Neutrosophic Fuzzy MAGDM using Maximal Entropy OWA Weights

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Abstract

The attribute weights calculated from completely unknown information using the Maximal Entropy OWA weights. The neutrosophic fuzzy ordered weighted averaging (NFOWA) operator is utilized to aggregate all individual neutrosophic fuzzy decision matrices provided by the decision-makers into the collective neutrosophic fuzzy decision matrix, and then we use the obtained attribute weights and the neutrosophic fuzzy hybrid averaging (NFHA) operator to fuse the neutrosophic fuzzy information in the collective neutrosophic fuzzy decision matrix to get the overall neutrosophic fuzzy values of alternatives, and then rank the alternatives, and select the most desirable alternative.

Keywords:*MAGDM; Ordered Weighted Averaging; Hybrid Averaging Operator; Correlation of neutrosophic fuzzy sets; Entropy weights.*

I. INTRODUCTION

Atanassov [1986, 1989] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set. He also [1994] discussed decision making operators in interval valued intuitionistic fuzzy sets. Yager [1988] developed the ordered weighted averaging (OWA) operator and applied problems with alternatives and attributes in fuzzy decision making problems. Herrera et al., [1999] developed an aggregation process for combining calculating for interval valued and linguistic information, and then proposed different extensions of this process to deal with contexts in which can appear other type of information such as IFSs or multi-granular linguistic information.

Szmidt&Kacprzyk [2002] proposed few solution for a problem with operators and variables in terms of intuitionistic (individual and social) fuzzy preference relations in group decision making. They [2003]analyzed the consensus-reaching process in group decision making based on individual intuitionistic fuzzy preference relations. Xu &Yager [2006] developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave anusage for the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information.

Xu [2007] and Xu & Chen [2007] also developed some arithmetic aggregation operators, like as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid averaging (IFHA) operator. The interval-valued intuitionistic fuzzy sets (IVIFSs), introduced by Atanassovand Gargov [1989], which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers, are a very useful means to describe the decision information in the process of decision making.

Wei and Wang [2007], respectively, developed few geometric aggregation operators, such as the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator and interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and applied them to MAGDM with interval-valued intuitionistic fuzzy information. However, they used the IIFWG operator, the IIFOWG operator and the IIFHG operator in the situation where the information about attribute weights is completely known.

Based on the theory of ordered weighted averaging (OWA) operators suggested by O'Hagan [1988], a special class of OWA operators having maximal entropy of the OWA weights for a given level of *orness* is utilized. Using the method of Lagrange multipliers, Fuller &Majlender [2001] solved the constrained optimization problem of OWA operators having maximal entropy analytically and derived a polynomial equation which is then solved to determine the optimal weighting vector. They also investigated MAGDM problems in which all the information provided by the decision-makers is presented as interval valued intuitionistic fuzzy decision matrices where each of its elements is characterized by interval valued intuitionistic fuzzy number (IVIFN).

Park et al. [2009] proposed an ordered weighted geometric (OWG) model to aggregate all individual interval valued intuitionistic fuzzy decision matrices provided by the decision makers into the collective interval valued intuitionistic fuzzy decision matrix. In the proposed model, from the maximal entropy attribute weight information, an optimization model is established to determine the unknown weights. Then the obtained attribute weights and the interval valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operators are used to fuse the interval valued intuitionistic fuzzy information in the collective interval valued intuitionistic fuzzy decision matrix to get the overall interval valued intuitionistic fuzzy values of the alternatives. Correlation coefficient is used as a tool to rank alternatives since it preserves the linear relationship between the variables.

Robinson &Amirtharaj, (2011a; 2011b; 2012a; 2012b; 2013) defined correlation coefficient for interval vague sets and triangular and trapezoidal intuitionistic fuzzy sets and proposed different MAGDM algorithms. Solairajuet. al. [2013, 2014] have worked on the decision making problems with vague sets. Wei et al. [2011] and Park et al. [2009] have also adopted correlation coefficient as a ranking tool for deciding the best alternatives. In this paper, the correlation coefficient proposed by Park et al. [2009] for IVIFSs is utilized for ranking the alternatives.

Amirtharaj& Robinson [2013] proposed MAGDM models for IVIFSs with a novel method of attributes weight determination especially when the weights are completely unknown. Correlation coefficient of the overall Interval Valued Intuitionistic Fuzzy values and the ideal interval valued intuitionistic fuzzy numbers (IVIFN) value is calculated and the ranking of the most desirable alternatives is done based on the obtained correlation coefficients. A MAGDM model based on the maximal entropy weights [Fuller &Majlender, 2001] is presented for computing the attributes weights, and a numerical illustration is given.

II. CORRELATION COEFFICIENT OF NEUTROSOPHIC FUZZY SETS

Definition 2.1: A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, an indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as A = { < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > : x $\in$X }, where T_A , I_A ,F_A : X→ [0. 1] and $0 \le T_A(x) \le 1$; $0 \le I_A(x) \le 1$; $0 \le F_A(x) \le 1$, for all x $\in$ X.

For each IFS *A*in *X* ,$\pi_A(x) = 3 - T_A(x) - I_A(x) - F_A(x)$ is called as the neutrosophic index of *x* in *A*. It is otherwise called as the hesitation degree of *x* to *A*. It is obvious that $0 \le \pi_A(x) \le 1$ for each x in X. For neutrosophic fuzzy sets A, B, define A $\subseteq$ B if $T_A(x) \le T_B(x)$; $I_A(x) \le I_B(x)$; $F_A(x) \le F_B(x)$ for all x in X.

Let $X = \{x_1, x_2,\ldots,x_n\}$ be a finite universal set and A, B in NFS(X). Now, the method of calculating the covariance and the correlation coefficient between two NFSs is utilized proposed by us. For each A in NFS (X), the informational neutrosophic energy of A is defined as follows:

$E_{NFS}(A) = \sum_{i=1}^{n} \frac{{T_A}^2(x) + {I_A}^2(x) + {F_A}^2(x)}{3}$.

The function *E* satisfies the following conditions:

(1). $E_{NFS}(A) = E_{NFS}(A^c)$ for any A in NFS(X). (2). $E_{NFS}(A) \le n$ for any A in NFS(X).

The covariance of *A* and *B* is defined by the formula:

$C_{NFS}(A, B) = \frac{1}{3}\sum_{i=1}^{n}[\, T_A(x)T_B(x) + T_A(x)T_B(x) + T_A(x)T_B(x)]$.

Furthermore, the correlation coefficient of A and B is defined by the formula:

$R_{NFS}(A, B) = \frac{C_{NFS}(A, B)}{\sqrt{E_{NFS}(A,).E_{NFS}(B)}}$

Theorem 1: For each A,B in NFS (X), the correlation coefficient satisfies:

(a). $R_{NFS}(A, B) = R_{NFS}(B, A)$; (b).$0 \le R_{NFS}(A, B) \le 1$. (c). A = B iff $R_{NFS}(A, B) = 1$.

III. MAXIMAL ENTROPY OWA OPERATOR WEIGHTS

Yager, (1988) introduced two characterizing measures associated with the weighting vector *W* of an OWA operator. The first one, the measure of *orness* of the aggregation, is given as *orness*(*W*) = 1/(n-1) $\sum_{i=1}^{n}(n-i)w_i$ and it characterizes the degree to which the aggregation is like an *or* operation. It is clear that *orness*(*W*)$\in [0,1]$ holds for any weighting vector. The second one, the measure of dispersion of the aggregation, is defined as:*dis p*(*W*) = (-)$\sum_{i=1}^{n} w_i \, In\, w_i$, and it measures the degree to which *W* takes into account all information in the aggregation.

Another approach, suggested by O'Hagan, (1988) determines a special class of OWA operators having maximal entropy of the OWA weights for the given level of *orness*. This approach is based on the solution of the following mathematical programming problem:Maximize (-)$\sum_{i=1}^{n} w_i \, In\, w_i$ subject to 1/(n-1) $\sum_{i=1}^{n}(n-i)w_i$ = α, 0

≤α≤ 1, and $\sum_{i=1}^{n} w_i = 1$, $0 \le w_i \le 1$ for i varies from 1 to n… (1).

Using the method of Langrangemultipliers Fuller &Majlender(2001) transferred problem (1) to a polynomial equation which is then solved to determine the optimal weighting vector.

First it can be noted that *disp*(*W*) is meaningful if $w_i > 0$, and by letting w_i In (w_i) to zero if $w_i = 0$ problem (1) turns into *disp*(*W*) → max; subject to {*orness* (*W*) = $c_1w_1 + c_2w_2 + \ldots + c_nw_n : 0 \le c_i \le 1$ for all i }.

If *n* = 2, then from *orness*{w_1 , w_2} = c gives that w_1 = c and w_2 = 1 – c. Furthermore, if c =0 or c = 1, then the associated weighting vectors are uniquely defined as $(0, 0, \ldots, 1)^T$ respectively, with value of dispersion zero.

Suppose now that n≥ 3, and 0 ≤ c ≤ 1. ThenL (W, λ_1, λ_2) = (-) $\sum_{i=1}^{n} w_i\,(In\,w_i)$ + $\lambda_1\left(\sum_{i=1}^{n} \frac{n-i}{n-1} w_i - \alpha\right)$ + $\lambda_2(\sum_{i=1}^{n} w_i - 1)$denote the Lagrange function of constrained optimization problem (4), where λ_1 and λ_2 are real numbers. Then the partial derivatives of *L* are computed as

$\frac{\partial L}{\partial w_j}$ = (-) In w_j – 1 + λ_1 + [(n - j) / (n - 1)]λ_2 = 0 for all j.; $\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{n} w_i - 1 = 0$ …(2);

$\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i - \alpha = 0$.

For *j*=*n*, the equation (2) turns into:(-) Inw_n – 1 + λ_1 = 0iff In w_n + 1 = λ_1

and for *j*=1, (2) implies that (-) In w_1 – 1 + λ_1 + λ_2 = 0

impliesλ_2 = In w_1 + 1 - λ_1= In w_1 + 1- In w_2 -1 = In w_1 - In w_2

For 1 < j < n, it finds that In w_j = [(j-1) / (n-1)] in w_n + (n-j) / (n-1)] In $w_1$gives that implies that $w_i = \sqrt[(n-1)]{w_1^{(n-j)} w_n^{(j-1)}}$ …(3).

If $w_1 = w_n$, then (3) gives $w_1 = w_2 = \ldots = w_n$ = (1/ n) gives that disp (W) = In (n) which is the optimal solution to (1) for α = 0.5 (actually, this is the global optimal value for the dispersion of all OWA operators of dimension *n*). Suppose now that $w_1 \neq w_n$.

Let$u_1 = w_1^{1/(n-1)}$; $u_n = w_n^{1/(n-1)}$; Then it may rewrite (3) as $w_j = u_1^{(n-j)} u_n^{(j-1)}$ for all j with 1 ≤ j ≤ n. From the first condition, *orness*(*W*) = α,

find$\sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha$ iff $\sum_{i=1}^{n} (n-i)\, u_1^{(n-j)} u_n^{(j-1)}$ = (n-1)α.

And from $\sum_{i=1}^{n} (n-i)\, u_1^{(n-j)} u_n^{(j-1)} = \frac{1}{u_1 - u_n}\left[(n-1)u_1^{\,n} - \sum_{i=1}^{n-1} u_1^{\,i} u_n^{\,n-i}\right]$

$$= \frac{1}{u_1 - u_n}\left[(n-1)u_1^{\,n} - u_1 u_n \frac{u_1^{\,n-1} - u_n^{\,n-1}}{u_1 - u_1}\right]$$

Then (n-1) $u_1^{n+1} - nu_1^n u_n + u_1 u_n^n = (n-1)\,\alpha\,(u_1 - u_n)^2$;

$nu_1^n - u_1 = (n-1)\,\alpha\,(u_1 - u_n)$.

Therefore u_n = [1/(n-1)α)] [((n-1)α + 1)u_1 – nu_1^n]; u_n / u_1 = [(n-1)α + 1 – nw_1) / (n-1)α] …(4).

From the second condition,

$\sum_{j=1}^{n} u_1^{(n-j)} u_n^{(j-1)} = 1$ iff $\frac{u_1^{\,n} - u_n^{\,n}}{u_1 - u_1} = 1$ iff $u_1^n - u_n^n = u_1 - u_n$iff $u_1^{n-1} - u_n^n/u_1 = 1 - u_n / u_1$ ...(5).

Comparing equations (4) and (5), it finds thatw_1 = [(n-1)α + 1 – nw_1) / (n-1)α] w_n= [(nw_1 -1) / (n-1)α]; w_n = [((n-1)α – n)w_1 + 1) / ((n-1)α +1-nw_1)] …(6).

Let us rewrite equation (5) as

$u_1^n - u_n^n = u_1 - u_n$; $u_1 (w_1-1) = u_n (w_n-1)$; $w_1 (w_1-1)^{n-1} = w_n (w_n-1)^{n-1}$.

$w_1 (w_1-1)^{n-1} = \left[\frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1}\right]\left[\frac{((n-1)\alpha)(w_1 - 1)}{(n-1)\alpha + 1 - nw_1}\right]^{n-1}$

$w_1[(n-1)\alpha + 1 - nw_1] = [(n-1)\alpha]^{n-1}$ [((n-1)-n)w_1 + 1] ….(7)

So the optimal value of $w_1$should satisfy equation (7). Once w_1 is computed then $w_n$can be determined from equation (9) and the other weights are obtained from equation (6).

Remark: If *n*=3 and (6), then $w_2 = \sqrt{w_1 w_3}$independently of the value of α, which means that the optimal value of $w_2$is always the geometric mean of w_1 and w_3

IV. COMPUTING THE OPTIMAL WEIGHTS

Suppose the functions, $f(w_1) = w_1\,[(n-1)\alpha + 1 - nw_1]^n$; $g(w_1) = [(n-1)\alpha]^{n-1}[((n-1)\alpha - n)w_1 + 1]$

Then to find the optimal value for the first weight we have to solve the following equation:

$f(w_1) = g(w_1)$, where,*g* is a line and f is a polynomial of w_1 of dimension (*n*+1).By solving the equation$f(w_1) = w_1$

$[(n-1)\alpha + 1 - nw_1]^n$ - $n^2 w_1 [(n-1)\alpha + 1 - nw_1]^{n-1} = 0$. We find that its unique solution is $\mathbf{w_1} = [(n-1)\alpha + 1] / n(n+1) < 1/n$ and its second derivative $f^n(\mathbf{w_1})$ is negative, which means that $\mathbf{w_1}$ is the only maximizing point of *f* on the segment [0, 1/n]. Then *g* can intersect *f* only once in the open interval (0, 1/n). It will guarantee the uniqueness of the optimal solution of problem (4).

From the equation $f''(\mathbf{w_1}) = (-)2n^2 [(n-1)\alpha + 1 - nw_1]^{n-1} + n^2(n-1)w_1[(n-1)\alpha+1- nw_1]^{n-2} = 0$, we find that its unique solution is $\mathbf{w_1} = (2)[(n-1)\alpha + 1] / n(n+1) < \mathbf{2w_1} < 1/n$ (since $\alpha < 0.5$) with the meaning that f is strictly concave on (0, $\mathbf{w_1}$) has an inflexion point at $\mathbf{w_1}$, and f is strictly convex on ($\mathbf{w_1}$, 1/n). Therefore the graph of g should lie below the graph of g if $\mathbf{w_1} < w_1 < 1/n$ and g can cross f only once in the interval (0, $\mathbf{w_1}$).

Example: Let us suppose that n = 5 and α = 0.6. Then from the equation

$$w_1 [4(0.6) + 1 - 5w_1] = (4(0.6))^4 [1 - (5 - 4(0.6)) w_1]$$

$w_1^* = 0.2884$; $w_5^* = [4(0.6)-5) w_1^* + 1] / [4(0.6) + 1 - 5w_1^*] = 0.1278$;

$w_2^* = = \sqrt[4]{(w_1^*)^3 w_5^*} = 0.2353$; $w_3^* = \sqrt[4]{(w_1^*)^2 (w_1^*)^2} = 0.1920$; $w_4^* = = \sqrt[4]{(w_5^*)^3 w_1^*} = 0.1566$;

Thus disp (W*) = 1.5692.

V. MAGDM PROBLEM WITH MAXIMAL ENTROPY OWA WEIGHTS

Let $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m \times n}$ be a neutrosophic fuzzy decision matrix, where $r_{ij}^{(k)} = (T_{ij}^{(k)}, I_{ij}^{(k)}, F_{ij}^{(k)})$ is a neutrosophic fuzzy matrices provided by the decision-maker $d_k \in D$ for an alternative O_j with respect to the attribute $u_i \in U$. Here $T_{ij}^{(k)}$ and $I_{ij}^{(k)}$ indicate the degrees of truth, and indeterminacy to the alternative $O_j \in O$ satisfy or partially satisfy respectively the attribute u_i, expressed by the decision-maker d_k, while $F_{ij}^{(k)}$ indicates the degree that the alternative O_j in O does not satisfy the attribute u_i, expressed by the decision-maker d_k, and $T_{ij}^{(k)} \subset [0, 1]$, $I_{ij}^{(k)} \subset [0, 1]$, and $F_{ij}^{(k)} \subset [0, 1]$, i = 1 to m, and j = 1 to n,. To make a final decision in the process of group decision making, we need to fuse all individual decision opinion into group opinion. To do this, we use the IIFHA operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m \times n}$ (k = 1, 2, 3,4) into the collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{m \times n}$.

Definition: Neutrosophic fuzzy ordered weighted average operator: r_{ij} = NFOWA($r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(s)}$) $=\left(\left(r_{ij}^{\sigma(1)}\right)^{\alpha_1} \otimes \left(r_{ij}^{\sigma(2)}\right)^{\alpha_2} \otimes \ldots \left(r_{ij}^{\sigma(s)}\right)^{\alpha_s}\right) = \left(1 - \prod_{k=1}^{n}\left(1 - T_{ij}^{(\sigma(k))}\right)^{a_k}, 1 - \prod_{k=1}^{n}\left(1 - I_{ij}^{(\sigma(k))}\right)^{a_k}, \prod_{k=1}^{n}\left(1 - F_{ij}^{(\sigma(k))}\right)^{a_k}\right)$ where $a = (a_1, a_2, \ldots, a_s)^T$ is a weight vector of NFOWA operator with $a_k > 0$ (k = 1, 2,…, s) and $\sum_{k=1}^{s} a_k = 1$. Here $r_{ij}^{(k)} = (T_{ij}^{(k)}, I_{ij}^{(k)}, F_{ij}^{(k)})$ and $r_{ij}^{\sigma k)} = (T_{ij}^{\sigma(k)}, I_{ij}^{\sigma(k)}, F_{ij}^{\sigma(k)})$ is the k^{th} largest of the weighted NFSs , and $r_{ij}^{\sigma(k)}$ is the image of r_{ij}^{k} under the permutation σ for i = 1 to m, and j = 1 to n.

Definition: The Neutrosophic fuzzy hybrid average operator r_j = NFHA ($r_{1j}, r_{2j}, \ldots, r_{mj}$) = $r_{1j}^{w1} \oplus r_{2j}^{w2} \oplus \ldots \oplus r_{mj}^{wm}$ = $[1 - \prod_{i=1}^{m} 1 - T_{ij})^{w_i}, 1 - \prod_{i=1}^{m} 1 - I_{ij})^{w_i}, \prod_{i=1}^{m} 1 - F_{ij})^{w_i}]$ where the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes can be completely determined in advance.

For the ranking order of the alternatives in accordance with the decision making problem, we give the largest NFS r* = < (1,0,0), (0,1,0), (0,0,1) > as the value of the ideal alternative.

VI. ALGORITHM I

Step 1: Utilize the NFOWA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$ (k=1 to 4) into a collective neutrosophic fuzzy decision matrix $R=(r_{ij})_{mxn}$.

Step: 2 To derive the unknown weights by Shannon Entropy methods by using

In w_j = (j-1)/(n-1 in w_n + (n-j)/(n-1) In w_1

$w_j = \sqrt[(n-1)]{w_1^{(n-j)} w_n^{(j-1)}}$

and $w_1[(n-1)\alpha + 1 - nw_1] = [(n-1)\alpha]^{n-1} [((n-1)-n)w_1 + 1]$.

Step: 3 Use the NFHA operator to get the overall values r_j of the alternatives O_j (j = 1, 2,…,n) using the weights 0.2717, 0.2254, 0.2608, 0.2421 by funding from Posssion distribution through a method of fitness.

Step: 4 Using r* = (1,0,0) = (T_A^*, I_A^*, F_A^*), find $d(r^*, r_j) = \sqrt{\left(T_A^* - T_{jA}\right)^2 + \left(I_A^* - I_{jA}\right)^2 + \left(F_A^* - F_{jA}\right)^2}$ to calculate the distances between informational neutrosophicvalues $r_j = (T_{jA}, I_{jA}, F_{jA})$ (j = 1,2,...,n).

Step 5: Rank the alternatives based on distances.

Step 6: Select the best alternative.

VII. ALGORITHM II

Step 1: Utilize the NFOWA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m \times n}$ (k = 1, 2, 3,4) into a collective neutrosophic fuzzy decision matrix $R = (r_{ij})_{m \times n}$.

Step 2: For every A ∈ NFS (X), E_{NFS} (A) = (1/n) $[\sum_{i=1}^{n} {T_A}^2(x) + (1 - I_A(x))^2 + (1 - F_A(x))^2]$ defined to calculate the informational neutrosophic energy of A.

Step 3 : For every B ∈ NFS (X), E_{NFS} (B) = (1/n) $[\sum_{i=1}^{n} {T_B}^2(x) + (1 - I_B(x))^2 + (1 - F_B(x))^2]$ is defined to calculate the informational neutrosophic energy of B.

Step 4: The covariance C_{NFS} (A, B) = $\frac{1}{n}[\sum_{i=1}^{n}[T_A(x)T_B(x) + (1 - I_A(x))(1 - I_B(x)) + (1 - F_A(x))(1 - F_B(x))]$ for all x in X to calculate the covariance between the neutrosophic values A , and B.

Step 5: The correlation coefficients R_{NFS} is calculated by equation $R_{NFS}(A, B) = \frac{C_{NFS}(A, B)}{\sqrt{E_{NFS}((A).E_{NFS}(B)}}$

VIII. NUMERICAL ILLUSTRATION

A problem concerning with a manufacturing company is discussed, searching the best global supplier for one of its most critical parts used in assembling process. The attributes which are considered here in selection of four potential global suppliers O_j (j=1, 2, 3, 4) are:

U_1: Overall cost of the product; U_2: Quality of the product; U_3: Service performance of supplier;
U_4: Supplier's profile; U_5: Risk factor.

Step 1: The experts $d_k (k = 1,2,3,4)$ represent, respectively, the characteristics of the potential global suppliers $O_j (j = 1,2,3,4)$ in terms of Neutrosophic fuzzy sets ($r_{ij}^{(k)}$) where i = 1,2,3,4; and j = 1, 2, 3, 4. An expert group is formed which consists of four experts $d_k (k = 1,2,3,4)$ (whose weight vector is $\lambda = (0.3, 0.2, 0.3, 0.2)^T$) from each strategic decision area.

$$R^1 = \begin{bmatrix} <0.25,0.54,0.8> & <0.3,0.4,0.9> & <0.7,0.35,0.5> & <0.9,0.2,0.8> \\ <0.6,0.5,0.5> & <0.6,0.2,0.3> & <0.2,0.4,0.9> & <0.6,0.23,0.7> \\ <0.3,0.45,0.9> & <0.7,0.1,0.4> & <0.6,0.5,0.5> & <0.4,0.2,0.9> \\ <0.45,0.38,0.27> & <0.37,0.68,0.16> & <0.6,0.25,0.3> & <0.1,0.4,0.8> \end{bmatrix}$$

$$R^2 = \begin{bmatrix} <0.1,0.3,0.7> & <0.6,0.6,0.5> & <0.4,0.2,0.1> & <0.3,0.7,0.6> \\ <0.3,0.55,0.37> & <0.75,0.42,0.1> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \\ <0.5,0.4,0.32> & <0.65,0.25,0.32> & <0.6,0.3,0.1> & <0.75,0.25,0.55> \\ <0.27,0.9,0.81> & <0.31,0.4,0.6> & <0.75,0.65,0.55> & <0.3,0.7,0.9> \end{bmatrix}$$

$$R^3 = \begin{bmatrix} <0.32,0.47,0.6> & <0.9,0.1,0.3> & <0.6,0.4,0.5> & <0.3,0.5,0.7> \\ <0.12,0.32,0.52> & <0.17,0.81,0.9> & <0.5,0.3,0.1> & <0.45,0.65,0.27> \\ <0.50,0.6,0.23> & <0.56,0.52,0.23> & <0.3,0.6,0.1> & <0.57,0.52,0.55> \\ <0.54,0.83,0.72> & <0.73,0.86,0.61> & <0.5,0.52,0.4> & <0.6,0.4,0.2> \end{bmatrix}$$

$$R^4 = \begin{bmatrix} <0.7,0.3,0.1> & <0.5,0.4,0.4> & <0.2,0.1,0.6> & <0.7,0.9,0.6> \\ <0.3,0.56,0.73> & <0.57,0.24,0.1> & <0.23,0.76,0.65> & <0.53,0.65,0.27> \\ <0.32,0.32,0.6> & <0.56,0.52,0.32> & <0.1,0.3,0.9> & <0.57,0.52,0.55> \\ <0.72,0.5,0.18> & <0.13,0.6,0.4> & <0.55,0.56,0.78> & <0.7,0.1,0.6> \end{bmatrix}$$

$$R^5 = \begin{bmatrix} <0.52,0.45,0.1> & <0.57,0.37,0.1> & <0.76,0.65,0.23> & <0.57,0.52,0.55> \\ <0.3,0.6,0.7> & <0.7,0.4,0.1> & <0.3,0.7,0.6> & <0.5,0.4,0.6> \\ <0.2,0.3,0.2> & <0.6,0.2,0.5> & <0.1,0.6,0.65> & <0.3,0.9,0.7> \\ <0.27,0.5,0.81> & <0.75,0.25,0.32> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \end{bmatrix}$$

Step 2: To derive the unknown weights by Shannon Entropy methods by using

In w_j = (j-1)/(n-1 in w_n + (n-j)/(n-1) In w_1

$w_j = \sqrt[(n-1)]{{w_1}^{(n-j)}{w_n}^{(j-1)}}$ and

$w_1[(n-1)\alpha + 1 - nw_1]^n = [(n-1)\alpha]^{n-1} [((n-1)-n)w_1 + 1]$.

To derive a weight vector *w* by using Shannon entropy methods.

Take n = 5, $\alpha = 0.4$.
$w_1[4(0.4) + 1 - 5w_1]^5 = [4(0.4)]^4 [(4(0.4) - 5)w_1 + 1]$.
$[-3125 w_1^6 + 8125 w_1^5 - 8450 w_1^4 + 4394 w_1^3 - 1142 w_1^3 + 141.09 w_1 - 6.5576] = 0$.

Solving the above equation by using Matlab, we get
$w_1^* = 0.1289$; $w_5^* = \frac{((n-1)\alpha - n)w_1^* + 1}{(n-1)\alpha + 1 - nw_1^*} = 0.2872$; $w_2^* = = \sqrt[4]{(w_1^*)^3 w_5^*} = 0.1567$;
$w_3^* = \sqrt[4]{(w_1^*)^2 (w_1^*)^2} = 0.1922$; $w_4^* = = \sqrt[4]{(w_5^*)^3 w_1^*} = 0.2359$;
Hence $w_1^* = 0.1289$; $w_2^* = 0.1567$; $w_3^* = 0.1922$; $w_4^* = 0.2359$; $w_1^* = 0.2872$.

Step 3: Utilize the IIFOWA operator (consider α = (0.1289, 0.1567, 0.1922, 0,2359, 0.2872)^{T}be its weight vector derived by the normal distribution based method) to aggregate the individual neutrosophic fuzzy decision matrices into the collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{mxn}$. The weights are arranged in the decreasing order.

R=

$$\begin{bmatrix} <0.0465,0.0252,0.0108> & <0.1043,0.0077,0.0102>< 0.0814,0.0064,0.0078> & <0.1016,0.212,0.0737> \\ <0.0418,0.0318,0.0395> & <0.0836,0.0162,0.0063>< 0.0306,0.0323,0.0116> & <0.0584,0.0221,0.0291> \\ <0.0379,0.0258,0.0198> & <0.0876,0.0060,0.0159>< 0.0480,0.0271,0.0081> & <0.0800,0.0179,0.0733> \\ <0.0643,0.0478,0.0194> & <0.0652,0.0277,0.0099>< 0.0862,0.0250,0.0293> & <0.0524,0.0089,0.0265> \end{bmatrix}$$

Step 4: Use the NFHA operator to get the overall values r_j of the alternatives O_j (j = 1, 2,…,n) using the weights 0.2717, 0.2254, 0.2608, 0.2421 by funding from Poisson distribution through a method of fitness. The NFHA operator is rj = $(1- \prod_{k=1}^{n}(1-T_{ij})^{w_i}$, $1- \prod_{k=1}^{n}(1-I_{ij})^{w_i}$, $\prod_{k=1}^{n}(1-F_{ij})^{w_i})$New reduced row Matrix R = (r_1, r_2, r_3, r_4) using the weights w = {0.2717, 0.2608, 0.2254, 0.2421} is R = (r_1, r_2, r_3, r_4); four neutrosophic fuzzy values are r_1 = (0.0476, 0.9674, 0.9775);r_2 = (0.0858, 0.9855, 0.9895); r_3 = (0.0621, 0.9776, 0.9858); and r_4 = (0.0738, 0.9260, 0.9491)

Step 5: Using r* = (1,0,0) = (T_A*, I_A*, F_A*), find d(r*, r_j) = $\sqrt{(T_A^* - T_{jA})^2 + (I_A^* - I_{jA})^2 + (F_A^* - F_{jA})^2}$to calculate the distances between informational neutrosophic values $r_j = (T_{jA}, I_{jA}, F_{jA})$ (j = 1, 2, 3, 4).
Thus d(r , r_1) = 1.1828 = A_1; d(r , r_2) = 1.1802 = A_2; d(r , r_3) = 1.1847 = A_3; d(r , r_4) = 1.1437 = A_4.
Step 5: Rank the alternatives based on distances.$A_2 > A_3 > A_4 > A_1$.
Step 6: Select the best alternative: A_2 is best alternative

IX. CONCLUSIONS

The MAGDM problems are investigated under neutrosophic fuzzy environment, and proposed an approach to handling the situations where the attribute values are characterized by NFSs, and the information about attribute weights completely unknown. The proposed approach first fuses all individual neutrosophic fuzzy decision matrices into the collective neutrosophic fuzzy decision matrix by using the NFOWA operator. Then the obtained attribute weights and the NFHA operator are used to get the overall neutrosophic fuzzy values of alternatives

Also it has proposed a method for calculating correlation coefficients between NFSs to rank the alternatives and then to select the most desirable one. The proposed approach in this work not only can comfort the influence of unjust arguments on the decision results, but also avoid losing or distorting the original decision information in the process of aggregation. Thus, the proposed approach provides us an effective and practical way to deal with multi-person multi-attribute decision making problems, where the attribute values are characterized by NFSs and the information about attribute weights is partially known. The suitable alternative is selected through the algorithm from the given neutrosophic information in which the unknown weights are derived the unknown weights by Shannon Entropy methods, and some weights from Poisson distribution.

REFERENCES

[1] Amirtharaj, E.C.H., & Robinson, J.P., (2013). Attributes weight determination for interval valued intuitionistic fuzzy multiple attribute group decision making problems, International Journal of Computing Algorithm. 2,339-353, October-2013.
[2] Atanassov .K, Intuitionistic Fuzzy sets, Fuzzy Sets and Systems, 20, 87-96, 1986.
[3] Atanassov .K, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems, 33, 37-46, 1989.

[4] Atanassov .K, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 64, 159-174, 1994.

[5] Atanassov .K, Gargov .G, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31, 343-349, 1989.

[6] Fuller, R &Majlender, P, An analytic approach for obtaining maximal entropy OWA operator weights, Fuzzy Sets and Systems, 124, 53-57, 2001.

[7] O'Hagan, M., Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic, in: Proc. 22nd Annual IEEE Asilomar Conf. Signals, Systems, Computers, Pacific Grove, CA, 81-689, 1988.

[8] Park .D.G., Kwun, Y. C., Park .J.H., Park .I.Y., Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. Mathematical and Computer Modelling, 50, 1279-1293, 2009.

[9] Robinson, J.P., Amirtharaj, E.C.H, A short primer on the correlation coefficient of vague sets, International Journal of fuzzy system applications, 1(2), 55-69, 2011a.

[10] Robinson, J.P., Amirtharaj, E.C.H, Extended TOPSIS with correlation coefficient of triangular intuitionistic fuzzy sets for multiple attribute group decision making, International Journal of Decision Support System Technology, 3(3), 15-40, 2011b.

[11] Robinson, J.P., Amirtharaj, E.C.H, Vague correlation coefficient of interval vague sets, International Journal of Fuzzy System Applications, 2(1), 18-34, 2012a.

[12] Robinson, J.P., Amirtharaj, E.C.H, A search for the correlation coefficient of triangular and trapezoidal intuitionistic fuzzy sets for multiple attribute group decision making, Communications in Computer and Information Science - 283, Springer-Verlag, 333-342, 2012b.

[13] Robinson, J.P.,&Amirtharaj, E.C.H., A Strategic TOPSIS algorithm with correlation coefficient of interval vague sets. International Journal of Computing Algorithm. 2,314-331, October 2013.

[14] Solairaju, A., Robinson, J.P., & Lenin, T., Applications of transforming vague sets into fuzzy sets for knowledge management. International Journal of Computing Algorithm. 2(2), 430-439, 2013.

[15] Solairaju, A., Robinson, J.P., & Lenin, T., A New horizon for multiple attribute group decision making problems with Vague sets. International Journal of Applied Mathematics and Statistical Sciences. 3(2), 1-10, 2014.

[16] Szmidt .E, Kacprzyk .J,,Using intuitionistic fuzzy sets in group decision- making. Control and Cybernetics, 31, 1037-1053, 2002.

[17] Szmidt, E., &Kacprzyk, J, A consensus-reaching process under intuitionistic fuzzy preference relations. International Journal of Intelligent Systems, 18, 837-852, 2003.

[18] Wei .G, Wang .X.R., Some geometric aggregation operators interval-valued intuitionistic fuzzy sets and their application to group decision making. InProc.2007, ICCIS-2007, 495-499, 2007.

[19] Wei,G, Wang, H-J, and Lin, R., Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information,Knowledge and Information Systems,26(2),337-349, 2011.

[20] Xu Z.S., An Overview of Methods for determining OWA weights, International Journal of Intelligent Systems, 20, 843-865, 2005.

[21] Xu .Z.S., Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Control and Decision, 22, 215-219, 2007.

[22] Xu .Z.S., Chen .J, An approach to group decision making based on interval-valued intuitionistic judgment matrices. Systems Engineering-Theory and Practice, 27, 126-133, 2007.

[23] Xu .Z.S., Yager .R.R., Some geometric aggregation operators based on intuitionistic fuzzy sets. International Journal of General Systems, 35, 417-433, 2006.

[24] Yager .R.R., On ordered weighted averaging aggregation operators in multi-criteria decision making. IEEE Trans. Systems, Man, Cybernetics., 18, 183-190, 1988.