On Quasi –Transitive Fuzzy Digraphs

Sarala N [1] Janaki P[2]
[1](Department of Mathematics, A.D.M.College for Women, Nagapattinam, Tamilnadu, India)
[2] (Department of Mathematics, RabiammalAhamedMaideen College for women,ThiruvarurTamilnadu, India)

Abstract:

In this paper we introduceQasi-Transitive Fuzzy Digraph. Also, some basic results of Quasi-Transitive FuzzyDigraph are discussed.

Keywords: *Fuzzy digraph,Quasi-Transitive Fuzzy Digraph,semicomplete fuzzyDigraph, Transitive Fuzzy Digraph.*

I. INTRODUCTION

The concept of fuzzy graph was introduced by Rosenfeld [1] in 1975. Fuzzy graph theory has a vast area of applications. It is used in evaluation of human cardiac function, fuzzy neural networks, etc. Fuzzy graphs can be used to solve traffic light problem, time table scheduling, etc. In fuzzy set theory, there are different types of fuzzy graphs which may be a graph with crisp vertex set and fuzzy edge set or fuzzy vertex set and crisp edge set or fuzzy vertex set and fuzzy edge set or crisp vertices and edges with fuzzy connectivity, etc. A lot of works have been done on fuzzy graphs [3], [4], [5]..

The Fuzzy Directed Graph, Fuzzy competition digraphs are well known topic. In this article Fuzzy Round digraphs are defined and their properties are investigated. Locally semicomplete fuzzy digraph, ordinary arc in a fuzzy round digraph are discussed

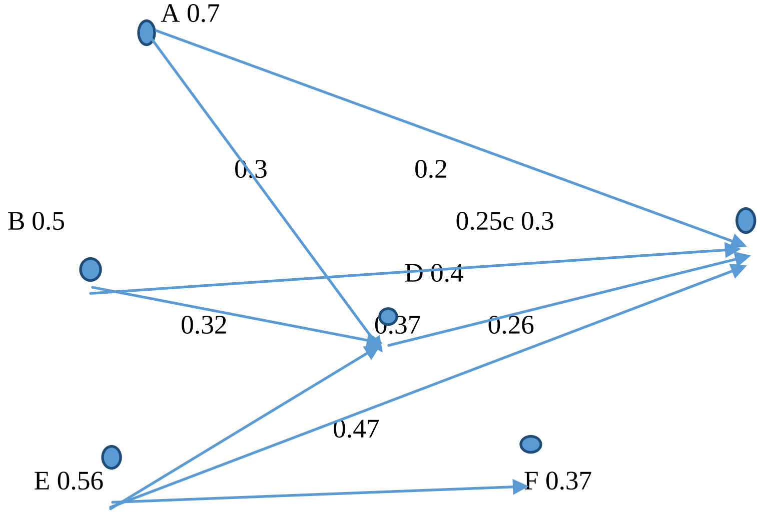
II. PRELIMINARIES

Definition 2.1:

Fuzzy digraph $\vec{\xi}$ = (V,σ, $\vec{\mu}$) is a non-empty set V together with a pair of functions σ : V → [0,1] and $\vec{\mu}$: V ×V → [0,1] such thatfor all x,y ∈ V , $\vec{\mu}$ (x,y) ≤ σ(x)∧σ(y).Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}$ (u,v) is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}$(x,x) ≠ 0. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}$(x,y) and $\vec{\mu}$ (y,x) may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges

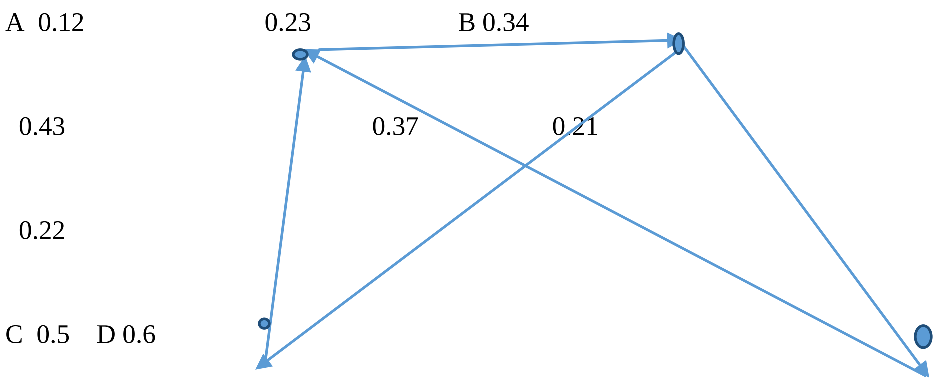
Definition:2.2

A Fuzzy digraph D is said to be ***transitive*** , if for every pair of arcs $\sigma(x)\sigma(y)$ and $\sigma(y)\sigma(z)$ in D such that $\sigma(x) \neq \sigma(z)$, the arc $\sigma(x)\sigma(z)$ is also in D.



Definition.2.3:

A fuzzy digraph D is ***quasi- transitive***, if for every triple $\sigma(x)\sigma(y)\sigma(z)$ of distinct vertices of D such that $\sigma(x)\sigma(y)$ and $\sigma(y)\sigma(z)$ are arcs of D , there is at least one arc between $\sigma(x)\ and \sigma(z)$.



III. QUASI-TRANSITIVE FUZZY DIGRAPH

Proposition.3.1:

Let D be a quasi-transitive fuzzy digraph. Suppose that P=$\sigma(x_1)\sigma(x_{2)}$........$\sigma(x_k)$ is a minimal path. Then the fuzzy digraph induced by V(P) is a semi complete fuzzy digraph and $\sigma(x_i)\sigma(x_j)$ for every 2≤ i+1 <j ≤ k, unless k=4 in which case the arc between x_i and x_k may be absent.

Proof:

The case k=2,3,4,5 are easily verified.As an example, let us consider the case k=5.If$\sigma(x_i)$ and $\sigma(x_j)$are adjacent and 2≤ i+1 <j ≤ 5, then $\sigma(x_i) \to \sigma(x_j)$since P is minimal. since D is fuzzy quasi transitive, $\sigma(x_i)$and $\sigma(x_{i+2})$are adjacent for i =1,2,3.This and the minimality of P imply that $\sigma(x_3) \to \sigma(x_1)$, $\sigma(x_4) \to \sigma(x_2)$ and$\sigma(x_5) \to \sigma(x_3)$. From these arcs the minimality of P we conclude that $\sigma(x_5) \to \sigma(x_1)$. Smilarly $\sigma(x_5) \to \sigma(x_1) \to \sigma(x_2)$ implies that $\sigma(x_5) \to \sigma(x_2)$.

The proof for the case k ≥ 6 is by in→duction on k with the case k=5as the basis. By induction, each of D ⟨{$\sigma(x_1)\sigma(x_2)\ldots\ldots\sigma(x_{k-1})$}⟩and D ⟨{$\sigma(x_2)\sigma(x_3)\ldots.\sigma(x_{k-1})$ i}⟩is a semicomplete digraph and$\sigma(x_j) \to \sigma(x_i)$ for any 1<j-i≤k-2. Hence $\sigma(x_3)$ dominates $\sigma(x_1)$ and $\sigma(x_k)$ dominates $\sigma(x_3)$ and the minimality of P implies that $\sigma(x_k)$ dominates $\sigma(x_1)$.

Result.3.2:

If a quasi-transitive fuzzy digraphD has an σ (x), σ (y) path but σ(x) does not dominates σ (y), then either σ (y) → σ(x), or there exist vertices σ (u), σ (v)∈ V(D)-{ $\sigma(x), \sigma(y)$} such that σ (x) → σ (u) → σ (v) → σ(y) and σ(y) → σ (u) → σ (v)→ σ (x).

Lemma.3.3:

Suppose that A and B are distinct strong components of a quasi –transitive fuzzy digraphs D with at least one arc from A to Bthen A⟼ B.

Proof:

Suppose that A and B are distinct strong components such that there exist an arc from A to B.Then for every choice ofσ (x)∈A andσ (y)∈ B there exist a path from σ (x) to σ (y) in D. Since A and B are distinct components , none of the alternatives in the above result can hold and hence σ (x) → σ (y).

Lemma.3.4:

Let D be a strong quasi-transitive fuzzy digraph on at least two vrtices. Then the following holds;

(a) $\overline{UG(D)}$ is disconnected;

(b) If S and S' are two fuzzy sub digraphs of D such that $\overline{UG(S)}$and $\overline{UG(S')}$ are distinct components of $\overline{UG(D)}$, then either S⟼ S' or S' ⟼ S, or both S⟼ S' andS' ⟼ S, in which case $|V\ (S)|$ = $|V(S')|$ =1

Proof:

The statement (b) can be easily verified from the definition of a quasi-trasitive fuzzy digraph and the fact that S and S' are completely adjacent in D. The statement (a) is trivially true when V(D) =2or 3. Assume that it holds when V(D) < n where n > 3.

Suppose that there is a vertex σ (z) such that D-σ(z)is not strong. Then there is an arc from (to) every terminal (initial) component of D –σ(z) to (from)σ (z).since D is quasi- transitive, by using the lemma 3.3 imply that X →Y for every initial (terminal) strong component X(Y) of D-σ (z). Similar argument shows that each strong component of D-σ (z) either dominates some terminal component or is dominated by some initial component of D-σ (z). Therefore $\overline{UG(D)}$contains a component consisting of the vertexσ (z) ,implying that $\overline{UG(D)}$ is disconnected and (a) follows.

Assume that there is a vertexσ (v) such that D-σ(v) is strong. Since D is strong, D contains an arc σ(v) σ(w) from σ(v) to D-σ(v). Since by induction $\overline{UG(D-\sigma(v))}$ is not connected. Let connected component S and S' of $\overline{UG(D-\sigma(v))}$ be chosen such that σ(w)∈ S,S→ S'in D. Then σ(v) is completely adjacent toS' in D. Hence $\overline{UG(S')}$ is a connected component of $\overline{UG(D)}$ and the proof is complete.

Theorem.3.5:

Let D be a Fuzzy Digraph which is quasi-transitive.

a) If D is not strong, then there is a fuzzy transitive oriented graph T with vertices $\{\sigma(u_1), \sigma(u_2) \dots \dots \sigma(u_t)\}$ and strong quasi –transitive fuzzy digraphs $H_1, H_2 \dots \dots H_t$ such that D= T$[H_1, H_2 \dots \dots H_t\,]$where $H_i$is substituted for u_i,i=1,2,…..t.

b) If D is strong then there exist a strong semicomplete fuzzy digraph S with vertices $\{\sigma(v_{1)}, \sigma(v_2), \dots \dots \sigma(v_s)\}$ and quasi transitive fuzzy digraphs $Q_1, Q_2 \dots . Q_s$such that Q_i is either a vertex or non strong and D= S$[Q_1, Q_2 \dots . Q_s]$, where Q_i substituted for v_i,i=1,2,….s.

Proof:

Suppose that Dis not strong and let $H_1, H_2 \dots \dots H_t$ be the strong components of D. According to Lemma 4.8.3., if there is an arc between$H_i$andH_j, then either$H_i \rightarrow H_j$or$H_j \rightarrow H_i$.Now if $H_i \rightarrow H_j \rightarrow H_k$ then by quasi -transitivity$H_i \rightarrow H_k$.So by contracting each H_i to a vertex$\sigma(h_i)$ we get a transitive oriented graph T with vertices $\sigma(h_1)\sigma(h_2), \dots \dots \sigma(h_t)$. This shows that D=$[H_1, H_2, \dots \dots H_t]$.

Suppose that D is strong. let $Q_1, Q_2, \dots \dots Q_s$ be the subdigraphs of D such that $\overline{UG(Q_i)}$ is a connected component of $\overline{UG(D)}$.According to lemma.3.4 (a) each Q_i is either non strong or just a single vertex. By lemma.3. 4. (b) we obtain a strong semicomplete fuzzy digraph S if each Q_i is contracted to a vertex. This shows that D=S[$Q_1, Q_2, \dots \dots Q_s$].

IV. CONCLUSION

Finally we have analyzed about the quasi-transitive fuzzy digraph and the conditions for the fuzzy digraph being a quasi- transitivefuzzys digraph and their properties. This study will help full in system where fuzzy digraphs are applied.

REFERENCES

[1] A.Rosenfeld, Fuzzygraphs, in:L.A.Zadeh, K.S. Fu, M. Shimura (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York, 77-95, 1975.

[2] S.Mathew and M.S. Sunitha, Types of arcs in a fuzzy graph, Information Sciences, 179, 1760-1768, 2009.

[3] SovanaSamantha,Mathumangal pal, Anitha pal, Some more results on fuzzy K- competition graphs IJARA, vol.3,2014.

[4] Jorgen Bang-Jensen, Gregory Gutin ,Digraphs Theory Algorithms and Applications,Springer- verlag 2007.

[5] S.Samanta and M. Pal, A new approach to social networks based on fuzzy graphs, To appear in Journal of Mass Communication and Journalism.