Intuitionistic L-Fuzzy Bi-ideals of a Ring

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Abstract

In this paper the notion of intuitionistic L-fuzzy bi-ideal of a ring are defined and discussed. Some of their Properties are studied.

Keywords: *Intuitionistic fuzzy set, Intuitionistic L-fuzzy subring, Intuitionistic fuzzy bi-ideal, Intuitionistic L-fuzzy bi-ideal.*

I. INTRODUCTION

Goguen in [4] generalized the zadeh [5] fuzzy subset of X, to L-fuzzy subset, as a mapping from x to a complete lattice L-Fuzzy ideals of rings were introduced by W. Liu [6] and it has been studied by several authors. In [3] N.Kuroki gave some Properties of fuzzy ideals and fuzzy bi-ideals in semigroups. The idea of intuitionistic fuzzy set was first published by Atanassov [1,2] ,as a generalization of the notion of fuzzy set. A detail work about bi-ideals and fuzzy bi-ideals in a ring can be found in [7]. Later P.K.Sharma and Aradhna Duggal define and study the notion of intuitionistic fuzzy Bi-ideal of a ring. Here in this paper, we introduce the notion of intuitionistic L-fuzzy bi-ideal in a ring and study some of their properties.

II. PRELIMINARIES

Definition: 2.1

Let R be a ring. An intuitionistic fuzzy set (IFS) A= $\{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ of R is said to be intuitionistic L-fuzzy subring of R (ILFSR) of R if

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
(iii) $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$
(iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ for all $x, y \in R$.

Definition: 2.2

An IFS A= $\{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ of a ring R said to be

(a) Intuitionistic L-fuzzy left ideal of R(ILFLI) of R if

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(y)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(y) \text{for all } x, y \in R.$$

(b) Intuitionistic L-fuzzy Right ideal of R (ILFRI) of R if

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(x)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(x) \text{for all } x, y \in R.$$

(c) Intuitionistic L-fuzzy ideal of R(ILFI) 0f R if,

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y) \text{for all } x, y \in R.$$

Definition: 2.3

Let A be Intuitionistic L-fuzzy set of a ring R. Then (α, β)-cut of A is a crisp subset $C_{\alpha,\beta}(A)$of the IFS A is given by $C_{\alpha,\beta}(A) = \{x \in R : \mu_A(x) \geq \alpha, \ \nu_A(x) \leq \beta\}$ where $\alpha, \beta \in [0,1]$

With $\alpha + \beta \leq 1$.

Definition: 2.4

A non-empty fuzzy subset μ of a ring R (ie, $\mu(x) \neq 0$ for some $x \in R$ is called an L-fuzzy bi-ideal of R if

(i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$

(ii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$

(iii) $\mu(xyz) \geq \mu(x) \wedge \mu(z) \ \forall \ x, y, z \in R$

Example: 2.5

Consider the fuzzy set μ of R by

$$\mu_A(x) = \begin{cases} 0.7 & if \ x \ is \ rational \\ 0.3 & if \ x \ is \ irrational \end{cases}$$

Then μ is a L-fuzzy bi-ideal of R.

III. INTUITIONISTIC L-FUZZY BI-IDEALS OF A RING

Definition: 3.1

An Intuitionistic Fuzzy set (IFS) A= $\{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ of a ring R is said to be Intuitionisitic L-fuzzy bi-ideal of R(ILFBI) of R if

(i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$

(iii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$

(iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$

(v) $\mu_A(xry) \geq \mu_A(x) \wedge \mu_A(y)$

(vi) $\nu_A(xry) \leq \nu_A(x) \vee \nu_A(y)$

Example: 3.2

Consider the ring R of real numbers under usual addition and multiplication operations.Define IFS A=(μ_A, ν_A) of R by,

$$\mu_A(x) = \begin{cases} 0.6 & \text{if x is rational} \\ 0.3 & \text{if x is irrational} \end{cases}$$

And

$$\nu_A(x) = \begin{cases} 0.2 & \text{if x is rational} \\ 0.7 & \text{if x is irrational} \end{cases}$$ A is a Intuitionistic L-fuzzy bi-ideal of R.

Theorem: 3.3

Let A= (μ_A, ν_A) be (ILFBI) of a field F. Then A is of the form,

$$\mu_A(x) = \begin{cases} \mu_A(1) & ; \text{if } x \neq 0 \\ \mu_A(0) & ; \text{if } x = 0 \end{cases} \text{and}$$

$$\nu_A(x) = \begin{cases} \nu_A(1) & ; \text{if } x \neq 0 \\ \nu_A(0) & ; \text{if } x = 0 \end{cases}$$

Where $\mu_A(1) \leq \mu_A(0)$and $\nu_A(1) \geq \nu_A(0)$.

Proof:

Let A= $\{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ be an ILFBI of a field F. Let $0 \neq x \in F$ be any element

Then, $\mu_A(x) = \mu_A(1.x.1) \geq \mu_A(1) \wedge \mu_A(1)$

$= \mu_A(1) = \mu_A(1.1)$

$= \mu_A[(x \cdot x^{-1})(x^{-1} \cdot x)]$

$= \mu_A[x(x^{-1}x^{-1})x]$

$\geq \mu_A(x) \wedge \mu_A(x)$

$= \mu_A(x)$.

ie,$\mu_A(x) = \mu_A(1)$. And

$\nu_A(x) = \nu_A(1.x.1) \leq \nu_A(1) \vee \nu_A(1)$

$= \nu_A(1)$
$= \nu_A(1.1) = \nu_A[(xx^{-1})(x^{-1}x)]$
$= \nu_A[x(x^{-1}x^{-1})x]$
$\leq \nu_A(x) \vee \nu_A(x)$
$= \nu_A(x).$

ie, $\nu_A(x) = \nu_A(1)$

Corollary: 3.4

If A is ILFBI of a field F such that $\mu_A(0) = \mu_A(1)$ and $\nu_A(0) = \nu_A(1)$ then A is Constant.

Theorem: 3.5

If A and B be two ILFBI'S of a ring R, then $A \cap B$ is ILFBI of ring R.

Proof:

Let A= (μ_A, ν_A) and B= (μ_B, ν_B) be two ILFBI's of a ring R.
Let $x, y \in A \cap B$ be any element. Then,

$\mu_{A\cap B}(x-y) = \mu_A(x-y) \wedge \mu_B(x-y)$
$\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]$
$= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}(x-y) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

Now, $\nu_{A\cap B}(x-y) \leq \nu_A(x-y) \vee \nu_B(x-y)$
$\leq [\nu_A(x) \vee \nu_A(y)] \vee [\nu_B(x) \vee \nu_B(y)]$
$\leq [\nu_A(x) \vee \nu_B(x)] \vee [\nu_A(y) \vee \nu_B(y)]$
$\leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$

Also,

$\mu_{A\cap B}(xy) = \mu_A(xy) \wedge \mu_B(xy)$
$\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]$
$= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Similarly We can show that,

$$\nu_{A\cap B}(xy) \leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$$

Next, let $x, y \in A \cap B$ and $r \in R$ be any element, then

$\mu_{A\cap B}(xry) = \mu_A(xry) \wedge \mu_B(xry)$
$\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]$
$= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Similarly We can show that ,
$\nu_{A\cap B}(xry) \leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$.
Hence $A \cap B$ is ILFBI of ring R.

Theorem: 3.6

Let A be a ILFLI and B be ILFRI of a ring R,then $A \cap B$ is ILFBI of ring R.

Proof:

$\mu_{A\cap B}(x-y) \geq \mu_A(x-y) \wedge \mu_B(x-y)$
$\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]$
$= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}(x-y) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

Similarly We can show that,

$$\nu_{A\cap B}(x-y) \leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$$

Also;

$$\mu_{A\cap B}(xy) \geq \mu_A(xy) \wedge \mu_B(xy) \longrightarrow (1)$$

Since A is ILFBI and B is ILFRI of the ring R.

Therefore We have

$$\mu_A(xy) \geq \mu_A(y) \text{ and } \mu_B(xy) \geq \mu_B(x)$$

$$\mu_A(xy) \wedge \mu_B(xy) \geq \mu_A(y) \wedge \mu_B(x) \longrightarrow (2)$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$ so ,

$$\mu_{A\cap B}(y) \leq \mu_A(y) \quad \mu_{A\cap B}(x) \leq \mu_B(x)$$

$$\mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y) \leq \mu_A(y) \wedge \mu_B(x) \longrightarrow (3)$$

Therefore from (1),(2) and (3) we have

$$\begin{aligned}\mu_{A\cap B}(xy) &\geq \mu_A(xy) \wedge \mu_B(xy)\\ &\geq \mu_A(y) \wedge \mu_B(x)\\ &\geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y).\end{aligned}$$

Thus,

$\mu_{A\cap B}(xy) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

Similarly we can show that

$$\nu_{A\cap B}(xy) \leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$$

Further let $x, y \in A \cap B$ and $r \in R$ then $\mu_{A\cap B}(xry) \geq \mu_A(xry) \wedge \mu_B(xry) \longrightarrow (4)$

But $\mu_A(xry) \geq \mu_A((xr)y) \geq \mu_A(y)$

$\mu_B(xry) \geq \mu_B\big(x(ry)\big) \geq \mu_B(x)$.

$\mu_A(xry) \wedge \mu_B(xry) \geq \mu_A(y) \wedge \mu_B(x) \longrightarrow (5)$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$ so

$\mu_{A\cap B}(y) \leq \mu_A(y)$ & $\mu_{A\cap B}(x) \leq \mu_B(x)$

Therorem: 3.7

If A= $\{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ be ILFBI of ring R, then

(i) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$

(ii) $\mu_A(-x) = \mu_A(x)$ and $\nu_A(-x) = \nu_A(x)$

(iii) $\mu_A(1) \geq \mu_A(x)$ and $\nu_A(1) \leq \nu_A(x)$

Proof:

$$\begin{aligned}\mu_A(0)=\mu_A(x-x) &\geq \mu_A(x) \wedge \mu_A(x)\\ &= \mu_A(x)\end{aligned}$$

$$\mu_A(0) \geq \mu_A(x)$$

$$\begin{aligned}\mu_A(-x) = \mu_A(0-x) &\geq \mu_A(0) \wedge \mu_A(x)\\ &=\mu_A(x)\end{aligned}$$

$$\begin{aligned}\mu_A(1) = \mu_A(xx^{-1}) &\geq \mu_A(x) \wedge \mu_A(x^{-1})\\ &\geq \mu_A(x)\end{aligned}$$

$$\mu_A(1) \geq \mu_A(x)$$

$$\begin{aligned}\nu_A(0) \leq \nu_A(x-x) &\leq \nu_A(x) \vee \nu_A(x)\\ &\leq \nu_A(x)\end{aligned}$$

$$\begin{aligned}\nu_A(-x) \leq \nu_A(0-x) &\leq \nu_A(0) \vee \nu_A(x)\\ &= \nu_A(x)\end{aligned}$$

$$\begin{aligned}\nu_A(1) = \nu_A(xx^{-1}) &\leq \nu_A(x) \vee \nu_A(x^{-1})\\ &\leq \nu_A(x).\end{aligned}$$

Theorem: 3.8

If A is an intuitionistic L-fuzzy bi-ideal of R, then $\mu_A(x-y) = \mu_A(0)$ and $\nu_A(x-y) = \nu_A(0)$ gives $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$ for x, y in R the identity 0 in R.

Proof:

$$\begin{aligned}\mu_A(x) = \mu_A(x-y+y) &\geq \mu_A[x-y-(-y)]\\ &\geq \mu_A(x-y) \wedge \mu_A(-y)\\ &\geq \mu_A(x-y) \wedge \mu_A(y)\\ &= \mu_A(0) \wedge \mu_A(y)\\ &= \mu_A(y)\end{aligned}$$

$$\nu_A(x) = \nu_A[x-y+y] = \nu_A[x-y-(-y)]$$

$\leq \nu_A(x-y) \vee \nu_A(-y)$
$\leq \nu_A(x-y) \vee \nu_A(y)$
$= \nu_A(0) \vee \nu_A(y)$
$= \nu_A(y)$

Theorem: 3.9

If A is an intuitionistic L-fuzzy bi-ideal of a ring R $H = \{x \backslash x \in R: \mu_A(x) = 1, \nu_A(x) = 1\}$is either emptyor is an intuitionistic L-fuzzy bi-ideal of R.

Operators:

The first two modal operators over IFSA $= \{< x, \mu_A(x), \nu_A(x) >: x \in R\}$ have the form

$\square A = \{< x, \mu_A(x), 1-\mu_A(x) > / \, x \in X\}$,

$\lozenge A = \{< x, \nu_A(x), 1-\nu_A(x) > / x \in X\}$.

Theorem: 3.10

If an IFS $A = (\mu_A, \nu_A)$ in X is an intuitionistic L-fuzzy bi-ideal of X, then so is

$\square A := (\mu_A, \overline{\mu_A})$, $\lozenge A := (\nu_A, \overline{\nu_A})$.

Proof:

It is sufficient to show that,$\overline{\mu_A}$ satisfies the condition

$\overline{\mu_A}(xy) = 1 - \mu_A(xy)$
$\geq 1 - [\mu_A(x) \wedge \mu_A(y)]$
$=(1-\mu_A(x)) \wedge (1-\mu_A(y))$
$=\overline{\mu_A}(x) \wedge \overline{\mu_A}(y)$

$\overline{\mu_A}(x-y) = 1 - \mu_A(x-y)$
$\geq 1 - [\mu_A(x) \wedge \mu_A(y)]$
$=(1-\mu_A(x)) \wedge (1-\mu_A(y))$
$\geq \overline{\mu_A}(x) \wedge \overline{\mu_A}(y)$

$\overline{\mu_A}(xyz) \geq 1 - \mu_A(xyz)$
$\geq 1 - [\mu_A(x) \wedge \mu_A(z)]$
$=(1-\mu_A(x)) \wedge (1-\mu_A(z))$
$\geq \overline{\mu_A}(x) \wedge \overline{\mu_A}(z)$

Also,

$\overline{\nu_A}(xy)=1-\nu_A(xy)$
$\leq 1 - [\nu_A(x) \vee \nu_A(y)]$
$\leq (1-\nu_A(x)) \vee (1-\nu_A(y))$
$\leq \overline{\nu_A}(x) \vee \overline{\nu_A}(y)$

$\overline{\nu_A}(x-y) = 1 - \nu_A(x-y)$
$\leq 1 - [\nu_A(x) \vee \nu_A(y)]$
$\leq (1-\nu_A(x)) \vee (1-\nu_A(y))$
$\leq \overline{\nu_A}(x) \vee \overline{\nu_A}(y)$

$\overline{\nu_A}(xyz) = 1 - \nu_A(xyz)$
$\leq 1 - [\nu_A(x) \vee \nu_A(z)]$
$\leq (1-\nu_A(x)) \vee (1-\nu_A(z))$
$\leq \overline{\nu_A}(x) \vee \overline{\nu_A}(z)$

IV. CONCLUSION

In this paper, concept of an intuitionistic L-fuzzy bi-ideal has been introduced. Using these, various results can be developed under the topic an intuitionistic L-fuzzy bi-ideals.

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