Generation of Infinite Set of Commutative Matrix on a Non-Singular Matrix (N×N)

Biswanath Rath
Department of physics, North Orissa University, Takatpur, Baripada-757003, Odisha, INDIA

Following the recent work of B. Rath Int. Jour. of Math. Trends and Tech 59(3),171(2018) we show that for a given non-singular square matrix (A), it is possible to generate an infinite class of commutative matrix $B_i i.e. [A, B_i] = 0 (i = 1,2,3 \ldots \ldots \infty)$

Mathematical Classification (2010):47A10;47B20

Key words - *fractional matrix, spectral analysis.*

I. INTRODUCTION

From the review of literature on matrix available in standard books, one can hardly find a commutative nature[1]. In fact it is true corresponding to different dimensions i.e. A(N×M), B(N×M).

$$AB \neq BA \tag{1}$$

However following the work of Rath[2] fractional matrix, it is possible to generate as many as commutative matrix corresponding to a non-singular matrix square matrix A(N×N). The procedure is as follows. In sec-II we suggest the method along with necessary proof. Sec-III deals with a suitable matrix A(N×N) (N=2,3). And sec-IV deals with the conclusion.

II. INFINITE GENERATION OF COMMUTATIVE MATRIX

Let A be a matrix and B is another matrix satisfying the relation

$$B = L + A \tag{2}$$

then A commutes with B i.e.

$$[A, B] = 0 \tag{3}$$

Further we can have

$$[B, B^{-1}] = 0 \tag{4}$$

then it is easy to find the commutative relation between A and B^{-1} more explicitly

$$[A, B^{-1}] = 0 \tag{5}$$

Mathematically this can be written as

$$AB^{-1} = B^{-1}A \to A\left(\frac{1}{L+A}\right) = \left(\frac{1}{L+A}\right)A = F \tag{6}$$

This fractional matrix was suggested recently by Rath [2] in calculating eigenvalues.

1. An alternate proof is an follows i.e. Let us consider the matrix relation

$$AB^{-1}A^{-1} = B^{-1} \tag{7}$$

Multiplying both sides with A we get

$$AB^{-1}A^{-1}A = B^{-1}A = AB^{-1} \tag{8}$$

2. Second alternate proof is an follows i.e. consider the eigenvalue relation as

$$AB^{-1}|\Psi> = \lambda|\Psi> \tag{9}$$

Now multiply A^{-1} from left then one will have

$$B^{-1}|\Psi> = \lambda(A^{-1}|\Psi>) = \lambda B^{-1}A(A^{-1}|\Psi>) \tag{10}$$

In other words we have

$$B^{-1}A = AB^{-1} \to \lambda \tag{11}$$

In this approach one has to vary $L \neq 0$ to any arbitrary values. Below we consider a simple (2x2) matrix and generate as many as commutative matrices as follows.

IIIA. INFINITE GENERATION OF COMMUTATIVE MATRIX: CASE STUDY (2×2)

Consider a simple (2x2) matrix A as

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \tag{12}$$

Now consider the matrix $B_L = L + A$ as

$$B_L = \begin{bmatrix} 2+L & 1 \\ 2 & 3+L \end{bmatrix} \tag{13}$$

Let us consider different values of L as follows **L=1** in this case we get the known matrix[1] i.e.

$$B_1 = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \tag{14}$$

and

$$B_1^{-1} = \begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix} \tag{15}$$

Then it easy to show that

$$AB_1^{-1} = B^{-1}A = \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.7 \end{bmatrix} \tag{16}$$

L=10 in this case we get the known matrix [1] i.e.

$$B_1 = \begin{bmatrix} 12 & 1 \\ 2 & 13 \end{bmatrix} \tag{17}$$

and

$$B_{10}^{-1} = \begin{bmatrix} \frac{13}{154} & \frac{-1}{154} \\ \frac{-1}{77} & \frac{6}{77} \end{bmatrix} \tag{18}$$

then it easy to show that

$$AB_{10}^{-1} = B_{10}^{-1}A = \begin{bmatrix} \frac{12}{77} & \frac{5}{77} \\ \frac{10}{77} & \frac{17}{77} \end{bmatrix} \tag{19}$$

L=100 in this case we get the known matrix [1] i.e.

$$B_{100} = \begin{bmatrix} 102 & 1 \\ 2 & 103 \end{bmatrix} \tag{20}$$

and

$$B_{100}^{-1} = \begin{bmatrix} \frac{103}{10504} & \frac{-1}{10504} \\ \frac{-1}{5252} & \frac{52}{5272} \end{bmatrix} \tag{21}$$

Then it easy to show that

$$AB_{100}^{-1} = B_{100}^{-1}A = \begin{bmatrix} \frac{51}{2626} & \frac{25}{2626} \\ \frac{25}{1313} & \frac{38}{1313} \end{bmatrix} \tag{22}$$

IIIB. COMMUTATIVE MATRICES: CASE STUDY (3×3)

Here we just consider a simple example of (3x3) matrix and generate suitable commutative counter part as follows. The explicit expression for A is

$$A = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & 3 & 3 \end{bmatrix} \tag{23}$$

Considering the value of **L=1** we get B_1 as

$$B_1 = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \tag{24}$$

$$B_1^{-1} = \begin{bmatrix} \frac{-7}{10} & \frac{1}{5} & \frac{3}{10} \\ \frac{-13}{10} & \frac{-1}{5} & \frac{7}{10} \\ \frac{4}{5} & \frac{1}{5} & \frac{-1}{5} \end{bmatrix} \tag{25}$$

$$AB_1^{-1} = B_1^{-1}A = \begin{bmatrix} \frac{17}{10} & \frac{-1}{5} & \frac{-3}{10} \\ \frac{13}{10} & \frac{6}{5} & \frac{-7}{10} \\ \frac{-4}{5} & \frac{-1}{5} & \frac{6}{5} \end{bmatrix} \tag{26}$$

IV. CONCLUSION

In this paper we have suggested a method to generate infinite no of commutative matrices to a given square matrix (N×N) (N = 2,3). One can consider any value of N. In other words this work will fill gap that exist in the literatures on matrix analysis. I hope very soon research in this area will motivate mathematical as well as other branches of physical science.

REFERENCE

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