Smarandache Anti Q-Fuzzy Semigroups

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Abstract

In this paper we introduce the concept of Smarandache anti Q-fuzzy semigroups, Smarandache anti Q-fuzzy normal semigroups and some of their properties are discussed.

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Keywords : *Q-fuzzy subset, anti Q-fuzzy group, Smarandache semigroup, Smarandache anti Q-fuzzy semigroups, Smarandache anti Q-fuzzy normal semigroups.*

I. INTRODUCTION

In 1965, L.A.Zadeh[10] introduced the concept of fuzzy subset. In 1971, A.Rosenfeld[7] introduced the notion of fuzzy group. R.Biswas[2] introduced the concept of anti fuzzy groups of in 1990. Padilla Raul[5] introduced the notion of Smarandache semigroup in the year 1998. Smarandache fuzzy semigroups were studied in 2003 by W.B.Vasantha kandasamy[9]. In 2008, A.Solairaju and R.Nagarajan[8] introduced a new algebraic structure namely Q-fuzzy groups. R.Arul doss and S.Suganya [1] introduced the idea of Smarandache Q-fuzzy semigroups and Smarandache Q-fuzzy normal semigroups in 2016. In this paper Smarandache anti Q-fuzzy semigroups, Smarandache anti Q-fuzzy normal semigroups are defined and some of their properties are discussed.

II. PRELIMINARIES

Definition 2.1. Let X be a non empty set. A fuzzy subset μ of the set

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X is a function $\mu : X \to [0, 1]$.

Definition 2.2. Let H be a Semigroup. H is said to be Smarandache semigroup(S-semigroup) if H has a proper subset G such that G is a Group under the operation of H.

Definition 2.3. Let X and Q be non empty sets. A Q-fuzzy subset A of X is a function $A : X \times Q \to [0, 1]$

Definition 2.4. Let G be a group and Q be any non empty set. A Q-fuzzy subset A of G is said to be Anti Q-fuzzy group of G if

(i) $A(xy, q) \le \max\{A(x, q), A(y, q)\}$

(ii) $A(x^{-1}, q) = A(x, q)$, for all $x, y \in G$ and $q \in Q$

Definition 2.5. Let G be a group and Q be any non empty subset. For any

$a \in G$, $a\mu a^{-1}$is an Anti Q-fuzzy middle coset of G, if $(a\mu a^{-1})(x, q) = \mu(a^{-1}xa)$, for all $x \in G$ and $q \in Q$.

III. SMARANDACHE ANTI Q-FUZZY SEMIGROUPS

Definition 3.1. Let S be an S-semigroup. A Q-fuzzy subset A of S is said to be a **Smarandache anti Q-fuzzy semigroup** if $A : S\times Q \to [0, 1]$ is such that A is restricted to at least one proper subset G of S which is a group and the restriction map $A_G : G\times Q \to [0, 1]$ is an anti Q-fuzzy group, that is for all x, y∈G and q∈Q, $A_G(xy, q) \leq \max\{A_G(x, q), A_G(y, q)\}$ and $A_G(x, q)=A_G(x^{-1}, q)$.

Example 3.2. Consider an S-semigroup Z_4 under multiplication modulo 4.

Let Q={1}.

Let A : $Z_4 \times Q \rightarrow [0, 1]$ be defined by,

$$A(x) = \begin{cases} 0.4 & if\ x = (0,1) \\ 0.5 & if\ x = (1,1),(3,1) \\ 0.6 & otherwise \end{cases}$$

Clearly A is a Q-fuzzy subset of Z_4.
Consider P = {1, 3} which is a proper subset of Z_4 and is also a group in Z_4.
It can be easily verified that $A_P : P \times Q \rightarrow [0, 1]$ is an anti Q-fuzzy group.
Therefore A is a Smarandache anti Q-fuzzy semigroup.

Remark 3.3. Throughout this paper we mention Smarandache Anti Q-Fuzzy Semigroup as **S-AQFS**.

Proposition 3.4. If A is an S-AQFS of an S-semigroup S relative to a group G and Q is a non empty set, then

(i) $A_G(e, q) \leq A_G(x, q)$, where e is the identity element of G.

(ii) $A_G(x^{-1}, q) \leq A(x, q)$ for all x ∈ G and q ∈Q.

Proof. Let A be an S-AQFS of an S-semigroup S. Then A is restricted to at least one proper subset G of S which is a group and $A_G : G \times Q \rightarrow [0, 1]$ is an anti Q-fuzzy group.
Therefore $A_G(x, q) = A(x, q), \forall x \in G$ and q∈Q

(i) Let e∈G, where e is the identity element of G. Now,

$$\begin{aligned} A_G(e, q) &= A_G(xx^{-1}, q) \\ &\leq \max\{A_G(x, q), A_G(x^{-1},q)\} \\ &= \max\{A_G(x, q), A_G(x, q)\} \\ &= A_G(x, q) \end{aligned}$$

Therefore $A_G(e, q) \leq A_G(x, q)$.

(ii) $$\begin{aligned} A_G(x^{-1}, q) &= A_G(ex^{-1}, q) \\ &\leq \max\{A_G(e, q), A_G(x^{-1}, q)\} \\ &= \max\{A_G(e, q), A_G(x,q)\} \\ &= A_G(x, q) \text{(by (i))} \end{aligned}$$

$$A_G(x^{-1}, q) \leq A_G(x, q)$$

Therefore $A_G(x^{-1}, q) \leq A_G(x, q)$.

□

Theorem 3.5. Let S be an S-semigroup, Q any nonempty set and let G be a proper subset of S which is a group in S. $\mu : S\times Q \to [0, 1]$ is an S-AQFS relative to G if and only if $\mu_G(xy^{-1}, q) \leq \max\{\mu_G(x, q), \mu_G(y, q)\}$, for all $x, y \in G$ and $q \in Q$.

Proof. Assume that $\mu:S\times Q\to[0,1]$ is an S-AQFS relative to G. Then μ is restricted to G and $\mu_G : G\times Q \to [0, 1]$ is an anti Q-fuzzy group. Then

$\mu_G(x, q) = \mu(x, q), \forall x \in G$ and $q \in Q$.

Let $x, y^{-1} \in G$ and $q \in Q$.

$$\text{Then } \mu_G(xy^{-1}, q) \leq \max\{\mu_G(x, q), \mu_G(y^{-1}, q)\}$$

$$= \max\{\mu_G(x, q), \mu_G(y, q)$$

Therefore, $\mu_G(xy^{-1}, q) \leq \max\{\mu_G(x, q), \mu_G(y, q)\}$

Conversely, assume that

$$\mu_G(xy^{-1}, q) \leq \max\{\mu_G(x, q), \mu_G(y, q)\} \qquad \text{(i)}$$

for all $x, y \in G$ and $q \in Q$

Put $y = x$ in (i).

$$\text{Then } \mu_G(xx^{-1}, q) \leq \max\{\mu_G(x, q), \mu_G(x, q)\}$$

$$\mu_G(e, q) \leq \mu_G(x, q) \qquad \text{(ii)}$$

iii)Now,

$$\mu_G(y^{-1}, q) = \mu_G(ey^{-1}, q)$$

$$\leq \max\{\mu_G(e, q), \mu_G(y, q)\} \text{ (by (i))}$$

$$\mu_G(y^{-1}, q) \leq \mu_G(y, q) \text{(by(ii))}$$

Also,

$$\mu_G(y, q) = \mu_G(e(y^{-1})^{-1}, q)$$

$$\leq \max\{\mu_G(e, q), \mu_G(y^{-1}, q)\} \text{(by (i))}$$

$$\mu_G(y, q) \leq \mu_G(y^{-1}, q) \text{ (by (ii))}$$

Therefore $\mu_G(y, q) = \mu_G(y^{-1}, q)$

iv)

$$\mu_G(xy, q) = \mu_G(xy)^{-1}, q)$$

$$= \mu_G(y^{-1}x^{-1}, q)$$

$$\leq \max\{\mu_G(y^{-1}, q), \mu_G(x^{-1}, q)\}$$

$$\leq \max\{\mu_G(y, q), \mu_G(x, q)\} \text{(by (i))}$$

$$\mu_G(xy, q) \leq \max\{\mu_G(x, q), \mu_G(y, q)\}$$

From (iii) and (iv) μ is an S-AQFS relative to a group G. □

Theorem 3.6. The union of two S-AQFS of an S-semigroup S relative to a group G is also an S-AQFS of S.

Proof. Let A and B be any two S-AQFS of an S-semigroup S relative to a group G.
Then $A_G : G\times Q \to [0, 1]$ and $B_G : G\times Q \to [0, 1]$ are anti Q-fuzzy groups.

Let $x, y \in G$ and $q \in Q$.

(i) $(A_G \cup B_G)(xy, q) = \max\{A_G(xy, q), B_G(xy, q)\}$

$$\leq \max\{\max\{A_G(x, q), A_G(y, q)\}, \max\{B_G(x, q), B_G(y, q)\}\}$$
$$= \max\{\max\{A_G(x, q), B_G(x, q)\}, \max\{A_G(y, q), B_G(y, q)\}\}$$
$$= \max\{(A_G \cup B_G)(x, q), (A_G \cup B_G)(y, q)\}$$

$$(A_G \cup B_G)(xy, q) \leq \max\{(A_G \cup B_G)(x, q), (A_G \cup B_G)(y, q)\}$$

(ii) $(A_G \cup B_G)(x^{-1}, q) = \max\{A_G(x^{-1}, q), B_G(x^{-1}, q)\}$

$$= \max\{(A_G(x, q), B_G(x, q)\}$$
$$(A_G \cup B_G)(x^{-1}, q) = (A_G \cup B_G)(x, q)$$

From (i) and (ii), the union of two S-AQFS is also an S-AQFS of S relative a group G.
□

Theorem 3.7. The union of a family of S-AQFS of an S-semigroup S relative to a group G is an S-AQFS.

Proof. Let $\{A_i\}_{i\in I}$ be a family of S-anti Q fuzzy semigroups of S relative to a group G.

That is, $A = \bigcup_{i\in I} A_i$

Then for $x, y \in G$ and $q \in Q$,

$$A(xy, q) = \sup_{i\in I} A_i(xy, q)$$
$$\leq \sup_{i\in I} \max\{A_i(x, q), A_i(y, q)\}$$
$$= \max\{\sup A_i(x, q), \sup A_i(y, q)\}$$
$$= \max\{A(x, q), A(y, q)\}$$

$$A(xy, q) = \max\{A(x, q), A(y, q)\} \forall\, x, y \in G \text{ and } q \in Q. \qquad (i)$$

$$A(x^{-1}, q) = \sup_{i\in I} A_i(x^{-1}, q)$$
$$\leq \sup_{i\in I} A_i(x, q)$$
$$= A(x, q)$$

$$A(x^{-1}) \leq A(x) \forall\, x \in G \text{ and } q \in Q. \qquad (ii)$$

From (i) & (ii) the union of family of S-AQFS is an S-AQFS. □

Theorem 3.8. Let μ be an S-AQFS relative to a group $G \subset S$, where S is an S-semigroup. If $\mu_G(xy^{-1}, q) = 0$, then $\mu_G(x, q) = \mu_G(y, q)$ for all $x, y \in G$ and $q \in Q$.

Proof. Let μ be an S-AQFS relative to a group G⊂S, where S is an S-semigroup. Then the restriction map μ_G: G × Q →[0, 1] is an anti Q-fuzzy group.

Let x, y∈G and q∈Q.

$\mu_G(x, q) = \mu_G(xy^{-1}y, q)$

$\leq \max\{\mu_G(xy^{-1}, q), \mu_G(y, q)\}$(since μ is an S− AQFS) = max{0, $\mu_G(y, q)$

$= \mu_G(y, q)$

$\mu_G(x, q) \leq \mu_G(y, q)$ (i)

$\mu_G(y, q) = \mu_G(y^{-1}, q)$(By the definition of S −AQFS)

$= \mu_G(x^{-1}xy^{-1}, q)$

$\leq \max\{\mu_G(x^{-1}, q), \mu_G(xy^{-1}, q)\}$

$= \max\{\mu_A(x^{-1}, q), 0\}$

$= \mu_G(x^{-1}, q) =$

$\mu_G(x, q)$

$\mu_G(y, q) \leq \mu_G(x, q)$ (ii)

From (i) and (ii), if $\mu_G(xy^{-1}, q) = 0$, then $\mu_G(x, q) = \mu_G(y, q)$ for all x, y∈G and q∈Q. □

IV. SMARANDACHE ANTI Q-FUZZY NORMAL SEMIGROUPS

Definition 4.1. Let A be an S-AQFS of an S-semigroup S relative to a group G. Then A is called Smarandache anti Q-fuzzy normal semigroups (S-AQFNS) if $A_G(xy, q) = A_G(yx, q)$, that is $A_G(x, q) = A_G(yxy^{-1}, q)$ for all x, y∈G⊂S and q∈ Q.

Theorem 4.2. If A is an S-AQFNS of an S-semigroup S relative to a group G, then $A_G(xy, q) = A_G(yx, q)$ if and only if $A_G(x, q) = A_G(y^{-1}xy, q)$ for every x, y∈G.

Proof. Let A be an S-AQFNS of an S-semigroup S relative to a group G.

Let x, y∈G.

Assume that $A_G(xy, q) = A_G(yx, q)$.

We have $A_G(y^{-1}xy, q) = A_G(y^{-1}yx, q)$

$= A_G(ex, q)$

$= A_G(x, q)$

Therefore $A_G(y^{-1}xy, q) = A_G(x, q)$

Conversely, assume that $A_G(x, q) = A_G(y^{-1}xy, q)$.

$A_G(xy, q) = A_G(xyxx^{-1})$

$= A_G(yx, q)$ (by assumption)

$A_G(xy, q) = A_G(yx, q) \forall x, y \in G$.

□

Theorem 4.3. The union of two S-AQFNS of an S-semigroup S relative to a group G is also an S-AQFNS of S.

Proof. Let λ and μ be two S-AQFNS of an S-semigroup S relative to a same group G in S.

Then the restriction maps $\lambda_G : G\times Q \rightarrow [0, 1]$ and $\mu_G : G\times Q \rightarrow [0, 1]$ are anti Q-fuzzy groups. $\lambda_G \cup \mu_G$ is an S-AQFS of H relative to G.

For all x, y∈G, we have

$$(\lambda_G \cup \mu_G)(xyx^{-1}, q) = \max\{\lambda_G(xyx^{-1}, q), \mu_G(xyx^{-1}, q)\}$$

$$= \max\{\lambda_G(y, q), \mu_G(y, q)\}$$
$$= (\lambda_G \cup \mu_G)(y, q)$$

Therefore $(\lambda_G \cup \mu_G)(xyx^{-1}, q) = (\lambda_G \cup \mu_G)(y, q)$ Hence $(\lambda_G \cup \mu_G)$ is an S-AQFNS of S relative to G.

□

Theorem 4.4. Let A be an S-AQFNS of an S-semigroup S relative to a group G in S and Q be any non-empty set. Define a set $S_{AG}=\{x\in G|A_G(x, q) = A_G(e, q)$, e is the identity of G and $q \in Q\}$. Then S_{AG} is a normal subgroup of G.

Proof. Let A be an S-AQFNS of an S-semigroup S relative to a group G in S. Therefore A is restricted to G such that A_G is an anti-Q fuzzy group. Define $S_{AG}=\{x\in G|A_G(x, q) = A_G(e, q)\}$. Clearly $e\in S_{AG}$.

Therefore S_{AG} is a non empty subset of G.
Let x, y$\in S_{AG}$ and q∈Q.

Now $A_G(xy^{-1}, q) \leq \max\{A_G(x, q), A_G(y, q)\}$

$$= \max\{A_G(e, q), A_G(e, q)\}$$
$$= A_G(e,q)$$

Therefore $A_G(xy^{-1}, q) \leq A_G(e, q)$

Also, $A_G(xy^{-1}, q) \geq A_G(e, q)$

Therefore $A_G(xy^{-1}, q) = A_G(e, q)$

By the definition of S_{AG}, $(xy^{-1}, q) \in S_{AG}$. (i)

Therefore S_{AG} is a subgroup of G.
Now let $x\in S_{AG}$ and y∈G. Then $A_G(x, q) = A_G(e, q)$.
Since A is an S-fuzzy normal semigroup of G,

$$A_G(y^{-1}xy, q) = A_G(x, q)$$
$$= A_P(e, q)$$

Therefore $A_G(y^{-1}xy, q) = A_G(e, q)$

$(y^{-1}xy, q) \in S_{AG}$ (ii)

From (i) and (ii) S_{AG} is a normal subgroup of G. □

Theorem 4.5. If A is an S-AQFNS of an S-semigroup S relative to a group G and Q is any non-empty set, then gAg^{-1} is also an S-AQFNS.

***Proof*.** Let A be an S-AQFNS of an S-semigroup S relative to a group G and Q be any non-empty set. Therefore the restriction map $A_G : G\times Q \rightarrow [0, 1]$ is an anti Q-fuzzy group.

$$(gAg^{-1})(xyx^{-1}, q) = A(g^{-1}(xyx^{-1}, q)g) \text{ (by definition 2.5)}$$

$$= A(xyx^{-1}, q) \text{(since A is an S-AQFNS)}$$

$$= A(y, q)$$

$$= A(gyg^{-1}, q) \text{ for some } g \in G$$

$$(gAg^{-1})(xyx^{-1}, q) = gAg^{-1}(y)$$

Therefore gAg^{-1} is an S-AQFNS. □

V. CONCLUSION

In this paper, the concept of Smarandache anti Q-fuzzy semigroups and Smarandache anti Q-fuzzy normal semigroups are defined and some of their properties are discussed.

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