show that F_B is soft P-closed set which is a contradiction with theorem 4.3 (iii). This shows that (2) is true. Therefore (1) \Rightarrow (2).

(2) \Rightarrow (1).

Assume (2) is not true. Let F_A and F_B are two soft pre-disjoint non-empty and soft P-closed sets such that $F_E = F_A \tilde{\cup} F_B$. Then $p(\overline{F_A}) \tilde{\cap} F_B = F_A \tilde{\cap} p(\overline{F_B}) = F_A \tilde{\cap} F_B = F_\emptyset$. This contradicts the hypothesis of (2). This shows that (1) is true. Therefore (2) \Rightarrow (1).

4.9. Remark. If $F_\emptyset \cong F_A \cong (F_E, \tilde{\tau})$, we call F_A a soft P-connected set in F_E whenever $(F_E, \tilde{\tau})$ is a soft P-connected space.

4.10. Example. Soft P-connectivity is not a hereditary property.

In this example, we consider the soft topological space on $(F_A, \tilde{\tau})$, where F_A and its soft subsets are considered as in Example 2.5. Consider $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}\}$. Here $\tilde{\tau}^c = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}\}$.

$$G_{Sp}(F_A) = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}, F_{A_3}, F_{A_7}, F_{A_{13}}, F_{A_{15}}\}.$$

$$F_{Sp}(F_A) = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}, F_{A_{10}}, F_{A_4}, F_{A_{12}}, F_{A_{16}}\}.$$

It is clear that F_A is soft P-connected space since the only soft P-clopen sets are F_A and F_\emptyset . Let

$$F_{A_7} \cong F_A, \quad \text{where} \quad F_{A_7} = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\}. \quad \text{Then} \quad \tilde{\tau}_{A_7} = \{F_{A_7}, F_\emptyset, F_{A_5}, F_{A_8}\},$$

$$\tilde{\tau}_{A_7}^c = \{F_\emptyset, F_{A_7}, F_{A_6}, F_{A_{11}}\}. \quad \text{Here} \quad G_{Sp}(F_{A_7}) = \{F_{A_7}, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{15}}, F_{A_{10}}\} \quad \text{and} \quad F_{Sp}(F_{A_7}) =$$

$$\{F_{A_7}, F_\emptyset, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_5}\}. \quad \text{It is clear that } (F_{A_7}, \tilde{\tau}_{A_7}^c) \text{ is not soft P-connected space. Since } F_{A_{10}}, F_{A_5}$$

are soft P-clopen sets other than F_\emptyset and F_{A_7} . Thus a soft P-connectivity is not a hereditary property.

4.11. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space and let F_A be a soft P-connected set. Let F_B and F_C are soft P-separated sets. If $F_A \cong F_B \tilde{\cup} F_C$. Then either $F_A \cong F_B$ or $F_A \cong F_C$.

Proof:

Suppose F_A is soft P-connected set and F_B, F_C are soft P-separated sets such that $F_A \cong F_B \tilde{\cup} F_C$.

Let $F_A \not\cong F_B$ and $F_A \not\cong F_C$. Suppose $F_{A_1} = F_B \tilde{\cap} F_A \neq F_\emptyset$ and $F_{A_2} = F_C \tilde{\cap} F_A \neq F_\emptyset$. Then $F_A =$

$$F_{A_1} \tilde{\cup} F_{A_2}. \quad \text{Since } F_{A_1} \cong F_B. \quad \text{Hence } p(\overline{F_{A_1}}) \cong p(\overline{F_B}). \quad \text{Since } p(\overline{F_B}) \tilde{\cap} F_C = F_\emptyset \text{ then } p(\overline{F_{A_1}}) \tilde{\cap} F_{A_2} = F_\emptyset.$$

$$\text{Since } F_{A_2} \cong F_C. \quad \text{Hence } p(\overline{F_{A_2}}) \cong p(\overline{F_C}). \quad \text{Since } p(\overline{F_C}) \tilde{\cap} F_B = F_\emptyset. \text{ Then } p(\overline{F_{A_2}}) \tilde{\cap} F_{A_1} = F_\emptyset. \text{ But}$$

$F_A = F_{A_1} \tilde{\cup} F_{A_2}$, therefore F_A is not soft P-connected space. This is a contradiction. Then either

$$F_A \cong F_B \text{ or } F_A \cong F_C.$$

4.12. Theorem. If F_A is soft P-connected set then $p(\overline{F_A})$ is soft P-connected.

Proof: Suppose F_A is soft P-connected set and $p(\overline{F_A})$ is not. Then there exist two soft P-separated sets F_B and F_C such that $p(\overline{F_A}) = F_B \tilde{\cup} F_C$. But $F_A \cong p(\overline{F_A})$, then $F_A = F_B \tilde{\cup} F_C$ and since F_A is soft P-connected set then by Theorem 4.11 either $F_A \cong F_B$ or $F_A \cong F_C$.

(i): If $F_A \cong F_B$ then $p(\overline{F_A}) \cong p(\overline{F_B})$. But $p(\overline{F_B}) \tilde{\cap} F_C = F_\emptyset$. Hence $p(\overline{F_A}) \tilde{\cap} F_C = F_\emptyset$. Since $F_C \cong p(\overline{F_A})$, then $F_C = F_\emptyset$ this is a contradiction.

(ii): If $F_A \cong F_C$ then the same way we can prove that $F_B = F_\emptyset$ which is a contradiction. Therefore $p(\overline{F_A})$ is soft P-connected.

4.13. Theorem. If F_A is soft P-connected set and $F_A \cong F_B \cong p(\overline{F_A})$ then F_B is soft P-connected.

Proof:

If F_B is not soft P-connected, then there exist two soft sets F_C and F_D such that $p(\overline{F_C}) \tilde{\cap} F_D = F_C \tilde{\cap} p(\overline{F_D}) = F_\emptyset$ and $F_B = F_C \tilde{\cup} F_D$. Since $F_A \cong F_B$, thus either $F_A \cong F_C$ or $F_A \cong F_D$. Suppose $F_A \cong F_C$ then $p(\overline{F_A}) \cong p(\overline{F_C})$, thus $p(\overline{F_A}) \cong F_D = p(\overline{F_C}) \tilde{\cap} F_D = F_\emptyset$. But $F_D \cong F_B \cong p(\overline{F_A})$, thus $p(\overline{F_A}) \tilde{\cap} F_D = F_D$. Therefore $F_D = F_\emptyset$ which is a contradiction. Thus F_B is soft P-connected set.

If $F_A \cong F_B$, then by the same way we can prove that $F_C = F_\emptyset$. This is a contradiction. Then F_B is soft P-connected.

4.14. Theorem. The soft union F_A of any family $\{F_{A_i}; i \in I\}$ of soft P-connected sets having a non – empty soft intersection is an soft P-connected set.

Proof: Let F_A be a soft union of any family of soft P-connected sets having a non-empty soft intersection. Suppose that $F_A = F_B \tilde{\cup} F_C$, where F_B and F_C form a soft P-separation of F_A . by hypothesis, we may choose a soft point $(x_{f_{A_i}}) = (x, f_{A_i}(x)) \in \tilde{\cap}_{i \in I} F_{A_i}$. Then $(x_{f_{A_i}})$ must belong to either a soft subset of F_B or a soft subset of F_C . Since F_B and F_C are soft disjointed, we must have $F_{A_i} \cong F_B$ for all $i \in I$, and so $F_A \cong F_B$. From this we obtain that $F_C = F_\emptyset$. Which is a contradiction. This proves the theorem.

4.15. Theorem. Let (F_E, τ) be a soft topological space such that any two point (x_{f_A}) and (x_{f_B}) of F_E are contained in some soft subspace of F_E . Then F_E is soft P-connected.

Proof:

Suppose F_E is not soft P-connected space. Then F_E is the soft union of two soft P-separated sets F_A and F_B . Since F_A, F_B are non –empty soft sets, there exist (x_{f_A}) and (x_{f_B}) such that $(x_{f_A}) \tilde{\in} F_A$ and $(x_{f_B}) \tilde{\in} F_B$. Let (F_{E_1}, τ_1) be a soft P-connected soft subspace of F_E , which contains (x_{f_A}) and (x_{f_B}) . Therefore by Theorem 4.11 either $F_{E_1} \cong F_A$ or $F_{E_1} \cong F_B$ which is a contradiction. Since $F_A \tilde{\cap} F_B = F_\emptyset$. Then $(F_E, \tilde{\tau})$ is soft P-connected space.

4.16. Corollary. If a soft topological space $(F_E, \tilde{\tau})$ contains a soft P-connected subspace F_{E_1} such that $p(\overline{F_{E_1}}) = F_E$, then $(F_E, \tilde{\tau})$ is soft P-connected.

Proof: Suppose F_{E_1} is soft P-connected subspace of $(F_E, \tilde{\tau})$ such that $p(\overline{F_{E_1}}) = F_E$. Since $F_{E_1} \cong F_E = p(\overline{F_{E_1}})$ then by Theorem 4.13 $(F_E, \tilde{\tau})$ is soft P-connected.

4.17. Theorem. If $(F_{E_1}, \tilde{\tau}_1)$ and $(F_{E_2}, \tilde{\tau}_2)$ are soft P-connected subspace of soft topological space $(F_E, \tilde{\tau})$ such that $F_{E_1} \tilde{\cap} F_{E_2} \neq F_\emptyset$ then $F_{E_1} \tilde{\cup} F_{E_2}$ is soft P-connected subspace.

Proof: Suppose that $F_{E_1} \tilde{\cup} F_{E_2}$ is not soft P-connected subspace. Then there exist two soft P-separated sets F_A and F_B such that $F_{E_1} \tilde{\cup} F_{E_2} = F_A \tilde{\cup} F_B$. Since $F_{E_1} \cong F_{E_1} \tilde{\cup} F_{E_2} = F_A \tilde{\cup} F_B$ and F_{E_1} is soft P-connected, then by Theorem 4.11 either $F_{E_1} \cong F_A$ or $F_{E_1} \cong F_B$. Since $F_{E_2} \cong F_{E_1} \tilde{\cup} F_{E_2} = F_A \tilde{\cup} F_B$ and F_{E_2} is soft P-connected, then either $F_{E_2} \cong F_A$ or $F_{E_2} \cong F_B$.

- (i): If $F_{E_1} \cong F_A$ and $F_{E_2} \cong F_A$, then $F_{E_1} \tilde{\cup} F_{E_2} \cong F_A$. Hence $F_B = F_\emptyset$. This is a contradiction.
- (ii): If $F_{E_1} \cong F_A$ and $F_{E_2} \cong F_B$, then $F_{E_1} \tilde{\cap} F_{E_2} \cong F_A \tilde{\cap} F_B = F_\emptyset$. Therefore $F_{E_1} \tilde{\cap} F_{E_2} = F_\emptyset$. Which is a contradiction. By the same way we can get a contradiction if $F_{E_1} \cong F_B$ and $F_{E_2} \cong F_A$ or if $F_{E_1} \cong F_A$ and $F_{E_2} \cong F_B$. Therefore $F_{E_1} \tilde{\cup} F_{E_2}$ is soft P-connected subspace of $(F_E, \tilde{\tau})$.

4.18. Corollary. Let $(F_E, \tilde{\tau}_1)$ be a soft P-connected space and $\tilde{\tau}_2 \cong \tilde{\tau}_1$. Then $(F_E, \tilde{\tau}_2)$ is soft P-connected.

Proof: Suppose to the contrary F_A and F_B is a soft P-separation of F_E with soft topology $\tilde{\tau}_2$. Since $\tilde{\tau}_2 \cong \tilde{\tau}_1$ then F_A and F_B is a soft P-separation of F_E with soft topology $\tilde{\tau}_1$. This is a contradiction. Therefore $(F_E, \tilde{\tau}_2)$ is soft P-connected.

5. Conclusion.

We have introduced soft pre-neighbourhood system which is defined over soft topological spaces. The notions of soft pre-open, soft pre-closed sets, soft pre-interior, soft pre-closure, soft pre-neighbourhood system, soft pre-connectedness are introduced and their basic properties are investigated. In the end, we must say that, this paper is just a beginning of a new structure and we have studied a few ideas only, Further, there are scopes for researcher in this field.

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