A New Property Related to the Roots of an Equation

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Abstract

If all roots of the n degree equation $a_0x^n + a_1x^{(n-1)} + a_2x^{(n-2)} + \cdots + a_n = 0$ [Where $n \in \mathbb{N}$ and $a_0, a_1, \ldots, a_n$ are real numbers and $a_0 > 0$] are positive real numbers then,
$\frac{a_i a_{(n-i)}}{a_n} \geq [\binom{n}{i}]^2 a_0$ *[For all $i \in \mathbb{N}$ and $1 \leq i \leq n$]*

Keywords: *Cauchy-Schwarz inequality,Roots,Equation*

I. INTRODUCTION

Apart from some well known properties of equations, in this article I am proposing a new condition related to the roots of an n degree equation.

II. PROPOSAL

If all roots of the n degree equation $a_0x^n + a_1x^{(n-1)} + a_2x^{(n-2)} + \cdots + a_n = 0$ [Where $n \in \mathbb{N}$ and $a_0, a_1, \ldots, a_n$ are real numbers and $a_0 > 0$] are positive real numbers then,

$\frac{a_i a_{(n-i)}}{a_n} \geq [\binom{n}{i}]^2 a_0$ [For all $i \in \mathbb{N}$ and $1 \leq i \leq n$]

III. EXAMPLE

When $n = 2$

The given equation is $a_0x^2 + a_1x + a_2 = 0$ [a_0, a_1, a_2 are real numbers and $a_0 > 0$]

Let, roots of the quadratic equation are α, β

$$\therefore \alpha\beta = \frac{a_2}{a_0}$$

If α, β are positive real numbers then,

$$\frac{a_2}{a_0} > 0$$

So, $a_2 > 0$ [As $a_0 > 0$]

So according to my proposal if $i = 1$ we get,

$$\Rightarrow \frac{a_1 a_{(2-1)}}{a_2} \geq [\binom{2}{1}]^2 a_0$$

$$\Rightarrow \frac{{a_1}^2}{a_2} \geq 4a_0$$

$\Rightarrow {a_1}^2 \geq 4a_0a_2$ [$a_2 > 0$]

When $i = 2$ then according to my proposal we get,

$$\frac{a_2 a_{(2-2)}}{a_2} \geq [\binom{2}{2}]^2 a_0$$

Which is also true as equality occurs**.**

III. PROOF

Let, roots of the n degree equation $a_0x^n + a_1x^{(n-1)} + a_2x^{(n-2)} + \cdots + a_n = 0$[Where $n \in \mathbb{N}$ and $a_0, a_1, \ldots, a_n$ are real numbers and $a_0 > 0$] are $b_1, b_2, \ldots, b_n$

Let, p denote the sum of products of $b_1, b_2, \ldots, b_n$ taken i at a time.

Let, q denote the sum of products of $\frac{1}{b_1}, \frac{1}{b_2}, \ldots, \frac{1}{b_n}$ taken i at a time.

Let, r denote the sum of products of $b_1, b_2, \ldots, b_n$ taken $(n-i)$ at a time.

Clearly, $p = (-1)^i \frac{a_i}{a_0}$

And $\quad r = (-1)^{(n-i)} \left[\frac{a_{(n-i)}}{a_0}\right]$

And $\quad q = \frac{r}{b_1 b_2 \ldots b_n} = \frac{(-1)^{(n-i)}\left[\frac{a_{(n-i)}}{a_0}\right]}{(-1)^n \frac{a_n}{a_0}}$

If all $b_1, b_2, \ldots, b_n$ are positive real numbers then,

$p \times q \geq [\sum_{k=1}^{\binom{n}{i}} 1]^2$ [By using **Cauchy-Schwarz inequality**]

$$\Rightarrow (-1)^i \frac{a_i}{a_0} \times \frac{(-1)^{(n-i)}\left[\frac{a_{(n-i)}}{a_0}\right]}{(-1)^n \frac{a_n}{a_0}} \geq \left[\binom{n}{i}\right]^2$$

$$\Rightarrow \frac{a_i a_{(n-i)}}{a_0 a_n} \geq \left[\binom{n}{i}\right]^2$$

$$\Rightarrow \frac{a_i a_{(n-i)}}{a_n} \geq \left[\binom{n}{i}\right]^2 a_0 \quad [a_0 > 0]$$

V. CONCLUSION

Thus apart from some well known properties of equations in this article I have proposed a new property related to the roots of an n degree equation.

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