Correlation Technique for Solving Multi - Objective Programming (MOP) Problems - An Evaluation

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Abstract
The paper evaluates the correlation technique for solving Multi-Objective Programming (MOP) problems. The correlation technique is proposed by Nawkhass and Birdawod[2] with its demonstration using examples. The technique becomes inefficient to solve the same problem with minor modification. The technique was also found unsuitable to solve other examples of MOP problems [1].

Keywords: *Coefficient of correlation, Multi-Objective Programming, Weighted sum method.*

I. INTRODUCTION

The weighted sum method of solving multi-objective programming problems is very popular. A combined objective function is formulated by manipulating multiple objective functions using different methods and optimized under common constraints. Mean, median average and new average methods [1] have been used frequently for the purpose. The correlation technique proposed by Maher [2] to formulate the combined objective function for solving MOP problems is evaluated in this paper.The examples used by the above scientists have been reproduced here to examine the suitability of the correlation method for solving MOP problems.The correlation method may be explained as below:

Let;

Optimize Z= [Max. Z_1, Max. Z_2.....Max. Z_r Min. Z_{r+1} ....Min. Z_s]
Subject to:
AX = b and X≥ 0

The combined objective function is constructed by weighting the objective functions by the inverse of the coefficient of correlation 'r'.

$$Max. Z = \frac{\sum_{j=1}^{r} Zj}{r} - \frac{\sum_{j=r+1}^{s} Zj}{r}$$

Where, 'r' is the coefficient of correlation between individual optima and respective maximum values of decision variables.

The application of the technique was demonstrated with following example [2]:

Example 1:

Max. $Z_1 = 3X_1 + 2X_2$
Max. $Z_2 = 4X_1 + 7X_2$
Max. $Z_3 = 5X_1 - 12X_2$
Min. $Z_4 = 4X_1 - 3X_2$
Max. $Z_5 = -6X_1 + 9X_2$
Max. $Z_6 = -4X_1 + 3X_2$

Subject to;

$4X_1 + 5X_2 \leq 20$
$X_1 - 2X_2 \leq 3$
$1.3X_1 - X2 \leq 4.5$
$X_1\ X_2 \geq 0$

A. Solution

All the objective functions have been optimized individually subject to the constraints. The results are given in Table I.

Table I: Solution of Individual optimization

No.	X_1 , X_2	Max. Z	Max. X_i	Absolute Max. Z_i
1	4, 0.76	13.52	4	13.52
2	0, 4	28	4	28
3	3, 0	15	3	15
4	0, 4	-12	4	12
5	3.75, 0.37	-19.17	3.75	19.17
6	4, 0.76	-13.72	4	13.72

With help of an above table, the coefficient of correlation 'r' between Max.X_i and absolute Max.Z_i was calculated which was 0.95. The combined objective function was constructed by the method explained above. The combined objective function was maximized and generated the following solution:

Max. Z= 2.09 at X_1 = 3.74 and X_2 = 0.37

The method was declared successful for solving multi objective programming problems.

II. LIMITATIONS OF THE CORRELATION TECHNIQUE

Combining the objective functions by scalarizing them by the coefficient of correlation as described above or by any constant will generate the same solution (Values of decision variables). And the solution shall also remain unchanged if objective functions are combined without scalarization. Combining objective functions with this method seems not logical when objective functions are of different dimensions.

Further, the applicability of the technique was assessed with minor modification in the above example. If the objectives 3rd and 5th are dropped, the correlation coefficient 'r' becomes Zero and correlation technique cannot be applied. The technique was furtherevaluated for its suitability with other MOP examples [1]:

Example 2:

$$Max. Z_1 = \frac{(2X1 + X2 + 1)(2X1 + X2 + 2)}{(3X1 + 3X2 + 3)}$$

$$Max. Z_2 = \frac{(6X1 + 3X2 + 3)(4X1 + 2X2 + 4)}{(2X1 + 2X2 + 2)}$$

$$Max. Z_3 = \frac{(8X1 + 4X2 + 4)(6X1 + 3X2 + 6)}{(5X1 + 5X2 + 5)}$$

$$Max. Z_4 = \frac{(10X1 + 5X2 + 5)(-8X1 - 4X2 - 8)}{(7X1 + 7X2 + 7)}$$

$$Max. Z_5 = \frac{(-4X1 - 2X2 - 2)(6X1 + 3X2 + 6)}{(6X1 + 6X2 + 6)}$$

$$Max. Z_6 = \frac{(-2X1 - X2 - 1)(4X1 + 2X2 + 4)}{(9X1 + 9X2 + 9)}$$

Subject to:

$X_1 + 2X_2 \leq 4$
$3X_1 + X2 \leq 6$

$X_1, X_2 \geq 0$

Example 3:

$$Max. Z_1 = \frac{(3X1 + 3X2 + 2)(2X1 + 3X2 + 3)}{(5X1 + 5X2 + 5)}$$

$$Max. Z_2 = \frac{(6X1 + 6X2 + 4)(6X1 + 9X2 + 9)}{(6X1 + 6X2 + 6)}$$

$$Max. Z_3 = \frac{(12X1 + 12X2 + 8)(4X1 + 6X2 + 6)}{(8X1 + 8X2 + 8)}$$

$$Max. Z_4 = \frac{(9X1 + 9X2 + 6)(8X1 + 12X2 + 12)}{(7X1 + 7X2 + 7)}$$

$$Min. Z_5 = \frac{(15X1 + 15X2 + 10)(-12X1 - 18X2 - 18)}{(3X1 + 3X2 + 3)}$$

$$Min. Z_6 = \frac{(-21X1 - 21X2 - 28)(10X1 + 15X2 + 15)}{(4X1 + 4X2 + 4)}$$

$$Min. Z_7 = \frac{(-18X1 - 18X2 - 12)(14X1 + 21X2 + 21)}{(6X1 + 6X2 + 6)}$$

Subject to:

$X_1 + 4X_2 \leq 4$
$2X_1 + X_2 \leq 2$

$X_1, X_2 \geq 0$

A. Solution

Both the above problems have been solved by individual optimization. The solution is presented in Table II.

Table II: Optimal values of individual objective functions and values of decision variables.

Zi	Example 2		Example 3	
	Optimal Value of Zi	Values of Xi X_1, X_2	Optimal Value of Zi	Values of Xi X_1, X_2
Z_1	3.33	2, 0	3.46	4/7 6/7
Z_2	30	2, 0	17.37	4/7 6/7
Z_3	24	2, 0	17.37	4/7 6/7
Z_4	-28.57	2, 0	29.79	4/7 6/7
Z_5	-10	2, 0	-173.79	4/7 6/7
Z_6	-2.22	2, 0	-200.44	4/7 6/7
Z_7			-121	4/7 6/7

The results of Table 2 clearly indicate that all the objectives of example 2 are optimized individually at a unique solution of Xi (2, 0).The 3rd example has also got a single solution at 4/7 of X_1 and 6/7 of X_2. The coefficients of correlation between absolute values of individual optima and maximum values of Xi are zero for both the examples. Therefore the correlation technique cannot be used to solve such MOP problems. Further, the existence of unique solutions in both the examples indicates the absence of conflicts amongst objectives. The optimal solution of any single objective optimizes all the other objectives simultaneously.These examples are inferior for evaluating the suitability of any MOP technique.

III. CONCLUSION

The solution of optimization refers to the values of real decision variablesand respective values of the real objective functions. When the solutions of all the individual optimizations are unique, there is the absence of conflicts among objectives. Hence, the multi-objective optimization is not required.The Correlation technique of MOP cannot be used if all the individual optimizations have a unique solution with respect to the values of decision variables or the values of objective functions.The multi-objective function with or without scalarization by any constant has no effect on the solution. The problem of multidimensional aggregation may also arise in certain applications.

REFERENCES

[1] A.Nejmaddin, Suleiman, and M.A. Nawkhass, "Transforming and Solving Multi-objective Quadratic Fractional Programming Problems by Optimal Average of Maximin&Minimax Techniques,". American Journal of Operational Research 3(3): 92-98, 2013.

[2] M.A.Nawkhassand Q.BHawkar, "Transformed and Solving Multi- Objective Linear Programming Problems to Single Objective by Using Correlation Technique," Cihan International Journal of Social Science Vol. 1, No. 1 pp.0-36, 2017.