Intuitionistic Fuzzy Join Semi L-Filter of Lattice Homomorphism

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Abstract

In this paper contains the properties of Intuitionistic fuzzy join semi L-Filter of Lattice homomorphism .Also we defined g-invariant and established a correspondence between the Intuitionistic fuzzy join semi L-Filters of a lattice which are g-invariant and intuitionistic fuzzy join semi L-filter of its homomorphic image.

Keywords: *Intuitionistic fuzzy join semi L-Filter, Intuitionistic fuzzy join semi L-Filter of lattice homomorphism.*

I. INTRODUCTION

In 1965 Lofti A.Zadeh introduced the notion of fuzzy subset of a set as a method for representing uncertainty in real physical world .The concept of intuitionistic fuzzy set was introduced by Atanassov. KT, N.Ajmal [2] discuss the homomorphism of fuzzy subgroups and fuzzy quotient groups . Kavitha . A and Chellappa.B discuss the homomorphism on fuzzy meet semi L-Filter.In this paper we introduced intuitionistic fuzzy join semi L-Filter of lattice homomorphism.

Definition:
Let $L_l$and L_1^1be a Lattices . A mapping $g:L_1 \rightarrow L_1^1$ is called a homomorphism if $g(a \vee b)= g(a) \vee g(b)$ and $g(a \wedge b)= g(a) \wedge g(b)$, for all $a,b \in L$.
Definition:
A mapping $g:L_1 \rightarrow L_1^1$ is called a isomorphism if g is an one – one and onto homomorphism.
Definition:
A homomorphism from $L_1 \rightarrow L_1^1$ is called endomorphism. Onto homomorphism from $L_1 \rightarrow L_1^1$ is called endomorphism.
Definition:
A mapping g from L_1 *to* $L_1^1$and A_1be an Intuitionistic fuzzy set on L then the image of A_1 is denoted by $g(A_1)$ and is defined by
$g(A_1) = \{< z, g(\mu)(z), g(\gamma)(z) > / z \in L^1\}$

$$\text{Where} g(\mu)(z) = \begin{cases} \sup\{\mu(\varkappa)/\varkappa \in g^{-1}(z)\} & \text{if } g^{-1}(z) \text{ is non–empty} \\ O & \text{if } g^{-1}(z) \text{ empty} \end{cases}$$

$$\text{and } g(\gamma)(z) = \begin{cases} \inf\{\gamma(\varkappa)/\varkappa \in g^{-1}(z)\} & \text{if } g^{-1}(z) \text{ is non - empty} \\ O & \text{if } g^{-1}(z) \text{ empty.} \end{cases}$$

If A_1^1 is an Intuitionistic fuzzy set in L_1^1. Then inverse image of A_1^1 is defined by
$g^{-1}(A_1^1)= \{<\varkappa, g^{-1}(\mu)(\varkappa), g^{-1}(\gamma)(\varkappa)>/ \varkappa \in L\}$
where $g^{-1}(\mu)(\varkappa)= \mu(g(\varkappa))$ *and* $g^{-1}(\gamma)(\varkappa) = \gamma(g(\varkappa))$.

Theorem :1
If $g: L_1 \rightarrow L_1^1$ is a lattice epimorphism and A_1 is an IFJSLF of L_1 then $g(A_1)$ is an IFJSLF of L_1^1 [(ie) image of an IFJSLF is also an IFJSLF].
Proof :
Let $A_1= \{ <\varkappa_1\, \mu(\varkappa), \gamma(\varkappa)>/ \varkappa \in L\}$ be an IFJSLF of L_1 .Then image of A_1 is defined by
$g(A_1) = \{ <y, g(\mu)(y), g(\gamma)(y)>/ y \in L_1^1\}$.

Let $y_1,z_1 \in L_1^1$.

Then

$g(\mu)(y_1 \vee z_1) = \sup\{\mu(\varkappa) / \varkappa \in g^{-1}(y_1 \vee z_1)\}$

$\leq \sup\{\mu(u_1 \vee v_1) / u_1 \in g^{-1}(y_1), v_1 \in g^{-1}(z_1)\}$

$\leq \sup\{\max \mu(u_1), \mu(v_1)\} u_1 \in g^{-1}(y_1)\ v_1 \in g^{-1}(z_1)\}$

$= \max\{\sup \mu(u_1) / u_1 \in g^{-1}(y_1)\},$
$\sup\{\mu(v_1) / v_1 \in g^{-1}(z_1)\} = \max\{g(\mu(y_1), g(\mu(z_1))\}$
$\therefore g(\mu(y_1 \vee z_1) \leq \max\{g(\mu(y_1), g(\mu(z_1))\}$

Also
$g(\gamma)(y_1 \vee z_1) = \inf\{\gamma(\varkappa) / \varkappa \in g^{-1}(y_1 \vee z_1)\}$
$\geq \inf\{\gamma(u_1 \vee v_1) / u_1 \in g^{-1}(y_1), v_1 \in g^{-1}(z_1)\}$
$\geq \inf\{\min\{\gamma(u_1), \gamma(v_1)\} / u_1 \in g^{-1}(y_1), v_1 \in g^{-1}(z_1)\}$
$= \min\{\inf\{\gamma(u_1) / u_1 \in g^{-1}(y_1)\},$
$\inf\{\gamma(v_1) / v_1 \in g^{-1}(z_1)\}\} = \min\{g(\gamma)(y_1), g(\gamma)(z_1)\}$
Hence
$g(\gamma)(y_1 \vee z_1) \geq \min\{g(\gamma)(y_1), g(\gamma)(z_1)\}$
Hence image of a IFJSLF is an IFJSLF.

Theorem : 2
If $g: L_1 \rightarrow L_1^1$ is a lattice homomorphism and A_1^1 is an IFJSLF of L_1^1 then inverse image of A_1^1 is an IFJSLF of L_1.
Proof :
Let $A_1^1 = \{<y, \mu(y), \gamma(y) / y \in L_1^1\}$ be an IFJSLF of L_1^1.
To prove that inverse image of A_1^1 is an IFJSLF of L_1. For any $\varkappa_1, y_1 \in L_1$,
$g^{-1}(\mu)(\varkappa_1 \vee y_1) = \mu(g(\varkappa_1 \vee y_1))$
$= \mu(g(\varkappa_1) \vee g(y_1))$
$\leq \max\{\mu(g(\varkappa_1)), \mu(g(y_1))\}$
$= \max\{g^{-1}(\mu)(\varkappa), g^{-1}(\mu)(y)\}$
Hence
$g^{-1}(\mu)(\varkappa_1 \vee y_1) \leq \max\{g^{-1}(\mu)(\varkappa_1), g^{-1}(\mu)(y_1)\}$
Also,
$g^{-1}(\varkappa)(\varkappa_1 \vee y_1) = \gamma((g)(\varkappa_1 \vee y_1))$
$= \gamma(g(\varkappa_1) \vee g(y_1))$
$\geq \min\{\gamma(g(\varkappa_1)), \gamma(g(y_1))\}$
$= \min\{g^{-1}(\gamma)(\varkappa_1), g^{-1}(\gamma)(y_1)\}$
Hence
$g^{-1}(\gamma)(\varkappa_1 \vee y_1) \geq \min\{g^{-1}(\gamma)(\varkappa_1), g^{-1}(\gamma)(y_1)\}$
Hence inverse image of A_1^1 is an IFJSLF of L_1.

Theorem :3
If $g: L_1 \rightarrow L_1^1$ is an onto mapping and A_1 and A_1^1 are IFJSLF$_s$ of the lattices L_1 & L_1^1 respectively. Then a) $g(g^{-1}(A_1^1)) = A_1^1$, b) A_1 is contained in $g^{-1}(g(A_1))$.
Proof: For(a)
Let $y \in L_1^1$.
Then we have $g(g^{-1}(\mu)(y) = \sup\{g^{-1}(\mu)(x_1) / x_1 \in g^{-1}(y)\}$
$= \sup\{\mu(g(x_1) / x_1 \in L_1, g(x_1) = y\}$
$g(g^{-1}(\mu))(y) = \mu(y)$

Since g is an onto mapping for every $y \in L_1^1$

There exist $x_1 \in L_1$ such that $g(x_1) = y$

$g(g^{-1}(\gamma))(y) = \inf\{g^{-1}(\gamma)(\varkappa_1) / x_1 \in g^{-1}(y)\}$
$= \inf\{\gamma(g(\varkappa_1)) / g(\varkappa_1) = y\}$
$g(g^{-1}(\gamma))(y) = \gamma(y)$
Hence $g(g^{-1}(A_1^1)) = A_1^1$
For (b) :
Let $x_1 \in L_1$. Then we have
$g^{-1}(g(\mu)(\varkappa_1) = g(\mu)(g(\varkappa_1))$

$= \sup\{ \mu(\varkappa_1) / x_1 \in g^{-1}(g(\varkappa_1))\}$

$g^{-1}(g(\mu)(\varkappa_1) \geq \mu(\varkappa_1)$ and

$g^{-1}(g(\gamma)(\varkappa_1) = g(\gamma)(g(\varkappa_1))$

$= \inf\{ \gamma(\varkappa_1) / \varkappa_1 \in g^{-1}(g(\varkappa_1))\}$

$g^{-1}(g(\gamma)(\varkappa_1 \leq \gamma(\varkappa_1)$

Hence A_1 is contained in $g^{-1}(g(A_1))$

Definition :

If $g:x \rightarrow y$be any function from a set X to another set Y and A_1 be an IFJSLF of X. Then A_1 is said to be g-invariant if $\varkappa_1, \varkappa_2 \in X$ s.t $g(\varkappa_1) = g(\varkappa_2) \Rightarrow \mu(\varkappa_1) = \mu(\varkappa_2)$ and $\gamma(\varkappa_1) = \gamma(\varkappa_2)$.

Note :4

If on IFJSLF A_1 is g-invariant Then $g^{-1}(g(A_1)) = A_1$.

Theorem :5

If $g:x \rightarrow y$is any function from a set X onto another set Y and A_1, B_1 are $IFJSLF_s$ of X and A_1^1, B_1^1 are IFJSLFs of Y.

Then a) $A_1 \subseteq B_1 \Rightarrow g(A_1) \subseteq g(B_1)$ and

b) $A_1^1 \subseteq B_1^1 \Rightarrow g^{-1}(A_1^1) \subseteq g^{-1}(B_1^1)$

Proof:

Let A_1 and B_1 be $IFJSLF_s$ of X.

Then A_1 is contained in $B_1 \Rightarrow \mu_{A1}(x) \leq \mu_{B1}(x)$ and

$\gamma_{A1}(x) \geq \gamma_{B1}(x)$

Image of A_1 and B_1 is defined by

$g(A_1) = \{<y, g(\mu_A)(y), g(\gamma_A)(y)> / y \in Y\}$ and

$g(B_1) = \{<y, g(\mu_B)(y), g(\gamma_B)(y)> / y \in Y\}$

For all $x \in X$, we have

$g(\mu_{A1})(x) = \sup\{ \mu_{A1}(z) / z \in g^{-1}(x)\}$

$\leq \sup\{ \mu_{B1}(z) / z \in g^{-1}(x)\}$

$= g(\mu_{B1})(x)$.

$\therefore g(\mu_{A1})(x) \leq g(\mu_{B1})(x)$.

Also , $g(\gamma_{A1})(x) = \inf\{ \gamma_{A1}(z) / z \in g^{-1}(x)\}$

$\geq \inf\{ \gamma_{B1}(z) / z \in g^{-1}(x)\}$

$= g(\gamma_{B1})(x)$

Hence $g(\gamma_{A1})(x) \geq g(\gamma_{B1})(x)$

Hence A_1 contained in B_1 which implies

$g(A_1)$ contained in $g(B_1)$

Also , Inverse image of A_1 and B_1 is defined by

$g^{-1}(A_1) = \{ <x, g^{-1}(\mu_{A1})(x) , g^{-1}(\gamma_{A1})(x) > / x \in L_1\}$

$g^{-1}(B_1) = \{ <x, g^{-1}(\mu_{B1})(x) , g^{-1}(\gamma_{B1})(x) > / x \in L_1\}$

$g^{-1}(\mu_{A1})(x) = \mu_{A1}(g(x)) \leq \mu_{B1}(g(x)) = g^{-1}(\mu_{B1})(x)$

Hence $g^{-1}(\mu_{A1})(x) \leq g^{-1}(\mu_{B1})(x)$

Also , $g^{-1}(\gamma_{A1})(x) = \gamma_{A1}(g(x)) \geq \gamma_{B1}(g(x))$

$= g^{-1}(\gamma_{B1})(x)$

There fore $g^{-1}(\gamma_{A1})(x) \geq g^{-1}(\gamma_{B1})(x)$

Hence A_1^1 contained in B_1^1 which implies $g^{-1}(A_1^1)$ contained in $g^{-1}(B_1^1)$

Theorem :6

If $g: L_1 \rightarrow L_1^1$ is a lattice homomorphism, Then there is one- one order preserving correspondence between the $IFJSLF_S$ of L_1^1 and those of L_1 which are g-invariant .

<u>Proof :</u>

Let $J(L_1^1)$ denote the set of all $IFJSLF_S$ of L_1^1 and$J(L_1)$ denote the set of all $IFJSLF_S$ of L_1 which are g-invariant.

Define $f : J(L_1) \rightarrow J(L_1^1)$ and $h: J(L_1^1) \rightarrow J(L_1)$ such that $f(A_1) = g(A_1)$ and $h(A_1^1) = g^{-1}(A_1^1)$

By theorem 1 and 2 , f and h are well- defined

Also by thorem 3 and note 4 , f and h inverse to each other which gives the one to one correspondence,

Also by theorem 5, we have A_1contained in B_1 which implies $g(A_1)$ contained in $g(B_1)$. Thus the correspondence is order preserving ,

Theorem :7

If $g: L_1 \rightarrow L_1^1$ is a lattice epimorphism and A_1 and B_1 areIFJSLFs of L then $g(A_1) \cup g(B_1) \subseteq g(A_1 \cup B_1)$

Proof :

Since $A_1 \subseteq A_1 \cup B_1$ and $B_1 \subseteq A_1 \cup B_1$
By theorem 10, $g(A_1) \subseteq g(A_1 \cup B_1)$, $g(B_1) \subseteq g(A_1 \cup B_1)$.
$g(A_1 \cup B_1) = \{ <y, g(\mu_{A1\cup B1})(y), g(\gamma_{A1\cup B1})(y)> / y \in L_1^1\}$
$g(A) = \{<y, g(\mu_{A1})(y), g(\gamma_{A1})(y)> / y \in L_1^1\}$
Since $g(A_1) \subseteq g(A_1 \cup B_1)$, we have
$g(\mu_{A1})(y) \leq g(\mu_{A1\cup B1})(y)$ and $g(\gamma_{A1})(y) \geq g(\gamma_{A1\cup B1})(y)$
$g(\mu_{B1})(y) \leq g(\mu_{A1\cup B1})(y)$ and $g(\gamma_{B1})(y) \geq g(\gamma_{A1\cup B1})(y)$
Now $g(\mu_{A1\cup B1})(y) \leq \max\{ g(\mu_{A1})(y) , g(\mu_{B1})(y)\}$
$= (g(\mu_{A1}) \vee g(\mu_{B1}))(y)$
Also $g(\gamma_{A1\cup B1})(y) \geq \min \{g(\gamma_{A1})(y) , g(\gamma_{B1})(y)\}$
$= [g(\gamma_{A1}) \vee g(\gamma_{B1})](y)$
Hence $g(A_1) \cup g(B_1) \subseteq g(A_1 \cup B_1)$

CONCLUTION

Correspondence between the Intuitionistic fuzzy join semi L-Filters of a lattice which are g-invariant and intuitionistic fuzzy join semi L-filter of its homomorphic image**.**

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