

Exact Value of pi $\pi (17 - 8\sqrt{3})$

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Abstract

In this paper I have shown the proof of exact value of pi. The derivation of this value is supported by number of geometrical constructions, arithmetic calculations and use of some simple algebraic formulae.

I. INTRODUCTION

Pi $\pi = C$ = circumference of circle, D = diameter of circle, A = area of circle, R = radius of circle,
We know that: $(C \div D) = \pi$, Area of circle = πr^2 , Area of circle $\div r^2 = \pi$

i.e. $(C \div D) = (A \div R^2)$

If we calculate $(C \div D)$ we cannot measure end point of circumference. So it will give approximate results.

I started research to find exact area of circle using $(A \div R^2)$ method. This method gives me exact area of circle. = $(17 - 8\sqrt{3}) r^2$

We know that problem of exact value of pi is not new to this world. It has been discussed for more than a thousand years ago. Now days we can find exact value of pi more than 13 trillion digits with the help of computer to achieve more accuracy. Even then we can't reach up to exact value of pi which will give area of circle equal to area of square.

Numbers of mathematician have tried to divide circle into n-sides polygon in order to get more accurate value. As per this concept Higher the number of sides of polygon more will be the accuracy of value of pi. But this will never give exact results [all the results are approaching towards exactness]. Due to this method all the world assumed that pi is transcendental number.

If pi is transcendental I.e. $\pi = 3.1415926535897\dots$ infinite digit old value

Then $(\pi - 3) = 0.1415926535897\dots$ & $(4 - \pi) = 0.8584073464102\dots$ infinite digit

$(\pi - 3) + (4 - \pi) = 1$ = area of (circumscribed square – inscribed dodecagon)

$1 = (0.1415926535897\dots + 0.8584073464102\dots) = 0.999999999999\dots$ infinite digit

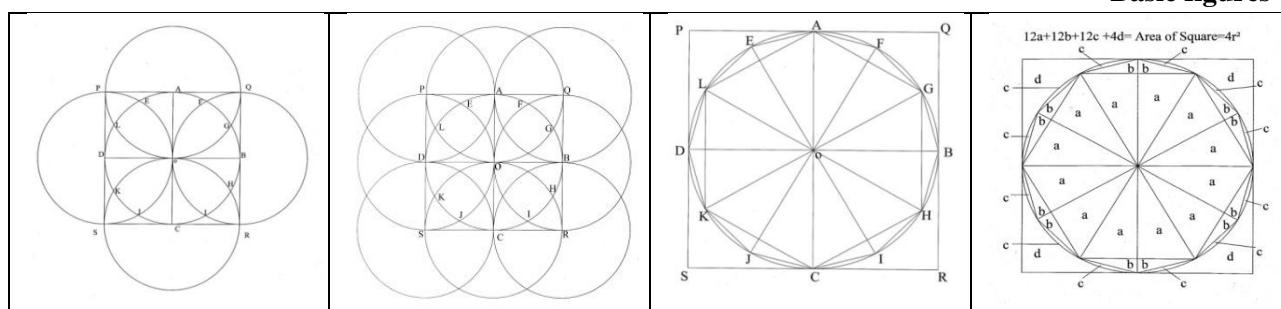
this is not exactly equal to 1. If pi is algebraic i.e. $(17 - 8\sqrt{3})$

Then $[4 - (17 - 8\sqrt{3})] = (8\sqrt{3} - 13)$, $[\pi - 3 = (17 - 8\sqrt{3}) - 3] = (14 - 8\sqrt{3})$, $(4 - \pi) + (\pi - 3) = 1$

$1 = [(8\sqrt{3} - 13) + (14 - 8\sqrt{3})]$

I have prepared many proofs & here I am giving one of them.

Basic figures



Basic information: Note: let a, b, c & d each part shows area in following figures

$\begin{aligned} \text{Area of inscribed dodecagon} \\ = (12a + 12b) &= 3r^2 \\ = (\pi r^2 - 12c) \end{aligned}$	$\begin{aligned} \text{Area of circumscribed square} \\ = (12a + 12b + 12c + 4d) &= 4r^2 \\ = (\pi r^2 + 4d) \end{aligned}$	$\begin{aligned} (4 - \pi)r^2 &= 4d \\ (\pi - 3)r^2 &= 12c \\ (4d + 12c) &= [(4 - \pi) + (\pi - 3)] = 1 \end{aligned}$

As we know, the exact area of inscribed dodecagon = $3r^2$. In order to calculate exact area of circle, we have to calculate exact area of $12c$. Hence there is no need to divide whole circle into infinite number of parts to calculate its accurate area. How to estimate the exact area of part $12c$ & part $4d$?

(Area of circumscribed square – area of inscribed dodecagon)

$$= (12a + 12b + 12c + 4d) - (12a + 12b) = (12c + 4d)$$

$$= (4 - 3)r^2 = r^2 \quad \text{area of } \frac{1}{3} \text{ inscribed dodecagon} = (4a + 4b)$$

$$\text{i.e. } (4a + 4b = 12c + 4d) \quad (4a + 4b - 12c - 4d = 0) \quad (a + b - 3c - d = 0) \quad \text{equation no. 1}$$

$$\text{I found that } (14b - 2a - 3c = 0) \quad \text{equation no. 2} \quad (3a - 13b - d = 0) \quad \text{equation no. 3}$$

Above equations are equal & have value equal to zero which helps in determining value of $3c$ & d in terms of a & b

$$\text{i. e. } 3c = (14b - 2a) = [14(0.25 - 0.125\sqrt{3})r^2 - 2(0.125\sqrt{3})r^2] = (3.5 - 2\sqrt{3})r^2$$

$$d = (3a - 13b) = [3(0.125\sqrt{3})r^2 - 13(0.25 - 0.125\sqrt{3})r^2] = (2\sqrt{3} - 3.25)r^2$$

$$a = (0.125\sqrt{3})r^2 \quad b = (0.25 - 0.125\sqrt{3})r^2 \quad 3c = (3.5 - 2\sqrt{3})r^2 \quad d = (2\sqrt{3} - 3.25)r^2$$

Area of circumscribed square = $(12a + 12b + 12c + 4d)$

$$= 12(0.125\sqrt{3})r^2 + 12(0.25 - 0.125\sqrt{3})r^2] + 4(3c = 3.5 - 2\sqrt{3})r^2 + 4(2\sqrt{3} - 3.25)r^2$$

$$= (1.5\sqrt{3})r^2 + (3 - 1.5\sqrt{3})r^2] + (14 - 8\sqrt{3})r^2 + (8\sqrt{3} - 13)r^2$$

$$= 4r^2$$

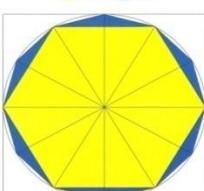
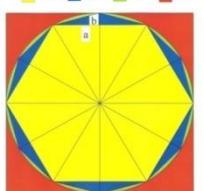
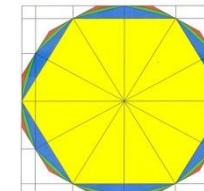
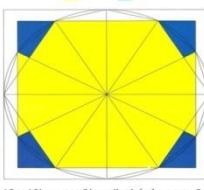
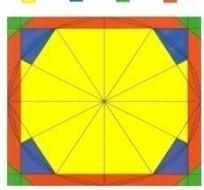
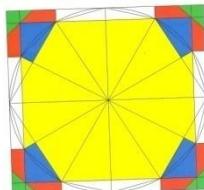
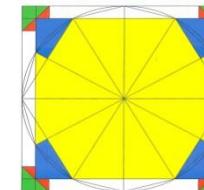
Area of circle = $(12a + 12b + 12c)$

$$= 12(0.125\sqrt{3})r^2 + 12(0.25 - 0.125\sqrt{3})r^2] + 4(3c = 3.5 - 2\sqrt{3})r^2$$

$$= (1.5\sqrt{3})r^2 + (3 - 1.5\sqrt{3})r^2] + (14 - 8\sqrt{3})r^2$$

$$= (17 - 8\sqrt{3})r^2$$

II. SUBSTITUTION OF VALUES IN EQUATIONS

$a = (0.125\sqrt{3}) r^2$	$b = (0.25 - 0.125\sqrt{3}) r^2$	$3c = (3.5 - 2\sqrt{3}) r^2$	$d = (2\sqrt{3} - 3.25)r^2$	
By using values of a, b, 3c, & d in following equations we get appropriate answer.				
 Inscribed dodecagon = $12a + 12b = 3r^2$	 $12a + 12b + 12c + 4d = \text{area of square}$	 $12a + 12b + 12c + 6d = \text{Area of Circle} + 2d - 9c$	 $12a + 12b + 18c = \text{area of cir.dodecagon}$	
 $12a + 12b = \text{area of inscribed dodecagon} = 3r^2$	 $12a + 12b + 12c + 4d = \text{area of square}$	 $12a + 12b + 3c + 2d = \text{area of hexagon}$	 $12a + 12b + 18c = \text{area of cir.dodecagon}$	
Area of inscribed dodecagon = $3r^2$ = $(12a + 12b)$ = $(\pi r^2 - 12c)$	Area of circumscribed square = $4r^2$ = $(12a + 12b + 12c + 4d)$ = $(\pi r^2 + 4d)$	Area of circumscribed hexagon = $(2\sqrt{3}) r^2$ = $16a$	Area of circumscribed Dodecagon = $12(2 - \sqrt{3}) r^2$ = $96b$	Area of circle = $(17 - 8\sqrt{3}) r^2$ = $(12a + 12b + 12c)$
= $36c + 12d$ = $16a - 16b + 6c$ = $64b + 4d$ = $20a - 44b + 12c$ = $8a + 40b - 6c$ = $96b - 18c$ = $14a - 2b + 3c$ = $24a - 72b + 18c$ = $4a + 68b - 12c$ = $6a + 38b + 2d$ $92a + 172b + 39c + 18d$	= $16a + 16b$ = $48c + 16d$ = $64b + 12c + 8d$ = $128b - 24c$ = $4a + 68b + 4d$ = $18a + 2b + 3c$ = $96b - 6c + 4d$ = $120b - 8a + 8d$ = $20a - 12b + 6c$ = $30a - 82b + 21c$ $80a + 400b + 60c + 40d$	= $(12a + 12b + 3c + 2d)$ = $39c + 14d$ = $13a + 13b + d$ = $14a + 14b - 3c$ = $10a + 42b - 9c$ = $24a - 56b + 12c$ = $112b - 24c$ = $7a + 39b + 3d$ = $18a - 14b + 3c$ = $4a + 52b + 4d$ $102a + 214b + 21c + 24d$	= $12a + 12b + 18c$ = $4a + 68b + 6c$ = $54c + 12d$ = $8a + 40b + 12c$ = $2a + 82b + 3c$ = $16a - 16b + 24c$ = $12a + 44b - 4d$ = $16a + 15c - 2d$ = $18a + 18b - 6d$ = $-6a + 122b + 2d$ $82a + 370b + 132c + 2d$	= $(64b + 12c + 4d)$ = $(4a + 68b)$ = $(16a + 16b - 4d)$ = $(16a - 16b + 18c)$ = $(96b - 6c)$ = $(120b - 8a + 4d)$ = $(128b - 24c - 4d)$ = $(12d + 48c)$ = $(10a + 42b - 2d)$ = $8a + 40b + 6c$ $46a + 558b + 54c + 10d$

Total area of [10 inscribed dodecagon + circumscribed (10 square + 10 hexagon + 10 dodecagon)]

$$\begin{aligned}
 &= 10[3r^2 + 4r^2 + (2\sqrt{3})r^2 + (24 - 12\sqrt{3})r^2] = [30r^2 + 40r^2 + (20\sqrt{3})r^2 + (240 - 120\sqrt{3})r^2] = (310 - 100\sqrt{3})r^2 \\
 &= [(92a + 172b + 39c + 18d) + (80a + 400b + 60c + 40d) + (102a + 214b + 21c + 24d) + (82a + 370b + 132c + 2d)] \\
 &= (356a + 1156b + 252c + 84d) \\
 &= 356(0.125\sqrt{3})r^2 + 1156(0.25 - 0.125\sqrt{3})r^2 + 84(3c = 3.5 - 2\sqrt{3})r^2 + 84(2\sqrt{3} - 3.25)r^2 \\
 &= (44.5\sqrt{3})r^2 + (289 - 144.5\sqrt{3})r^2 + (294 - 168\sqrt{3})r^2 + (168\sqrt{3} - 273)r^2 \\
 &= (310 - 100\sqrt{3})r^2
 \end{aligned}$$

Equations & value	Figures value
$(356a + 1156b + 252c + 84d) - \text{area of 21 square}$	$= (310 - 100\sqrt{3})r^2 - 21(4r^2)$
$= (356a + 1156b + 252c + 84d) - 21(12a + 12b + 12c + 4d)$	$= (310 - 100\sqrt{3})r^2 - 84r^2$
$= (356a + 1156b + 252c + 84d) - (252a + 252b + 252c + 84d)$	$= (226 - 100\sqrt{3})r^2$
$= (104a + 904b)$	
$= 104(0.125\sqrt{3})r^2 + 904(0.25 - 0.125\sqrt{3})r^2$	
$= (13\sqrt{3})r^2 + (226 - 113\sqrt{3})r^2$	
$= (226 - 100\sqrt{3})r^2$	

Total area of [10 inscribed dodecagon + circumscribed (10 hexagon + 10 dodecagon)]

$$\begin{aligned}
 &= 10[3r^2 + (2\sqrt{3})r^2 + (24 - 12\sqrt{3})r^2] = [30r^2 + (20\sqrt{3})r^2 + (240 - 120\sqrt{3})r^2] = (270 - 100\sqrt{3})r^2 \\
 &= [(92a + 172b + 39c + 18d) + (102a + 214b + 21c + 24d) + (82a + 370b + 132c + 2d)] \\
 &= (276a + 756b + 192c + 44d)
 \end{aligned}$$

Equations & value	= Figures value
$(276a + 756b + 192c + 44d) + \text{area of 5 ins. dodecagon}$	$= (270 - 100\sqrt{3})r^2 + 5(3r^2)$
$= (276a + 756b + 192c + 44d) + 5(4a + 68b - 12c)$	$= (270 - 100\sqrt{3})r^2 + (15r^2)$
$= (276a + 756b + 192c + 44d) + (20a + 340b - 60c)$	$= (285 - 100\sqrt{3})r^2$
$= (296a + 1096b + 132c + 44d)$	$(285 - 100\sqrt{3})r^2 - 11(4r^2)$
$(296a + 1096b + 132c + 44d) - \text{area of 11 square}$	$= (241 - 100\sqrt{3})r^2$
$= (296a + 1096b + 132c + 44d) - 11(12a + 12b + 12c + 4d)$	
$= (164a + 964b)$	
$= 164(0.125\sqrt{3})r^2 + 964(0.25 - 0.125\sqrt{3})r^2$	
$= (20.5\sqrt{3})r^2 + (241 - 120.5\sqrt{3})r^2$	
$= (241 - 100\sqrt{3})r^2$	

Total area of [10 inscribed dodecagon + circumscribed (10 square + 10 hexagon + 10 dodecagon)]

$$\begin{aligned}
 &= 10[3r^2 + 4r^2 + (2\sqrt{3})r^2 + (24 - 12\sqrt{3})r^2] = (310 - 100\sqrt{3})r^2 \\
 &= [(92a + 172b + 39c + 18d) + (80a + 400b + 39c + 39d) + (102a + 214b + 21c + 24d) + (82a + 370b + 132c + 2d)] \\
 &= (356a + 1156b + 252c + 84d)
 \end{aligned}$$

$$\begin{aligned}
 &(356a + 1156b + 252c + 84d) + \text{Area of 21 Ins. dodecagon} + 14 \text{ cir dodecagon} \\
 &= (356a + 1156b + 252c + 84d) + 21(4a + 68b - 12c) + 14(18a + 18b - 6d) \\
 &= (356a + 1156b + 252c + 84d) + (84a + 1428b - 252b) + (252a + 252b - 84d) \\
 &= (692a + 2856b) = (310 - 100\sqrt{3})r^2 + 21(3r^2) + 14(24 - 12\sqrt{3})r^2 \\
 &= 692(0.125\sqrt{3})r^2 + 2836(0.25 - 0.125\sqrt{3})r^2 = (310 - 100\sqrt{3})r^2 + 21(3r^2) + 14(24 - 12\sqrt{3})r^2 \\
 &= (86.5\sqrt{3})r^2 + (709 - 354.5\sqrt{3})r^2 = (310 - 100\sqrt{3})r^2 + (63r^2) + (336 - 168\sqrt{3})r^2 \\
 &= (709 - 268\sqrt{3})r^2 = (709 - 268\sqrt{3})r^2
 \end{aligned}$$

Total area of [10 inscribed dodecagon + circumscribed (10 square + 10 hexagon + 10 dodecagon)]

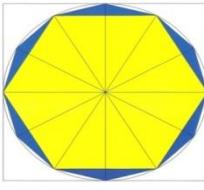
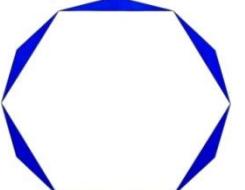
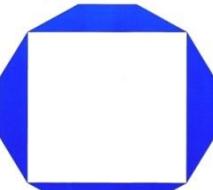
$$\begin{aligned}
 &= 10[3r^2 + 4r^2 + (2\sqrt{3})r^2 + (24 - 12\sqrt{3})r^2] = (310 - 100\sqrt{3})r^2 \\
 &= [(92a + 172b + 39c + 18d) + (80a + 400b + 39c + 39d) + (102a + 214b + 21c + 24d) + (82a + 370b + 132c + 2d)] \\
 &= (356a + 1156b + 252c + 84d)
 \end{aligned}$$

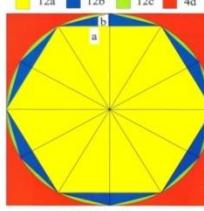
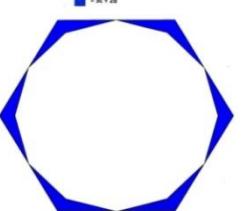
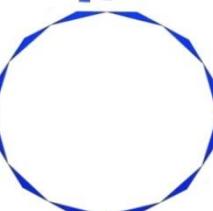
$$\begin{aligned}
 &(356a + 1156b + 252c + 84d) + \text{Area of 28 hexagon} + \text{area of 21 cir. Dodecagon} \\
 &= (356a + 1156b + 252c + 84d) + 28(10a + 42b - 9c) + 21(12a + 44b - 4d) \\
 &= (356a + 1156b + 252c + 84d) + (280a + 1176b - 252c) + (252a + 924b - 84d) \\
 &= (888a + 3256b) = (310 - 100\sqrt{3})r^2 + 28(2\sqrt{3})r^2 + 21(24 - 12\sqrt{3})r^2 \\
 &= 888(0.125\sqrt{3})r^2 + 3256(0.25 - 0.125\sqrt{3})r^2 = (310 - 100\sqrt{3})r^2 + (56\sqrt{3}r^2) + (504 - 252\sqrt{3})r^2 \\
 &= (111\sqrt{3})r^2 + (814 - 407\sqrt{3})r^2 = (310 - 100\sqrt{3})r^2 + (504 - 196\sqrt{3})r^2 \\
 &= (814 - 296\sqrt{3})r^2 = (814 - 296\sqrt{3})r^2
 \end{aligned}$$

$$\text{Area of 10 circle} = (46a + 558b + 54c + 10d) + \text{area of } (9 + 5) \text{ circle}$$

$$\begin{aligned}
 \text{Area of 24 circle} &= (46a + 558b + 54c + 10d) + 9(96b - 6c) + 5(10a + 42b - 2d) \\
 &= (46a + 558b + 54c + 10d) + (864b - 54c) + (50a + 210b - 10d) = (96a + 1632b) = 24(4a + 68b) \\
 &= 24[4(0.125\sqrt{3})r^2 + 68(0.25 - 0.125\sqrt{3})r^2] = 24[(0.5\sqrt{3})r^2 + (17 - 8.5\sqrt{3})r^2] = 24(17 - 8\sqrt{3})r^2
 \end{aligned}$$

III. SUPPORTED WORK: INSCRIBED DODECAGON METHOD

Area of inscribed figures			
 Inscribed dodecagon = $12a + 12b = 3r^2$	 $= 21.375c + 6.75d$	 $= 6.75c + 1.5d$	 $= 12c + 4d$
Area of ins. Dodecagon $= 3r^2 = (12a + 12b)$	Area of ins. Triangle $= (0.75\sqrt{3}) r^2 = 6a$ $= (12a + 12b) - 6.75d - 21.375c$	Area of ins. Hexagon $= (1.5\sqrt{3}) r^2 = 12a$ $= (12a + 12b) - 1.5d - 6.75c$	Area of ins. Square $= 2r^2 = (8a + 8b)$ $= (12a + 12b) - 4d - 12c$

Area of inscribed figures			
 area of circle + 4d	 $= 22.5c + 9d$	 $= 3c + 2d$	 $= 36c$
Area of cir. Square $= 4r^2 = (16a + 16b)$ $(12a + 12b) + 12c + 4d$	Area of cir. Triangle $= (3\sqrt{3}) r^2 = 24a$ $= (12a + 12b) + 22.5c + 9d$	Area of cir. Hexagon $= 2\sqrt{3} r^2 = 16a$ $= (12a + 12b) + 3c + 2d$	Area of cir. Dodecagon $= 12(2 - \sqrt{3}) r^2 = 96b$ $= (12a + 12b) + 18c$

Exact equations	Derived equations
Area of circumscribed square $= 12(a + b + c) + 4d$	Area of Inscribed Triangle $= 12(a + b) - (21.375c + 6.75d)$
Area of inscribed dodecagon $= 12(a + b)$	Area of inscribed square $= 12(a + b) - (12c + 4d)$
Area of circle $= 12(a + b + c)$	Area of inscribed hexagon $= 12(a + b) - (6.75c + 1.5d)$

IV. PROOF OF EXACT EQUATION EQUAL TO DERIVED EQUATIONS

$$\begin{aligned} & (\text{Area of circumscribed 5 square} + \text{area of 2 inscribed dodecagon}) = [5(4r^2) + 2(3r^2)] = 26r^2 \\ & = [\text{area of cir. (2 triangle} + 3 \text{ hex.} + 1 \text{ Dodd.)} + 1 \text{ ins. square}] = \{2(3\sqrt{3}) r^2 + 3(2\sqrt{3}) r^2 + 1(24 - 12\sqrt{3})\} r^2 + 2r^2 = 26r^2 \end{aligned}$$

Exact equations	= Derived equations
$\begin{aligned} & [\text{Area of circumscribed 5 square} \\ & \quad = 5(12a + 12b + 12c + 4d) \\ & \quad + \text{area of inscribed 2 dodecagon} \\ & \quad = 2(12a + 12b)] \\ & = (60a + 60b + 60c + 20d) + (24a + 24b) \\ & = (84a + 84b + 60c + 20d) \\ & = 84(a + b) + 5(12c + 4d) \\ & = 84(0.25) r^2 + 5(1) r^2 = (21 + 5) r^2 = 26r^2 \end{aligned}$	$\begin{aligned} & [\text{area of circumscribed (2 triangle} + 3 \text{ hexagon} + 1 \text{ dodecagon)} \\ & \quad + 1 \text{ ins. Square}] \\ & = [2(12a + 12b + 22.5c + 9d) + 3(12a + 12b + 3c + 2d) \\ & \quad + 1(12a + 12b + 18c) + 1(12a + 12b - 12c - 4d)] \\ & = [(24a + 24b + 45c + 18d) + (36a + 36b + 9c + 6d) + (12a + 12b + 18c) \\ & \quad + (12a + 12b - 12c - 4d)] \\ & = (84a + 84b + 60c + 20d) \\ & = 84(a + b) + 5(12c + 4d) = 84(0.25) r^2 + 5r^2 = (21 + 5) r^2 = 26r^2 \end{aligned}$

I got method by using it we can solve infinite examples similar to above. Let us see following table.

Proof of exact equations equal to derived equations $a = (0.125\sqrt{3}) r^2$ $b = (0.25 - 0.125\sqrt{3}) r^2$ ($a + b = 0.25r^2$)

S. R.	Figures	Values	Exact equations	Derived equations
1	Area of inscribed dodecagon	$3r^2$	$= (12a + 12b)$	$= (12a + 12b)$
2	Area of circumscribed dodecagon	$(24 - 12\sqrt{3}) r^2$	$= 96b$	$= (12a + 12b) + 18c$
3	Area of inscribed hexagon	$(1.5\sqrt{3}) r^2$	$= 12a$	$= (12a + 12b) - 6.75c - 1.5d$
4	Area of circumscribed hexagon	$(2\sqrt{3}) r^2$	$= 16a$	$= (12a + 12b) + 3c + 2d$
5	Area of inscribed triangle	$(0.75\sqrt{3}) r^2$	$= 6a$	$= (12a + 12b) - 21.375c - 6.75d$
6	Area of circumscribed triangle	$(3\sqrt{3}) r^2$	$= 24a$	$= (12a + 12b) + 22.5c + 9d$
7	Area of inscribed square	$2r^2$	$= (8a + 8b)$	$= (12a + 12b) - 12c - 4d$
8	Area of circumscribed square	$4r^2$	$= (16a + 16b)$	$= (12a + 12b) + 12c + 4d$

Using either exact or derived equations we get same answer in every example. [... $(12a + 12b) = \dots (3r^2)$]

S. R.	(Ins. Hexagon)	+ Cir. Hexagon	+ Ins. Triangle	+ Cir. Triangle	+ Cir. Dodecagon	+ Ins. Square	+ Cir. Square)	= ... Ins. Dodd. = ... $(3r^2 = 12a + 12b)$
1				(4	+ 1	+ 9)		$= 14(12a + 12b)$
2		(3		+ 2	+ 1	+ 6)		$= 12(12a + 12b)$
3	(2			+ 3	+ 1	+ 6)		$= 12(12a + 12b)$
4		(6			+ 1	+ 3)		$= 10(12a + 12b)$
5	(4			+ 2	+ 1	+ 3)		$= 10(12a + 12b)$
6	(1		+ 2	+ 3	+ 1	+ 3)		$= 10(12a + 12b)$
7	(2	+ 3		+ 1	+ 1	+ 3)		$= 10(12a + 12b)$
8	(6			+ 1	+ 1			$= 8(12a + 12b)$
9			(4	+ 3	+ 1			$= 8(12a + 12b)$
10	(4	+ 3			+ 1			$= 8(12a + 12b)$
11	(3		+ 2	+ 2	+ 1			$= 8(12a + 12b)$
12	(1	+ 3	+ 2	+ 1	+ 1			$= 8(12a + 12b)$
13	(8				+ 1		+ 3)	$= 12(12a + 12b)$
14	(3	+ 3	+ 2		+ 1		+ 3)	$= 12(12a + 12b)$
15	(5		+ 2	+ 1	+ 1		+ 3)	$= 12(12a + 12b)$
16	(2		+ 4	+ 2	+ 1		+ 3)	$= 12(12a + 12b)$
17		(3	+ 4	+ 1	+ 1		+ 3)	$= 12(12a + 12b)$
18	(7		+ 2		+ 1		+ 6)	$= 16(12a + 12b)$
19	(4		+ 4	+ 1	+ 1		+ 6)	$= 16(12a + 12b)$
20	(2	+ 3	+ 4		+ 1		+ 6)	$= 16(12a + 12b)$
21	(1		+ 6	+ 2	+ 1		+ 6)	$= 16(12a + 12b)$
22	(6		+ 4		+ 1		+ 9)	$= 20(12a + 12b)$
23			+ 8	+ 2	+ 1		+ 9)	$= 20(12a + 12b)$
24	(1	+ 3	+ 6		+ 1		+ 9)	$= 20(12a + 12b)$
25	(3		+ 6	+ 1	+ 1		+ 9)	$= 20(12a + 12b)$
26	(5		+ 6		+ 1		+ 12)	$= 24(12a + 12b)$
27		+ 3	+ 8		+ 1		+ 12)	$= 24(12a + 12b)$
28	(2		+ 8	+ 1	+ 1		+ 12)	$= 24(12a + 12b)$
29	(4		+ 8		+ 1		+ 15)	$= 28(12a + 12b)$
30	(1		+ 10	+ 1	+ 1		+ 15)	$= 28(12a + 12b)$
31	(3		+ 10		+ 1		+ 18)	$= 32(12a + 12b)$
32			+ 12	+ 1	+ 1		+ 18)	$= 32(12a + 12b)$
33	(2		+ 12		+ 1		+ 21)	$= 36(12a + 12b)$
34	(1		+ 14		+ 1		+ 24)	$= 40(12a + 12b)$
35			(16		+ 1		+ 27)	$= 44(12a + 12b)$

s. r.	figures	Area	Exact equations	Derived equations
1	Area of inscribed dodecagon	$3r^2$	$= (12a + 12b)$	$(12a + 12b)$
2	Area of inscribed square	$2r^2$	$= (8a + 8b)$	$(12a + 12b) - 12c - 4d$
3	Area of inscribed hexagon	$(1.5\sqrt{3})r^2$	$= 12a$	$(12a + 12b) - 6.75c - 1.5d$
4	Area of inscribed triangle	$(0.75\sqrt{3})r^2$	$= 6a$	$(12a + 12b) - 21.375c - 6.75d$
5	Area of circumscribed square	$4r^2$	$= (16a + 16b)$	$(12a + 12b) + 12c + 4d$
6	Area of circumscribed hexagon	$(2\sqrt{3})r^2$	$= 16a$	$(12a + 12b) + 3c + 2d$
7	Area of circumscribed triangle	$(3\sqrt{3})r^2$	$= 24a$	$(12a + 12b) + 22.5c + 9d$
8	Area of circumscribed dodecagon	$(24 - 12\sqrt{3})r^2$	$= 96b$	$(12a + 12b) + 18c$

From above table using exact equations or derived equations or putting value of area we get appropriate answer in every example.

Sum of each row Area of circle = ... $(17 - 8\sqrt{3})r^2$ = ... $(4a + 68b)$ = ... $(12a + 12b + 12c)$							
Sr. no.	(Ins. Dodd.)	+ Cir. square	+ Ins. Square	+ Cir. Hex.	+ Ins. Hex.	+ Cir. Dodd.)	= Total ... $(17 - 8\sqrt{3})r^2$
1	(1)					+ 2) =	$3(17 - 8\sqrt{3})r^2$
2		(1)	+ 1			+ 4) =	$6(17 - 8\sqrt{3})r^2$
3			(1)	+ 2		+ 7) =	$10(17 - 8\sqrt{3})r^2$
4		(2)	+ 2			+ 8) =	$12(17 - 8\sqrt{3})r^2$
5	(1)	+ 2	+ 2			+ 10) =	$15(17 - 8\sqrt{3})r^2$
6		(1)	+ 2	+ 2		+ 11) =	$16(17 - 8\sqrt{3})r^2$
7	(1)	+ 1	+ 2	+ 2		+ 13) =	$19(17 - 8\sqrt{3})r^2$
8		(1)		+ 1	+ 4	+ 14) =	$20(17 - 8\sqrt{3})r^2$
9	(3)	+ 2	+ 2			+ 14) =	$21(17 - 8\sqrt{3})r^2$
10	(1)	+ 1		+ 1	+ 4	+ 16) =	$23(17 - 8\sqrt{3})r^2$
11				+ 3	+ 4	+ 17) =	$24(17 - 8\sqrt{3})r^2$
12	(4)	+ 2	+ 2			+ 16) =	$24(17 - 8\sqrt{3})r^2$
13	(8)					+ 16) =	$24(17 - 8\sqrt{3})r^2$
14	(3)	+ 1	+ 2	+ 2		+ 17) =	$25(17 - 8\sqrt{3})r^2$
15	(2)	+ 1		+ 1	+ 4	+ 18) =	$26(17 - 8\sqrt{3})r^2$
16	(5)	+ 2	+ 2			+ 18) =	$27(17 - 8\sqrt{3})r^2$
17		+ 3	+ 4	+ 2		+ 19) =	$28(17 - 8\sqrt{3})r^2$
18	(5)	+ 1	+ 2	+ 2		+ 21) =	$31(17 - 8\sqrt{3})r^2$
19	(4)	+ 1		+ 1	+ 4	+ 22) =	$32(17 - 8\sqrt{3})r^2$
20	(6)	+ 1	+ 2	+ 2		+ 23) =	$34(17 - 8\sqrt{3})r^2$
21	(5)	+ 1		+ 1	+ 4	+ 24) =	$35(17 - 8\sqrt{3})r^2$
22	(4)			+ 3	+ 4	+ 25) =	$36(17 - 8\sqrt{3})r^2$
23	(6)	+ 1		+ 1	+ 4	+ 26) =	$38(17 - 8\sqrt{3})r^2$
24	(5)			+ 3	+ 4	+ 27) =	$39(17 - 8\sqrt{3})r^2$
25	(8)	+ 1	+ 2	+ 2		+ 27) =	$40(17 - 8\sqrt{3})r^2$
26	(1)	+ 1		+ 4	+ 8	+ 33) =	$47(17 - 8\sqrt{3})r^2$
27	(1)	+ 3	+ 2	+ 10	+ 16	+ 75) =	$107(17 - 8\sqrt{3})r^2$

For example no. 1

Area of 1 inscribed dodecagon + area of 2 circumscribed dodecagon

$$= 1(3r^2) + 2[12(2 - \sqrt{3})]r^2 = [3 + (48 - 24\sqrt{3})]r^2$$

$$= (51 - 24\sqrt{3})r^2 = 3(17 - 8\sqrt{3})r^2$$

$$= (12a + 12b) + 2(96b) = (12a + 204b) = 3(4a + 68b)$$

$$= (12a + 12b) + 2(12a + 12b + 18c) = (36a + 36b + 36c) = 3(12a + 12b + 12c)$$

V. CONCLUSIONS

Exact Area of circle = $(12a + 12b + 12c) = (4a + 68b) = (17 - 8\sqrt{3}) r^2$

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