Commutative Matrices

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Abstract
We propose a method to generate an infinite class of commutative matrices having dimension(NxN) (N=2,3) corresponding to different eigenvalues. Further we also correlate this product with spectral symmetry. However the product loses its symmetry nature in eigenfunction under the influence of a new matrix having the same dimension.

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I.INTRODUCTION

From the early development of mathematics in matrix theory it is known that matrices having different dimensions are no longer commutative[1]. For example consider two matrices having dimension A(MxN) and B(NxR) where $M \neq N \neq R$ having the behaviour

$$AB \neq BA \tag{1}$$

Further these type of matrices can hardly have physical relavances On the other hand square matrices have physical relavance . That may be the reason for growing interest[2,3] on square matrices in eigenvalue calculation. Further physicists give importance to square matrix on eigenvalue analysis in physical problems. Now question arises as to: if square matrices are widely used in physical sciences , can they reflect commutative behaviour in some sense ? Mathematically do the matrices X and Y belong to same eigen values?

$$X = D+AB \tag{2}$$

$$Y = D+BA \tag{3}$$

where B and A are certain typical matrices having the dimension same as that of D. In my understanding answer to this question is yes provided A, B are generated in a special way eventhough $A \neq B$. Recently it has been shown that on changing diagonal terms one can prove any non-singular square matrix can have infinite set of commutative matrix [3] i.e

$$[A, B_i] = 0 \tag{4}$$

where (i=1,2,3,4, ... 100, ... ∞). However in previous [2,3] approach one can not change the non-diagonal terms. Here we present a novel way to generate commutative matrix by changing non-diagonal terms as follows.

II. ASSUMPTIONS

Let the matrix A and B having same dimension be satisfy the condition

$$B_{i,j} = \pm A_{i,j} \, [i \neq j] \tag{5}$$

$$B_{i,i} = \pm B_{j,j} \, [i \neq j] \tag{6}$$

and

$$A_{i,i} = \pm A_{j,j} \, [i \neq j] \tag{7}$$

It should be remembered that certain matrices satisfying the above two condition may reflect commutative behaviour. Here matrices *A, B* corresponds to different eigenvalues.

III. EIGENVALUES, EIGENFUNCTIONS, SYMMETRY AND ASSYMETRY

If A is a square matrix having eigenvalue relation

$$A\xi = \lambda\xi \tag{8}$$

and *B* is another matrix (having same dimecsion as *A*) having eigenvalue relation

$$B\xi = \eta\xi \tag{9}$$

then it is obvious that the product must satisfy the relation

$$AB\xi = BA\xi = \Lambda\xi \tag{10}$$

even though $\lambda \neq \eta \neq \Lambda$. However symmetry point in these relations refer to ξ invariant. In other words if two matrices have the same eigenfunction then they must commute. Further the eigenfunction nature changes when one considers the relation

$$X\Psi = [D + AB]\,\Psi = E\Psi \tag{11}$$

$$Y\Psi = [D + BA]\,\Psi = \epsilon\Psi \tag{12}$$

The asymetry in the above relation is that $\Psi \neq \xi$. Further interested reader will find that swymmetry in above relation is that $E = \epsilon$

III. Examples of N =2

Below we consider different form of matrices from a general (2x2) matrix

$$A = \begin{bmatrix} \alpha + J & \pm L \\ \pm L & \alpha + J \end{bmatrix} \tag{13}$$

and

$$B = \begin{bmatrix} \beta - J & \pm L \\ \pm L & \beta - J \end{bmatrix} \tag{14}$$

where *J, L* are suitable variable constants. Below we consider different types of matrices following the above.

A. Real Non-Hermitian Matrix

Consider the case $\alpha = 10; \beta = 28$;L=3;J=19 ,the given matrix form of *A* and *B* are given below.

$$A = \begin{bmatrix} 29 & 3 \\ -3 & 29 \end{bmatrix} \tag{15}$$

and

$$B = \begin{bmatrix} 9 & -3 \\ 3 & 9 \end{bmatrix} \tag{16}$$

One can see that

$$AB = BA = \begin{bmatrix} 270 & -60 \\ 60 & 270 \end{bmatrix} \tag{17}$$

B. Real Hermitian Matrix

Consider the case $\alpha = 10; L = 3; \beta = 28; J = 19$ .the given matrix form of *A* and *B* are given below.

$$A = \begin{bmatrix} 29 & 3 \\ 3 & 29 \end{bmatrix} \tag{18}$$

and

$$B = \begin{bmatrix} 9 & 3 \\ 3 & 9 \end{bmatrix} \qquad (19)$$

One can see that

$$AB = BA = \begin{bmatrix} 270 & 114 \\ 114 & 270 \end{bmatrix} \qquad (20)$$

C. Complex Non-Hermitian Matrix

Consider the case $\alpha = 10; L = 3i; J = 19$ .the given matrix form of *A* and *B* are given below.

$$A = \begin{bmatrix} 29 & 3i \\ 3i & 29 \end{bmatrix} \qquad (21)$$

and

$$B = \begin{bmatrix} 9 & 3i \\ 3i & 9 \end{bmatrix} \qquad (22)$$

Then it is easy to verify the relation

$$AB = BA = \begin{bmatrix} 252 & 114i \\ 114i & 252 \end{bmatrix} \qquad (23)$$

D. Complex Hermitian Matrix

Consider the case $\alpha = 10; L = 3i; J = 19$ .the given matrix form of *A* and *B* are given below.

$$A = \begin{bmatrix} 29 & 3i \\ -3i & 29 \end{bmatrix} \qquad (24)$$

and

$$B = \begin{bmatrix} 9 & -3i \\ 3i & 9 \end{bmatrix} \qquad (25)$$

Similarly one will verify the relation

$$AB = BA = \begin{bmatrix} 252 & -60i \\ 60i & 252 \end{bmatrix} \qquad (26)$$

IV. EIGENVALUES INVARIANCE

Here we consider any arbitrary matrix ***D*** having eigenvalues $\lambda_{1,2}$ and show that the operation

$$D+AB - BA \qquad (27)$$

has the same eigenvalues as that of D Further we have

$$X = D+AB \qquad (28)$$

$$Y=D+BA \qquad (29)$$

For example if

$$D = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \qquad (30)$$

and A,B are the complex hermitian matrices as given above ,then one will see that both the matrices(X,Y) possess the same eigenvalues $\lambda_1 = \frac{5789}{30}; \lambda_2 = \frac{9391}{30}$

V.EXAMPLES OF N=3

We basically follow the above rule and construct the following matrices.

$$A = \begin{bmatrix} R\pm J & \pm L & \pm P \\ \pm L & R\pm J & \pm M \\ \pm P & \pm M & R\pm J \end{bmatrix} \quad (31)$$

and

$$B = \begin{bmatrix} a\pm J & \pm L & \pm P \\ \pm L & a\pm J & \pm M \\ \pm P & \pm M & a\pm J \end{bmatrix} \quad (32)$$

Below we consider the constants as :P=37;J=19;M=18;a=10;R=15;L=10;L=3.

A. Real Non-Hermitian

$$A = \begin{bmatrix} -9 & -3 & 37 \\ 3 & -9 & -18 \\ -37 & 18 & -9 \end{bmatrix} \quad (33)$$

and

$$B = \begin{bmatrix} 34 & 3 & -37 \\ -3 & 34 & 18 \\ 37 & -18 & 34 \end{bmatrix} \quad (34)$$

$$AB = BA = \begin{bmatrix} 1072 & -795 & 1537 \\ -537 & 27 & -885 \\ -1645 & 663 & 1387 \end{bmatrix} \quad (35)$$

B. Complex Non-Hermitian

$$A = \begin{bmatrix} -9 & 3i & 37i \\ 3i & -9 & 18i \\ 37i & 18i & -9 \end{bmatrix} \quad (36)$$

and

$$B = \begin{bmatrix} 34 & 3i & 37i \\ 3i & 34 & 18i \\ 37i & 18i & 34 \end{bmatrix} \quad (37)$$

It is easy to show that

$$AB = BA = \begin{bmatrix} -1684 & -666+75i & -54+925i \\ -666+75i & -639 & -111+450i \\ -54+925i & -111+450i & -1999 \end{bmatrix} \quad (38)$$

Similarly one can construct other operators like that of N=2

VI. CONCLUSION

In this we have suggested a method for generating commutative matrices and its generation for (2x2) and (3x3) dimensions. Folloing this one can generate (NxN) commutative matrices but its generation requires thorough understanding of the above matrices. It should be remembered that complex hermitian matrices considered above corresponds to different eigenvalues i.e (A[26,32];B[6,12]). We hope this paper will generate interest among many theoretical physicists and mathematician to generate new idea on matrix theory.

REFERENCES

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