Ideals and Separation Axioms in LSTBCH-Algebras

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Abstract

There has been some recent interest in applying topological notions to non-mainstream algebras [3, 4]. In our paper [12], we define the notion of topological BCH-algebras, give some examples and prove some important theorems. In this paper we discuss some algebraic properties such as ideals and topological properties such as T_0, T_1 & $T_2 -$ axioms in LS topological BCH-algebra.

Keywords: *Positive implicative TBCH-algebra, $LST_0BCH, LST_1BCH, LST_2BCH$-algebras.*

I. INTRODUCTION

Y.Imai and K.Iseki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [8, 9, 10, 11]. In [5,6] Q. P. Hu and X. Li introduced a wider class of abstract algebras: BCH-algebras. In [2], the authors studied topological BL-algebras and proved some theorems that determine the relationship between them. There has been some recent interest in applying topological notions to non-mainstream algebras [3,4]. In [1], Ahn and Kwon addresses the issue of attaching topologies to BCC-algebras in a natural manner.

In our paper [12], we define the notion of topological BCH-algebras, give some examples and prove some important theorems. Also we introduce the notion of topological BCH-algebras in the set of all left maps. In this paper we discuss ideals and separation axioms in LS-topological BCH-algebras.

II. PRELIMINARIES

In this section, we recall some basic definitions and results that are required for our work.

Definition 2.1:[6] A BCH-algebra which is not a BCI-algebra, then it is called a proper BCH-algebra.

Definition 2.2:[10] Let (X, $*$) be a BCH-algebra and a non empty subset I of X is called an ideal of X, if it satisfies the following conditions:

1. $0 \in I$
2. $(x * y) \in I$ & $y \in I \Rightarrow x \in I$.

Definition 2.3:[12] Let (X, $*$) be a BCH-algebra and ι a topology on X. Then X = (X, $*, \tau$) is called a topological BCH-algebra, if the operation '$*$' is continuous, or equivalently, for any x, y $\in$ X and for any x, y Є X and for any open set W of x$*$y there exist two open sets U and V respectively such that U$*$V is a subset of W.

Definition 2.4:[12] Let X be a TBCH-algebra. If X satisfies the condition,

(x$*$y)$*$z = (x$*$y)$*$(y$*$z), for all x, y, z in X

Then X is called positive implicative TBCH-algebra.

Definition 2.5: [12] Let (X , $*, \tau$) be a TBCH-algebra, and a Є X. Define a left map L_a : X→X by, L_a(x) = a$*$x, for all x $\in$ X.

Definition 2.6: [12] Let (X, $*, \tau$) be a TBCH-algebra. The set of all left maps on X is defined as, $L(X)$.

Definition 2.7:[12] Let X be a positive implicative BCH-algebra and A be any nonempty subset of $\mathbb{L}(X)$, then $L_A = \{ L_a \in \mathbb{L}(X), a \in$ A }

Definition 2.8: [12] Let $(X,*,\tau)$ be a positive implicative TBCH-algebra and the collection of subsets of $\mathbb{L}(X)$, $\tau' = \{\Phi(G) \subseteq \mathbb{L}(X) \backslash G \in \tau\}$ is called a LS-topology on the set $\mathbb{L}(X)$. Where $\Phi(G) = \{L_x \backslash x \in G\}$ and the collection $(\mathbb{L}(X), \circledast, \tau')$ is called the LS- topological BCH-algebra or LSTBCH-algebra.

III. IDEALS AND SEPARATION AXIOMS IN LSTBCH-ALGEBRAS

In this section we discuss about the ideals and separation axioms on a TBCH-algebra. Also we define ideals and separation axioms in LSTBCH-algebra. Finally, we discuss some properties on LSTBCH-algebra.

Definition 3.1: Let X be a positive implicative TBCH-algebra. A nonempty subset I of $\mathbb{L}(X)$ is called an ideal in $\mathbb{L}(X)$ if it satisfies the following condition :

1. $L_0 \in I$
2. $L_a \circledast L_b \in I$ & $L_b \in I \Rightarrow L_a \in I$.

Example 3.2: Consider a positive implicative TBCH-algebra $(X = \{0,1,2\},*,\tau)$ with the following cayley table,

$*$	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

And the topology $\tau = \{\phi, X, \{0,1\}, \{2\}\}$. The ideals of $\mathbb{L}(X)$ *are* $\mathbb{L}(X)$, $\{L_0\}$ & $\{L_0, L_1\}$.

Theorem 3.3: Let X be a positive implicative BCH-algebra, A subset I $\subseteq X$ is an ideal in X if and only if L_I is an ideal in $\mathbb{L}(X)$.

Proof: Let I be an ideal in X. Clearly, 0 $\in X \Rightarrow L_0 \in L_I$.
For $x, y \in X$, Let $x * y$ *and* $y \in I$. Since I is an ideal in X, and $x * y, y \in I \Rightarrow x \in I$.
Then $L_{x*y}, L_y \in L_I \Rightarrow L_x \in L_I$. That is, $L_x \circledast L_y,\ L_y \in L_I \Rightarrow L_x \in L_I$.$\Rightarrow L_I$ is an ideal in $\mathbb{L}(X)$.
Conversely, assume that L_I is an ideal in $\mathbb{L}(X)$. Then $L_0 \in \mathbb{L}(X)$, showing that $0 \in X$.
Assume that If $x * y \in I$ *and* $y \in I$. Then $L_{x*y} \in L_I$ *and* $L_y \in L_I$.$\Rightarrow L_x \circledast L_y \in L_I$ & $L_y \in L_I$.
Since L_I is an ideal in $\mathbb{L}(X)$, $L_x \in L_I$.$\Rightarrow x \in I$.$\Rightarrow I$ is an ideal in X.

Definition 3.4: Let $(X, *, \tau)$ be a positive implicative TBCH-algebra and A be any subset of X. Then $x \in A$ is called an interior point of A if there is an open set $U \in \tau$ such that, $U \subseteq A$.

Example 3.5: Consider the positive implicative TBCH-algebra ($X = \{0,1,2\}, *, \tau$) with the following cayley table,

$*$	0	1	2
0	0	0	0
1	1	0	0
2	2	1	0

And the topology $\tau = \{\phi, X\}$.
If $A = \{0,1\}$, no point of A is an interior point. If $A = \{0,1,2\}$, every point of A is an interior point of A.

Theorem 3.6: Let $(X, *, \tau)$ be a positive implicative TBCH-algebra and A be any ideal of X. If 0 is an interior point of A then L_A is open in LSTBCH- algebra.

Proof:

For every $x \epsilon A$, $L_x \circledast L_x = L_0$. Since 0 is an interior point of A then there exist an open set $U \epsilon \tau$ such that $U \subseteq A$.
Then $\Phi(U) \subseteq \Phi(A)$.
Since $\mathbb{L}(X)$ is a LSTBCH-algebra, there exist a neighbourhood $\Phi(V)$ *of* L_x such that $\Phi(V * V) \subseteq \Phi(U) \subseteq L_A$.
Now we claim that $V \subseteq A$.
If not, there is an element $a \epsilon V \,\&\, a \notin A. \Rightarrow y * a \in V * V \subseteq A. \Rightarrow a \in A$, which is contradicts to our assumption. Therefore, $V \subseteq A$.

Definition 3.7: Let $\mathbb{L}(X)$ be a LSTBCH-algebra. Then it is called a LST_0BCH -algebra, if for every $x, y \in X \,\&\, x \neq y$, there exist atleast one open neighbourhood U that contains one point of the pair (either x or y) excluding the point.

Definition 3.8: Let $\mathbb{L}(X)$ be a LSTBCH-algebra. Then it is called a LST_1BCH -algebra, if for every $x, y \in X \,\&\, x \neq y$, there exist open neighbourhoods $U_1 \,\&\, U_2$ such that U_1contains x but not y and U_2 contains y but not x.

Definition 3.9: Let $\mathbb{L}(X)$ be a LSTBCH-algebra. Then it is called a LST_2BCH -algebra, if for every $x, y \in X \,\&\, x \neq y$, both have disjoint open neighbourhoods U & V such that $x \in U \,\&\, y \in V$.

Example 3.10: : Consider the positive implicative LSTBCH-algebra ($X = \{0, 1, 2\}, \; *, \; \tau$) with the following cayley table,

*	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

And the topology $\tau = \{\phi, X, \; \{0,1\}, \{2\}, \{2,0\}, \{0\}\}$. & $\tau_1 = \{\phi, \; \mathbb{L}(X), \{L_0, L_1\}, \{L_2\}, \{L_2, L_0\}, \{L_0\}\}$ *and*
$\tau_2 = \{ \phi, \mathbb{L}(X), \{L_0, L_1\}, \{L_2\}, \{L_2, L_0\}, \{L_0\}, \{L_1\}, \{L_1, L_2\}\}$.
Then $(\mathbb{L}(X), \circledast, \tau_1)$ is called LST_0BCH-algebra and $(\mathbb{L}(X), \circledast, \tau_2)$ is called *a* LST_1BCH-algebra and LST_2BCH-algebra.

Theorem 3.11: Let X be a positive implicative TBCH-algebra. If $\{0\}$ is closed in X if and only if $(\mathbb{L}(X), \circledast, \; \tau')$ is a LST_2BCH-algebra.

Proof: Let $x, y \in X \,\&\, x \neq y$. Then we have $L_x \circledast L_y \neq L_0$ *or* $L_y \circledast L_x \neq L_0$.
Assume that, $L_x \circledast L_y \neq L_0$, *that is* $L_{x*y} \neq L_0$.
Then there exist some neighbourhood $\Phi(U) \,\&\, \Phi(V)$ of $L_x \,\&\, L_y$ respectively such that,

$$\Phi(U * V) \subseteq \mathbb{L}(X) \setminus \Phi(\{0\}).$$

Clearly, $\Phi(U) \cap \Phi(V) = \phi$. Hence, $\mathbb{L}(X)$ *is* LST_2BCH.
Conversely, we claim that, $X \setminus \{0\}$ is open in X. Let $x \in X \setminus \{0\}$,
Then there exist some neighbourhood $\Phi(U) \,\&\, \Phi(V)$ of $L_x \,\&\, L_0$ respectively such that, $\Phi(U) \cap \Phi(V) = \phi$.
$\Rightarrow 0 \notin U \Rightarrow U \subseteq X \setminus \{0\} \Rightarrow X \setminus \{0\}$ is open.
Thus {0} is closed in X.

Theorem 3.12: Let X be a proper positive implicative TBCH-algebra, then the following are equivalent

1. LST_0BCH –algebra
2. LST_1BCH –algebra
3. LST_2BCH – algebra.

Proof: From theorem 3.11, $LST_0BCH \Leftrightarrow LST_1BCH$. We need only show $(1) \Rightarrow (2)$.
Suppose that $L_x \neq L_y, L_{x*y} = L_0$ (or) $L_{y*x} = L_0$.

We may assume that $L_{x*y} \neq L_0$.

If there exist an open set $\Phi(U)$ such that $L_{x*y} \in \Phi(U)$ & $L_0 \notin \Phi(U)$, then we can find open neighbourhoods $\Phi(V)$ *of* L_x & $\Phi(W)$ *of* L_y such that $\Phi(V * W) \subseteq \Phi(U)$.

Then $L_x \notin \Phi(W)$ & $L_y \notin \Phi(V)$.

On the other hand, if there exist an open set $\Phi(U)$ such that $L_{x*y} \notin \Phi(U)$ & $L_0 \in \Phi(U)$, then we can find open neighbourhoods $\Phi(V)$ *of* L_x & $\Phi(W)$ *of* L_y such that $\Phi(V * W) \subseteq \Phi(U)$.

Then $L_x \notin \Phi(W)$ & $L_y \notin \Phi(V)$.

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IV. CONCLUSSION

In this paper, we worked on some algebric properties such as ideals and topological properties such as T_0, T_1 & $T_2 -$ *axioms* in LS topological BCH-algebra. If our BCH- algebra is associative, the same properties are true in RS- topological BCH-algebra, that is set of all right maping in TBCH-algebras with binary operation ⊛. Also we extend this paper by using other separation axioms like us T_3, T_4 & normal spaces.