Eulerian Integral Associated with Product of Two Prasad's Multivariable H-Functions, the Classes of Multivariable Polynomials

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ABSTRACT
Recently, Raina and Srivastava [2] and Srivastava and Hussain [7] have provided closed-form expressions for a number of a Eulerian integral about the multivariable H-functions.The present paper is evaluated a new Eulerian integral associated with the product of two modified multivariable I-functions defined by Prasad [1], a generalized Lauricella function an the classes of multivariable polynomials with general arguments . We shall study the case concerning the Srivastava-Daoust polynomial [4] and we shall give few remarks..

Keywords: Eulerian integral, modified multivariable H-function, generalized Lauricella function of several variables, generalized hypergeometric function, class of polynomials

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1. Introduction

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. The authors Raina and Srivastava [2], Saigo and Saxena [3], Srivastava and Hussain [7], Srivastava and Garg [6] and other have studied the Eulerian integral. In this paper, we consider a general class of Eulerian integral concerning the product of two multivariable I-functions, defined by Prasad [1], the generalized hypergeometric function and the classes of multivariable polynomials.
The generalized polynomials of multivariables defined by Srivastava [4], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}$$

$$A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{1.1}$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex. Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.2}$$

The coefficients are $B[E;R_1,\ldots,R_u]$ arbitrary constants, real or complex.

We shall note $a_v = \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]$ and

$$b_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{1.3}$$

The multivariable I-function of r-variables defined by Prasad [1] is an extension of the multivariable H-function defined by Srivastava and Panda [8,9]. It is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{(1)}_j,\alpha^{(1)}_j)_{1,p^{(1)}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{(1)}_j,\beta^{(1)}_j)_{1,q^{(1)}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right) - \left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)} + \sum_{k=1}^{q_3}\beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.10}$$

where $i = 1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,, max(\,|z_1|,\cdots,|z_r|\,) \rightarrow 0$$

$$I(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,, min(\,|z_1|,\cdots,|z_r|\,) \rightarrow \infty$$

where $k = 1,\cdots,r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1,\cdots,m^{(k)}$ and

$$\beta'_k = max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j = 1,\cdots,n^{(k)}$$

Condider a second multivariable I-function defined by Prasad [1]

$$I(z'_1, z'_2, ...z'_s) = I^{0,n'_2;0,n'_3;\cdots;0,n'_r:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}} \left(\begin{array}{c|c} z'_1 & (a'_{2j};\alpha'^{(1)}_{2j},\alpha'^{(2)}_{2j})_{1,p'_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_{2j};\beta'^{(1)}_{2j},\beta'^{(2)}_{2j})_{1,q'_2};\cdots; \end{array} \right.$$

$$\left. \begin{array}{c} (a'_{sj};\alpha'^{(1)}_{sj},\cdots,\alpha'_{sj}{}^{(s)})_{1,p'_s} : (a'^{(1)}_j,\alpha'^{(1)}_j)_{1,p'^{(1)}};\cdots;(a'_j{}^{(s)},\alpha'_j{}^{(s)})_{1,p'^{(s)}} \\ \\ (b'_{sj};\beta'^{(1)}_{sj},\cdots,\beta'_{sj}{}^{(s)})_{1,q'_s} : (b'^{(1)}_j,\beta'^{(1)}_j)_{1,q'^{(1)}};\cdots;(b'_j{}^{(s)},\beta'_j{}^{(s)})_{1,q'^{(s)}} \end{array} \right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \psi(t_1,\cdots,t_s) \prod_{i=1}^{s} \xi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|arg z'_i| < \frac{1}{2}\Omega'_i \pi$,

$$\Omega'_i = \sum_{k=1}^{n'^{(i)}} \alpha'^{(i)}_k - \sum_{k=n'^{(i)}+1}^{p'^{(i)}} \alpha'^{(i)}_k + \sum_{k=1}^{m'^{(i)}} \beta'^{(i)}_k - \sum_{k=m^{(i)}+1}^{q'^{(i)}} \beta'^{(i)}_k + \left(\sum_{k=1}^{n'_2} \alpha'^{(i)}_{2k} - \sum_{k=n_2+1}^{p'_2} \alpha'^{(i)}_{2k} \right)$$

$$+ \cdots + \left(\sum_{k=1}^{n'_s} \alpha'_{sk}{}^{(i)} - \sum_{k=n'_s+1}^{p'_s} \alpha'_{sk}{}^{(i)} \right) - \left(\sum_{k=1}^{q'_2} \beta'^{(i)}_{2k} + \sum_{k=1}^{q'_3} \beta'^{(i)}_{3k} + \cdots + \sum_{k=1}^{q'_s} \beta'_{sk}{}^{(i)} \right) \tag{1.13}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s) = 0(\,|z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}\,), max(\,|z'_1|,\cdots,|z'_s|\,) \to 0$$

$$I(z'_1,\cdots,z'_s) = 0(\,|z'_1|^{\beta'_1},\cdots,|z'_s|^{\beta'_s}\,), min(\,|z'_1|,\cdots,|z'_s|\,) \to \infty$$

where $k = 1,\cdots,z : \alpha''_k = min[Re(b'^{(k)}_j/\beta'^{(k)}_j)], j = 1,\cdots,m'^{(k)}$ and

$$\beta''_k = max[Re((a'^{(k)}_j - 1)/\alpha'^{(k)}_j)], j = 1,\cdots,n'^{(k)}$$

If $n_2 = \cdots = n_s = 0$, we shall **I** this function.

2 Rquired result

Lemma

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$\times F_D^{(k)}\left[\alpha,-\sigma_1,\cdots,-\sigma_k;\alpha+\beta;-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right] \tag{2.1}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i\in\mathbb{C},(i=1,\cdots,k)$; $min(Re(\alpha),Re(\beta))>0$ and

$$\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$$

$F_D^{(k)}$ is a Lauricella's function of k-variables, see Srivastava and Manocha ([8], page 60)

The formula (2.2) can be establish by expanding $\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.2}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the Lauricella function $F_D^{(k)}$ [4, page 454].

2. Eulerian integral

$$X=n^{(1)},m^{(1)},\cdots,n^{(r)},m^{(r)},n'^{(1)},m'^{(1)},\cdots,n'^{(s)\prime}m'^{(s)},;\underbrace{1,0;\cdots;1,0}_{l};\underbrace{1,0;\cdots;1,0}_{k} \tag{3.1}$$

$$Y=q^{(1)},p^{(1)};\cdots;q^{(r)},p^{(r)};q'^{(1)},p'^{(1)};\cdots;q'^{(s)},p'^{(s)};\underbrace{0,1;\cdots;0,1}_{l};\underbrace{0,1;\cdots;0,1}_{k} \tag{3.2}$$

$$\mathbf{Y}=q_2,p_2,\cdots,q_{r-1},p_{r-1}\ \ ;q_2',p_2',\cdots,q_{s-1}',p_{s-1}' \tag{3.3}$$

$$\mathbf{A}=(1-b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(1-b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};(1-b_{2k}';\beta_{2k}'^{(1)},\beta_{2k}'^{(2)})_{1,q_2'};\cdots;$$

$$(1-b_{(s-1)k}';\beta_{(s-1)k}'^{(1)},\beta_{(s-1)k}'^{(2)},\cdots,\beta_{(s-1)k}'^{(s-1)})_{1,q_{s-1}'};(1-b_{rk};\beta_{rk}'^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k})_{1,q_r},$$

$$(1-b_{sk}';\underbrace{0,\cdots,0}_{r}\beta_{sk}'^{(1)},\beta_{sk}'^{(2)},\cdots,\beta_{sk}'^{(r)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k})_{1,q_s},\quad(1-A_j;\underbrace{0,\cdots,0}_{r+s+k},\underbrace{1,\cdots,1}_{l})_{1,P}$$

$$A=(1-d_k^{(1)},\delta_k^{(1)})_{1,q^{(1)}};\cdots;(1-d_k^{(r)},\delta_k^{(r)})_{1,q^{(r)}}\ ;(1-d_k'^{(1)},\delta_k'^{(1)})_{1,q'^{(1)}};\cdots;(1-d_k'^{(s)},\delta_k'^{(s)})_{1,q'^{(s)}};$$

$$\underbrace{(0,1);\cdots;(0,1)}_{l};\underbrace{(0,1);\cdots;(0,1)}_{k} \tag{4.4}$$

$$A^*=(1-\alpha-\sum_{i=1}^{u}R_ia_i-\sum_{i=1}^{v}\eta_{G_i,g_i}a_i';\mu_1,\cdots,\mu_r,\mu_1',\cdots,\mu_s',h_1,\cdots,h_l,\underbrace{1,\cdots,1}_{k},\upsilon_1,\cdots,\upsilon_l),$$

$$(1-A_j;\underbrace{0,\cdots,0}_{r+s+k},\underbrace{1,\cdots,1}_{l})_{1,P},(1-\beta-\sum_{i=1}^{u}R_ib_i-\sum_{i=1}^{v}\eta_{G_i,g_i}b_i';\rho_1,\cdots,\rho_r,\rho_1',\cdots,\rho_s',\underbrace{0,\cdots,0}_{k},\tau_1,\cdots,\tau_l),$$

$$[1-A_j;\underbrace{0,\cdots,0}_{r},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{k},\underbrace{1,\cdots,1}_{l}]_{1,P}$$

$$[1+\sigma_j-\sum_{i=1}^{u}R_i\lambda_j^{\prime\prime(i)}-\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j^{\prime\prime\prime(i)};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j^{\prime(1)}\cdots,\lambda_j^{\prime(s)},\underset{\text{j}}{\underbrace{0\cdots,1,\cdots,0}_{k}},\zeta_j^{\prime},\cdots,\zeta_j^{(l)}]_{1,k} \tag{3.5}$$

$$B^*=(1-\alpha-\beta-\sum_{i=1}^{u}R_i(a_i+b_i)-\sum_{i=1}^{v}(a_i^{\prime}+b_i^{\prime})K_i;\mu_1+\rho_1,\cdots,\mu_r+\rho_r,\mu_1^{\prime}+\rho_1^{\prime},\cdots,\mu_r^{\prime}+\rho_r^{\prime},$$

$$\underbrace{1,\cdots,1}_{k},\upsilon_1+\tau_1,\cdots,\upsilon_l+\tau_l),[1-B_j;\underbrace{0,\cdots,0}_{r},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{k},\underbrace{1,\cdots,1}_{l}]_{1,Q},$$

$$[1+\sigma_j-\sum_{i=1}^{u}R_i\lambda_j^{\prime\prime(i)}-\sum_{i=1}^{v}\lambda_j^{\prime\prime\prime(i)}\eta_{G_i,g_i};\lambda_j^{(1)},\cdots,\lambda_j^{(r)},\lambda_j^{\prime(1)}\cdots,\lambda^{\prime(s)},\underbrace{0\cdots,0}_{k},\zeta_j^{\prime},\cdots,\zeta_j^{(l)}]_{1,k} \tag{3.6}$$

$$\mathbf{B}=(1-a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(1-a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};(1-b_{2k}^{\prime};\beta_{2k}^{\prime(1)},\beta_{2k}^{\prime(2)})_{1,q_2^{\prime}};$$

$$(1-a_{(s-1)k}^{\prime};\alpha_{(s-1)k}^{\prime(1)},\alpha_{(s-1)k}^{\prime(2)},\cdots,\alpha_{(s-1)k}^{\prime(s-1)})_{1,p_{s-1}^{\prime}};(1-a_{rk};\alpha_{rk}^{\prime\prime(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},\underbrace{0,\cdots,0}_{s},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k})_{1,p_r},$$

$$(1-a_{sk}^{\prime};\underbrace{0,\cdots,0}_{r}\alpha_{sk}^{\prime(1)},\alpha_{sk}^{\prime(2)},\cdots,\alpha_{sk}^{\prime(r)},\underbrace{0,\cdots,0}_{l},\underbrace{0,\cdots,0}_{k})_{1,p_s},(1-B_j;\underbrace{0,\cdots,0}_{r+s+k},\underbrace{1,\cdots,1}_{l})_{1,Q}$$

$$B=(1-c_k^{(1)},\gamma_k^{(1)})_{1,p^{(1)}};\cdots;(1-c_k^{(r)},\gamma_k^{(r)})_{1,p^{(r)}};\quad(1-c_k^{\prime(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(1-c_k^{\prime(s)},\alpha_k^{\prime(s)})_{1,p^{\prime(s)}};$$

$$\underbrace{(1,0);\cdots;(1,0)}_{l};\underbrace{(1.0);\cdots;(1.0)}_{k} \tag{3.7}$$

$$P_1=(b-a)^{\alpha+\beta-1}\left\{\prod_{j=1}^{h}(af_j+g_j)^{\sigma_j}\right\} \tag{3.8}$$

$$B_{u,v}=(b-a)^{\sum_{i=1}^{v}(a_i^{\prime}+b_i^{\prime})K_i+\sum_{i=1}^{u}(a_i+b_i)R_i}\left\{\prod_{j=1}^{h}(af_j+g_j)^{-\sum_{i=1}^{v}\lambda_i^{\prime\prime\prime}K_i-\sum_{i=1}^{u}\lambda_i^{\prime\prime}R_i}\right\} \tag{3.9}$$

We have the general Eulerian integral.

Theorem.

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}S_L^{h_1,\cdots,h_u}\left(\begin{array}{c}\mathrm{z}_1^{\prime\prime}(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{\prime\prime(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_u^{\prime\prime}(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{\prime\prime(u)}}\end{array}\right)$$

$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\left(\begin{array}{c} \mathrm{z}_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$${}_PF_Q\left[(A_p);(B_q);-\sum_{i=1}^{l}u_i(t-a)^{v_i}(b-t)^{\tau_i}\prod_{j=1}^{k}(f_jt+g_j)^{-\varsigma_j^{(i)}}\right]$$

$$\mathbf{I}\left(\begin{array}{c} \mathrm{z}_1(t-a)^{-\mu_1}(b-t)^{-\rho_1}\prod_{j=1}^k(f_jt+g_j)^{\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(t-a)^{-\mu_r}(b-t)^{-\rho_r}\prod_{j=1}^k(f_jt+g_j)^{\lambda_j^{(r)}} \end{array}\right)$$

$$\mathbf{I}\left(\begin{array}{c} \mathrm{z}_1'(t-a)^{-\mu_1'}(b-t)^{-\rho_1'}\prod_{j=1}^k(f_jt+g_j)^{\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s'(t-a)^{-\mu_s'}(b-t)^{-\rho_s'}\prod_{j=1}^k(f_jt+g_j)^{\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$=P_1\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{k=1}^{u}z''^{R_k}\prod_{l=1}^{v}z'''^{K_l}\ a_vb_uB_{u,v}$$

$$I^{0,l+k+2:X}_{\mathbf{Y};q_r+q_s'+l+k+2,p_r+p_s'+l+k+1:Y}\left(\begin{array}{c|c} \frac{(b-a)^{\mu_1+\rho_1}}{z_1\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}^*,\mathbf{A},A \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)^{\mu_r+\rho_r}}{z_r\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{(b-a)^{\mu_1'+\rho_1'}}{z_1'\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)^{\mu_s'+\rho_s'}}{z_s'\prod_{j=1}^k(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)f_k}{af_k+g_k} & \cdot \\ \frac{u_1(b-a)^{\tau_1+v_1}}{\prod_{j=1}^k(af_j+g_j)^{\varsigma_j'}} & \cdot \\ \cdots & \mathbf{B},\mathrm{B}^*,\mathrm{B} \\ \cdots & \\ \frac{u_l(b-a)^{\tau_l+v_l}}{\prod_{j=}^k(af_j+g_j)^{\varsigma_j^{(l)}}} & \end{array}\right) \quad (3.10)$$

We obtain the I-function of $r+s+k+l$ variables.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu'_u,\rho_i,\rho'_u,\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+$, $f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}$ $(i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$
$a'_i,b'_i,\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $a_{ij},b_{ik},\in\mathbb{C}\,(i=1,\cdots,r;j=1,\cdots,p_i;k=1,\cdots,q_i);a_j^{(i)},b_j^{(k)}\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p^{(i)};k=1,\cdots,q^{(i)})$

$a'_{ij},b'_{ik},\in\mathbb{C}\,(i=1,\cdots,s;j=1,\cdots,p'_i;k=1,\cdots,q'_i);a_j'^{(i)},b_j'^{(k)},\in\mathbb{C}$

$(i=1,\cdots,r;j=1,\cdots,p'^{i)};k=1,\cdots,q'^{(i)})$

$\alpha_{ij}^{(k)},\beta_{ij}^{(k)}\in\mathbb{R}^+\,((i=1,\cdots,r,j=1,\cdots,p_i,k=1,\cdots,r)\,;\alpha_j^{(i)},\beta_i^{(i)}\in\mathbb{R}^+\,(i=1,\cdots,r;j=1,\cdots,p_i)$

$\alpha_{ij}'^{(k)},\beta_{ij}'^{(k)}\in\mathbb{R}^+\,((i=1,\cdots,s,j=1,\cdots,p'_i,k=1,\cdots,s)\,;\alpha_j'^{(i)},\beta_i'^{(i)}\in\mathbb{R}^+\,(i=1,\cdots,s;j=1,\cdots,p'_i)$

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $$Re\left(\alpha+\sum_{i=1}^{v}K_ia_i+\sum_{i=1}^{u}R_ia'_i\right)-\sum_{t=1}^{r}\mu_t\max_{1\leqslant k\leqslant n^{(i)}}Re\left(\frac{a_k^{(t)}-1}{\alpha_k^{(t)}}\right)-\sum_{t=1}^{s}\mu'_t\max_{1\leqslant k\leqslant n'^{(i)}}Re\left(\frac{a_k'^{(t)}-1}{\alpha_k'^{(t)}}\right)>0$$

and $$Re\left(\beta+\sum_{i=1}^{v}K_ib_i+\sum_{i=1}^{u}R_ib'_i\right)-\sum_{t=1}^{r}\rho_t\max_{1\leqslant k\leqslant n^{(i)}}Re\left(\frac{a_k^{(t)}-1}{\alpha_k^{(t)}}\right)-\sum_{t=1}^{s}\rho'_t\max_{1\leqslant k\leqslant n'^{(i)}}Re\left(\frac{a_k'^{(t)}-1}{\alpha_k'^{(t)}}\right)>0$$

(E) $$Re\left(\alpha+\sum_{i=1}^{v}K_ia'_i+\sum_{i=1}^{u}R_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$$

$$Re\left(\beta+\sum_{i=1}^{v}K_ib'_i+\sum_{i=1}^{u}R_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}K_i\lambda_j'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}K_i\lambda'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}-\sum_{k=1}^{p_2}\alpha_{2k}^{(i)}-\cdots-$$

$$\sum_{k=1}^{p_s}\alpha_{sk}^{(i)}-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right)-\mu_i-\rho_i-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\quad(i=1,\cdots,r)$$

$$\Omega_i' = \sum_{k=1}^{n'^{(i)}} \alpha_k'^{(i)} - \sum_{k=n'^{(i)}+1}^{p'^{(i)}} \alpha_k'^{(i)} + \sum_{k=1}^{m'^{(i)}} \beta_k'^{(i)} - \sum_{k=m^{(i)}+1}^{q'^{(i)}} \beta_k'^{(i)} - \sum_{k=1}^{p_2'} \alpha_{2k}'^{(i)} -$$

$$\cdots - \sum_{k=1}^{p_s'} \alpha_{sk}'^{(i)} - \left(\sum_{k=1}^{q_2'} \beta_{2k}'^{(i)} + \sum_{k=1}^{q_3'} \beta_{3k}'^{(i)} + \cdots + \sum_{k=1}^{q_s'} \beta_{sk}'^{(i)} \right) - \mu_i' - \rho_i' - \sum_{l=1}^{k} \lambda_j'^{(i)} - \sum_{l=1}^{l} \zeta_j'^{(i)} > 0 \quad (i = 1, \cdots, s)$$

(G) $\left| arg \left(z_i \prod_{j=1}^{k} (f_j t + g_j) \right)^{-\lambda_j^{(i)}} \right| < \frac{1}{2} \Omega_k \pi, \left| arg \left(z_i \prod_{j=1}^{k} (f_j t + g_j) \right)^{-\lambda_j'^{(i)}} \right| < \frac{1}{2} \Omega_k' \pi \; ,(a \leqslant t \leqslant b; i = 1, \cdots, r)$

(G) $P \leqslant Q + 1$. The equality holds, when , in addition,

either $P > Q$ and $\left| z_i \left(\prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right|^{\frac{1}{Q-P}} < 1 \qquad (a \leqslant t \leqslant b)$

or $P \leqslant Q$ and $\max_{1 \leqslant i \leqslant k} \left[\left| \left(z_i' \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| \right] < 1 \qquad (a \leqslant t \leqslant b)$

Proof

To prove (3.14), first, we express in serie the class of multivariable polynomials $S_{N_1, \cdots, N_v}^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v}[.]$ with the help of (1.1), the class of multivariable polynomials $S_L^{h_1, \cdots, h_u}[.]$ in serie with the help of (1.2) and we interchange the order of summations and t-integral (which is permissible under the conditions stated). Expressing the I-functions of r-variables and s-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.9) and (1.12) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process . Now collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Changing the order of integrations and summations (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) , using the lemma and interpreting $(r + s + k + l)-$Mellin-barnes integrals contour in modified multivariable -function I defined by Prasad. [1], we obtain the desired result (3.10).

4. Srivastava-Daoust polynomial

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta_j' + \cdots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b_j')_{R_1 \phi_j'} \cdots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{R_1 \psi_j' + \cdots + R_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d_j')_{R_1 \delta_j'} \cdots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}}$ **(4.1)**

then the general class of multivariable polynomial $S_L^{h_1, \cdots, h_u}[z_1, \cdots, z_u]$ reduces to generalized Lauricella function defined by Srivastava and Daoust [5], it's a particular case of generalized hypergeometric function defined by Srivastava and Panda [9,10].

$$F_{\bar{C}:D^{(1)}; \cdots; D^{(u)}}^{1+\bar{A}:B^{(1)}; \cdots; B^{(u)}} \left(\begin{array}{c|c} \mathrm{z}_1 & (\text{-L:R}_1, \cdots, R_u), [(a); \theta', \cdots, \theta^{(u)}] : [(b'); \phi']; \cdots; [(b^{()}); \phi^{(u)}] \\ \cdot & \cdot \\ \cdot & [(\mathrm{c}); \psi', \cdots, \psi^{(u)}] : [(d'); \delta']; \cdots; [(d^{(u)}); \delta^{(u)}] \\ \mathrm{z}_u & \end{array} \right) \quad \mathbf{(4.2)}$$

We have the following integral.

Corollary.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F^{1+\bar{A}:B';\cdots;B^{(u)}}_{\bar{C}:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{z}''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{array}\right)$$

$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\left(\begin{array}{c} \mathrm{z}'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(v)}_j} \end{array}\right)$$

$$\mathbf{I}\left(\begin{array}{c} \mathrm{z}_1(t-a)^{-\mu_1}(b-t)^{-\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{\lambda^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(t-a)^{-\mu_r}(b-t)^{-\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{\lambda^{(r)}_j} \end{array}\right)$$

$$\mathbf{I}\left(\begin{array}{c} \mathrm{z}'_1(t-a)^{-\mu'_1}(b-t)^{-\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{\lambda'^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s(t-a)^{-\mu'_s}(b-t)^{-\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{\lambda'^{(s)}_j} \end{array}\right)\mathrm{d}t$$

$$=P_1\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{k=1}^{u}z''^{R_k}\prod_{l=1}^{v}z'''^{K_l}\;a_vb_uB_{u,v}$$

$$I_{\mathbf{Y};q_r+q'_s+l+k+2,p_r+p'_s+l+k+1:Y}^{0,l+k+2:X}\left(\begin{array}{c|c} \frac{(b-a)^{\mu_1+\rho_1}}{z_1\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}^*,\mathbf{A},A \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)^{\mu_r+\rho_r}}{z_r\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{(b-a)^{\mu'_1+\rho'_1}}{z'_1\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)^{\mu'_s+\rho'_s}}{z'_s\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{(b-a)f_k}{af_k+g_k} & \cdot \\ \frac{u_1(b-a)^{\tau_1+\upsilon_1}}{\prod_{j=1}^k(af_j+g_j)^{\zeta'_j}} & \cdot \\ \cdots & \mathbf{B},\mathrm{B}^*,\mathrm{B} \\ \cdots & \\ \frac{u_l(b-a)^{\tau_l+\upsilon_l}}{\prod_{j=}^k(af_j+g_j)^{\zeta_j^{(l)}}} & \end{array}\right) \quad \textbf{(4.3)}$$

under the same conditions and notations that (3.10)

where $b_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(L;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B[L;R_1,\ldots,R_v]$ is defined by (4.1)

Remarks:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [1], classes of multivariable polynomials defined by Srivastava and Garg [5] and class of multivariable polynomials defined by Srivastava [4].
We have similar integrals concerning other multivariables special functions.

5. Conclusion.

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Prasad [1], a class of multivariable polynomials defined by Srivastava and Garg [5] and a class of multivariable polynomials defined by Srivastava [3] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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