Properties of Perfect Fuzzy Graphs

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Abstract

In our paper [6], we discussed strong regular fuzzy graphs. In this paper, some properties of perfect fuzzy graphs are studied. The effect of complement and complete regular fuzzy graphs on perfect fuzzy graphs is discussed.

Keywords: *Perfect fuzzy graphs, Strong fuzzy graph, Regular fuzzy graph, Complete regular fuzzy graph*

I. INTRODUCTION

Zadeh[7] introduced the notion of Fuzzy sets and Fuzzy relations to deal with the problems of uncertainty in real physical world. In 1975, Rosenfeld[5] introduced the concept of fuzzy graphs. The concept of perfect fuzzy graph was investigated by Basheer Ahmed and Nagoorgani[1]. In [4], the concept of regular fuzzy graph was investigated by Nagoorgani and Radha.

In [6], Seethalakshmi R and Gnanajothi R.B introduced strong regular fuzzy graph. In this paper, we derive that the complement of perfect fuzzy graph is perfect and every complete perfect fuzzy graph is a complete regular fuzzy graph.

II. PRELIMINARIES

Definition 1 [2]: A fuzzy graph G=(σ,μ) is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ with

$$\mu(u,v) \leq \sigma(u) \wedge \sigma(v), \forall\, u,v \in V,$$

where V is a finite nonempty set and $\wedge$ denote minimum.

Definition 2 [2]. A graph G*=(V,E) is called the underlying crisp graph of the fuzzy graph G, where

$$V = \{u/\sigma(u) \neq 0\} \text{ and}$$

$$E = \{(u,v) \in VXV / \mu(u,v) \neq 0\}.$$

Definition 3 : [3] A fuzzy graph $G = (\sigma, \mu)$ is defined to be a strong fuzzy graph if

$$\mu(u,v) = \sigma(u) \wedge \sigma(v), \forall\, (u,v) \in E.$$

Definition 4 : [5] A fuzzy graph $G = (\sigma, \mu)$ is defined to be a complete fuzzy graph if the underlying graph G*=(V,E) is complete and

$$\mu(u,v) = \sigma(u) \wedge \sigma(v), \forall\, u,v \in V.$$

Definition 5: [5] The complement of a fuzzy graph G=(σ,μ) on G*=(V,E) is a fuzzy graph $\bar{G} = (\bar{\sigma}, \bar{\mu})$, where

$$\bar{\sigma} = \sigma \text{ and}$$

$$\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v), \textit{for all } u,v \text{ in } V.$$

Definition 6:[4] Let G= (σ,μ) be a fuzzy graph. G*=(V,E). The fuzzy degree of a node u$\in V$ is defined as

$$(fd)(u) = \sum_{u \neq v, v \in V} \mu(u,v)$$

G is said to be a regular fuzzy graph if each vertex has same fuzzy degree.
If (fd)$(v) = k, \forall\, v \in V$, for some positive k, then G is called a $k-$ regular fuzzy graph.

Definition 7 : [1] Let G= (σ,μ) be a connected fuzzy graph on (V,E). A vertex is called perfect fuzzy vertex if

$$\sigma(v) = 1, \text{ for some } v \in V.$$

An edge $(u,v) \in E$ is called a perfect fuzzy edge if $\mu(u,v) = 1$, for some $(u,v) \in E$.

Definition 8 : [1] A fuzzy graph G= (σ,μ) on (V,E) is called a perfect fuzzy graph if

$$\sigma(v) = 1, \text{ for all } v \in V.$$

Definition 9 : [1] A fuzzy graph G= (σ, μ) on (V,E) is called a μ-perfect fuzzy graph if

$$\mu(u,v) = 1, \text{ for all } (u,v) \in E.$$

Definition 10 : [1] A perfect fuzzy graph G= (σ, μ) on (V,E) is a complete perfect fuzzy graph if the underlying graph of G is complete.

III . PROPERTIES OF PERFECT FUZZY GRAPHS

Theorem 1 Every μ-perfect fuzzy graph is a σ-perfect graph.

Proof :

Let G= (σ, μ) on (V,E) be a μ-perfect fuzzy graph.

Then $\mu(u,v) = 1$, for all $(u,v) \in E$.

We claim that $\sigma(v) = 1$, for all $v \in V$.
If possible, let $\sigma(v) \neq 1$, for some $v \in V$.

Then $0 < \sigma(v) < 1 \Rightarrow \sigma(u) \wedge \sigma(v) \leq \sigma(v)$

$< 1 = \mu(u,v)$

$$\leq \sigma(u) \wedge \sigma(v) \text{, a contradiction}$$

Therefore, . $\sigma(v) = 1$, for all $v \in V$.
Hence G is a σ-perfect graph.

Theorem 2 Every perfect fuzzy graph is a strong perfect graph.

Proof:

Let G= (σ, μ) on (V,E) be a perfect fuzzy graph.

$$\text{Then } \mu(u,v) = 1, \sigma(u) = 1 \text{ and}$$

$$\sigma(v) = 1 \text{ for all } u,v \in A \text{ and } (u,v) \in E.$$

Then $\mu(u,v) = 1 = \sigma(u) \wedge \sigma(v)$

Therefore, G is a strong perfect fuzzy graph.

Theorem 3 Complement of a perfect fuzzy graph is a perfect fuzzy graph.

Proof:

Let G= (σ, μ) on (V,E) be a perfect fuzzy graph.

Then $\mu(u,v) = 1, \sigma(u) = 1$ and

$$\sigma(v) = 1 \text{ for all } u,v \in V \text{ and } (u,v) \in E.$$

Let $\bar{G} = (\bar{\sigma}, \bar{\mu})$ be the complement of G.

For $(u,v) \in E$,

$$\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v) = 1 - 1 = 0.$$

For $(u,v) \notin E$,

$\bar{\mu}(u,v) = 1 - 0 = 1.$

Hence $\bar{G}$ is a perfect fuzzy graph.

Theorem 4 Every complete perfect fuzzy graph is a complete regular fuzzy graph.

Proof:

Let G= (σ,μ) on (V,E) be a complete perfect fuzzy graph on K_n, the complete graph with n vertices.

Then the underlying graph is complete with
$\mu(u,v) = 1, (u,v) \in E(K_n)$,.

(since for all vertices $v \in K_n, d(v) = n - 1.$)

Now, $(fd)(v) = \sum_{u \neq v, u \in V} \mu(u,v)$

$= (n-1).1 = n - 1$, a constant, for all $v \in V$.

Hence, G is a complete regular fuzzy graph.

IV. CONCLUSSION

Here, some properties of perfect fuzzy graphs are studied. The effect of complement and complete regular fuzzy graphs on perfect fuzzy graphs is also discussed. We can extend the result to strong regular fuzzy graph.

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