Fuzzy Regular Ternary Sub Semi Group of a Partial Ordered Ternary Semi Group and Fuzzy Simple Partial Ordered Ternary Semi Groups

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Abstract

In this paper the terms, fuzzy regularpo ternarysubsemigroup,λ-cut, fuzzy left regularTernary posubsemigroup, fuzzy right regularpoternary subsemigroup, fuzzy Intra regular po ternary subsemigroup, fuzzy completely regularpo ternary subsemigroup, ideal generated by the ordered fuzzy point a_λ, fuzzypoternarysemisimple, fuzzy left simplepoternarysemigroup, fuzzy right simplepoternary semigroup, fuzzy simplepoternary semigroup, fuzzy globally idempotent, maximal fuzzy ideal in a po ternarysemigroup are introduced. It is proved that, if f is fuzzy regular POternary subsemigroup of T then f is fuzzy idempotent. It is proved that, If f is fuzzy completely regular then f is regular, left regular and right regular. It is proved that, If a_λ is fuzzy regular then a_λ is fuzzy semisimple. It is proved that, If an ordered fuzzy point a_λ of T is left(lateral ,right) regular poternarysemigroup then a_λ is fuzzy semisimple. It is proved that, If a_λ is fuzzy intra regularPOternary semigroup then a_λ is fuzzy semisimple. It is also proved that$f_{(tta]}, f_{(att]}$are fuzzy left and fuzzy right ideals of T respectively then T is a fuzzy left (right) simple poternarysemigroup if and only if $f_{(tta]} = f_t = Tf_{(att]} = f_t = T)\ \forall a \in T$. It is proved that for anypo ternary semigroup T the following are equivalent(.a)Tis a left(lateral, right) simple po ternarysemigroup(b) T is a fuzzy left(lateral, right) simple po Ternary semigroup. Finally we proved that if T is a po ternarysemigroup with unity e then the union of all Proper fuzzy idealsofTistheuniquefuzzymaximalidealof T.

MATHEMATICAL SUBJECT CLASSIFICATION (2010) :20M07; 20M11; 20M12.

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I. INTRODUCTION

The algebraic theory of semigroups was widely studied by CLIFFORD [2],[3], PETRICH [5] and LJAPIN[4]. The ideal theory in general semigroups was developed by ANJANEYULU [1]. In this paper we introduce the notions of fuzzy regular element, fuzzy completely regular element, fuzzy intra regular element in a poternarysemigroup, fuzzy simplePo ternary semigroup and characterize fuzzy simplePo ternary semigroup. L ZADEH introduced the notion of fuzzy subset of a set in 1965.Since then,a series of research on fuzzy sets results fuzzy logic,fuzzy set theory,fuzzy algebra etc.ROSENFELD is the father of fuzzy abstract algebra. Kuroki and Xie are developed fuzzy theory ofsemigroups. In this paper we introduce the notions of fuzzy regular Po ternarysubsemigroup, fuzzy completely regularPo ternary subsemigroup,fuzzy Intra regular Po ternary subsemigroup and relates the fuzzy left simple, fuzzy right simple, fuzzy lateral simple and fuzzy simple Po Ternarysemigroup.

II. PRELIMINARIES

Definition 2.1: [5] A semigroup T with an ordered relation $\leq$ is said to be poternarysemigroupif T is a partially ordered set such that $a \leq b \Rightarrow aa_1a_2 \leq ba_1a_2, a_1a\,a_2 \leq a_1ba_2, a_1a_2\,a \leq a_1a_2b$ for all $a, b, a_1a_2 \in T$.

Definition 2.2: A function f from T to the closed interval [0,1] is called a fuzzy subset of T. The po ternarysemigroup T itself is a fuzzy subset of T such that T(x)=1, $\forall x \in T$. It is denoted by T or 1.

Definition 2.3: Let A be a non-empty subset of T. We denote f_A, the characteristic mapping of A. i.e., The mapping of T into [0,1] defined by $f_A(x)=\begin{cases}1 & \text{if } x \in A\\ 0 & \text{if } x \notin A\end{cases}$ Then f_A is a fuzzy subset of T.

Definition 2.4: Let f,g,h be fuzzy subsets of poternarysemigroup T. Then the inclusion relation $f\subseteq g\subseteq h$ is defined by $f(x)\leq g(x)\leq h(x)$, $\forall$ x. ∈T and $f\cup g\cup h$,$f\cap g\cap h$ are defined by

$(f\cup g\cup h)(x) = \max\{f(x), g(x), h(x)\} = (x)\vee g(x)\vee h(x)$,
$(f\cap g\cap h)(x) = \min\{f(x), g(x), h(x)\} = f(x)\wedge g(x)\wedge h(x)$ $\forall x\in T$

Definition 2.5: Let $(T,\leq)$ be a poternarysemigroup and f,g,h be fuzzy subsets of T. For $x\in T$ theproduct fogoh is defined by $(fogoh)(x) = \begin{cases}\vee_{x\leq pqr} f(p)\wedge g(q)\wedge h(r) & \text{if } x\leq pqr \text{ exists}\\ 0 & \text{otherwise}\end{cases}$

Definition 2.6: Let T be a po ternarysemigroup. For $H\subseteq T$
we define $(H]=\{t\in T / t\leq h$ forsome $h\in H\}$. For $H=\{a\}$ we write $(a]= (\{a\}] = \{ t\in T / t\leq a\}$

Definition 2.7: A fuzzy subset f of a po ternarysemigroup T is called fuzzyternary subsemigroupof T if $f(xyz)\geq f(x)\wedge f(y)\wedge f(z)\forall x, y, z\in T$.

Proposition 2.8:[9] A fuzzy subset f of a po ternarysemigroup T is fuzzyternary subsemigroup of T$\Leftrightarrow$ fof of $\subseteq$ f.

Definition 2.9: A fuzzy subset f of a po ternary semigroup T is called fuzzy poternarysubsemigroup of T if (i) $x\leq y$ then $f(x)\geq f(y)$ (ii) $f(xyz)\geq f(x)\wedge f(y)\wedge f(z)$, $\forall x, y,z\in T$.

Definition 2.10:[11]LetT be a poternarysemigroup. A fuzzy subset f of T is called a fuzzypo left idealof T if (i) $x\leq y$ then $f(x)\geq f(y)$ (ii) $f(xyz)\geq f(z)$, $\forall$ x,y, z $\in T$

Lemma 2.11: [10] Let T be a poternarysemigroup and f be a fuzzy subset of T. Then f is a fuzzypo left ideal of T if and only if f satisfies that (i) $x\leq y$ then $f(x)\geq f(y)$ $\forall$ x,y, z ∈T (ii) Tofof$\subseteq$f.

Definition 2.12: [11]Let T be a poternarysemigroup. A fuzzy subset f of T is called a fuzzy poright idealof T if (i) $x\leq y$ then $f(x)\geq f(y)$ (ii) $f(xyz)\geq f(x)$, $\forall$ x,y,z ∈T.

Lemma 2.13: [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy right ideal of T if and only if f satisfies that (i) $x\leq y$ then $f(x)\geq f(y)$ $\forall$ x,y, z ∈T (ii)fofoT$\subseteq$ f.

Definition 2.14: [11]Let T be a poternarysemigroup. A fuzzy subset f of T is called a fuzzy polateral idealof T if (i) $x\leq y$ then $f(x)\geq f(y)$ (ii) $f(xyz)\geq f(y)$, $\forall$ x,y, z ∈T
Lemma 2.15: [10] Let T be a poternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy lateral ideal of T if and only if f satisfies that (i) $x\leq y$ then $f(x)\geq f(y)$ $\forall$ x,y, z ∈T

(ii)foTof$\subseteq$f.

Definition 2.16: [11]Let T be a poternarysemigroup. A fuzzy subset f of Tis called a fuzzy idealofT if (i) $x\leq y$ thenf(x)≥f(y)(ii)f(xyz)≥f(z),(iii)f(xyz)≥f(x),f(xyz)≥ f(y) ∀x,,y, z ∈T.
Lemma 2.17: [10] Let T be a poternarysemigroup and f be a fuzzy subset of T. Then f is a fuzzy ideal of Tif and only if f satisfies that (i) $x\leq y$ then $f(x)\geq f(y)$ $\forall$ x,y, z ∈T (ii)fofoT$\subseteq$ f and Tofof$\subseteq$f andfoTof$\subseteq$f**.**

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Definition 2.18: Let T be a poternarysemigroup. A fuzzy ideal f of T is called idempotentif $f^3 = fofof = f$.

III.SPECIAL ELEMENTS IN FUZZY SEMIGROUP OF PO TERNARY SEMIGROUP

Definition 3.1: An element a of a poternarysemigroup T is said to be regula**r**if there exist $x\in T$such that

$a\leq axaxa$.
Definition 3.2**:** A poternarysemigroup T is said to be regular po ternary semigroupprovided every element is regular.
Definition 3.3: Let T be00 a poternarysemigroup and x∈T. Define $R_x = \{x'/x'\in T, x\leq xx'xx'x\}$. Let f be a fuzzy subsemigroup of T if $\forall x\in T$ there exist $x'\in R_x$ such that

$f(x)\leq f(x')$ provided $f(x)\neq 0$ then f is called fuzzy regular ternary subsemigroup of T.

Theorem 3.4:Let A be a non-empty subset of a poternarysemigroupT . A is regular ternary subsemigroup ofT if and only if f_A the characteristic function of A is a fuzzy regular ternary subsemigroup of T.
Proof: Suppose A is regular ternary subsemigroup of T and, $x, y, z\in A\Rightarrow xyz\in A$
Therefore $f_A(xyz)\geq f_A(x)\wedge f_A(y)\wedge f_A(z)$.

Let $x \in A \Rightarrow f_A(x) = 1$. Then from the regularity of A there exists $x' \in R_x$ such that $x' \in A$ and $f_A(x') = 1. \Rightarrow f_A(x) \leq f_A(x')$.

Therefore f_A is a fuzzy regular ternary subsemigroup of T.
Conversly suppose that f_A is a fuzzy regular ternary subsemigroup of T.
Let $x, y, z \in A \Rightarrow f_A(x) = f_A(y) = f_A(z) = 1$.

since f_A is fuzzy regular ternary subsemigroup of T.
$\Rightarrow f_A(xyz) \geq f_A(x) \wedge f_A(y) \wedge f_A(z) = 1 \Rightarrow f_A(xyz) = 1 \Rightarrow xyz \in A$

$\Rightarrow$ A is a regular ternary subsemigroup of T.
Let $x \in A \Rightarrow f_A(x) = 1$.

Since f_A is a fuzzy regular, there existsx$' \in R_x$ such that $f(x) \leq f_A(x')$.

$\Rightarrow f_A(x') \geq 1 \Rightarrow x' \in A$.Therefore A is regular ternary subsemigroup of T.

Definition 3.5: Let f be a fuzzy subset of a po ternary semigroup T. Let $\lambda \in [0,1]$.

Define$f_\lambda = \{x \in T / f(x) \geq \lambda\}$ is the λ-cut of f.

Theorem 3.6: Let T be a po ternarysemigroup. f is fuzzyternary subsemigroup of T if and only if$\forall \lambda \in [0,1]$, f_λ is a ternarysubsemigroup of T.
Proof: Assume that f is a fuzzyternary subsemigroup of T

. Let $a, b, c \in f_\lambda \Rightarrow f(a) \geq \lambda, f(b) \geq \lambda, f(c) \geq \lambda$
Since $f(abc) \geq f(a) \wedge f(b) \wedge f(c) \geq \lambda \wedge \lambda \wedge \lambda = \lambda \Rightarrow f(abc) = \lambda \Rightarrow abc \in f_\lambda$
$\Rightarrow f_\lambda$is ternary subsemigroup of T.
Converselyif possible suppose that f_λ is aternary subsemigroup of T. Suppose there exists one such that $f(x_1y_1z_1) < f(x_1) \wedge f(y_1) \wedge f(z_1) >< f(x_1) \Rightarrow f(x_1y_1z_1) < f(x_1)$
Define $\lambda = \frac{1}{2}[f(x_1) - f(x_1y_1z_1)]$ then $\lambda \in (0,1]$and $0 \leq f(x_1y_1z_1) < \lambda \leq 1, 0 < \lambda < f(x_1) \leq 1$.so that $x_1 \in f_\lambda$,similarly $y_1 \in f_\lambda$and $z_1 \in f_\lambda \Rightarrow x_1y_1z_1 \in f_\lambda \Rightarrow f(x_1y_1z_1) \geq \lambda$
But $f(x_1y_1z_1) < \lambda$ which is contradiction.
Therefore $f(x_1y_1z_1) \geq f(x_1) \wedge f(y_1) \wedge f(z_1)\ \forall x_1, y_{,1}\, z_1 \in T$

$\Rightarrow f_\lambda$isternary subsemigroup of T.

Theorem 3.7: Let f be a fuzzy subset of a po ternarysemigroup T. If f is a fuzzy regularternarysubsemi-group of T if and only if $\forall \lambda \in (0,1]$, f_λ is a regularternary subsemigroup of Tprovided$f_\lambda \neq \emptyset$.
Proof: Assume that f is fuzzy regular ternary subsemigroup of T.

From theorem 3.6, f_λ is ternary subsemigroup of T.

Let $x \in f_\lambda$ since f is fuzzy regular $\exists\, x' \in R_x$ such that $f(x) \leq f(x') \Rightarrow f(x') \geq \lambda$
$\Rightarrow x' \in f_\lambda$.Therefore $\forall x \in f_\lambda\ \exists\, x' \in f_\lambda$ such that $x \leq xx'xx'x$

$\Rightarrow f_\lambda$ is a regular subsemigroup of T.
Conversely, suppose that f_λ is a regular subsemigroup of T provided $f_\lambda \neq \emptyset$
Assume that f is not fuzzy regular

$\Rightarrow$ there exists $x \in T$ such that $f(x) \neq 0$ and $\forall x' \in R_x$, $f(x) > f(x')$

let $\lambda = f(x)$, clearly $x \in f_\lambda$ and $\forall x' \in R_x \Rightarrow \lambda = f(x) > f(x') \Rightarrow x' \notin f_\lambda$
which is a contradiction since f_λ is regular.

Therefore f is fuzzy regular ternary subsemigroup of T.

Theorem 3.8: Let Tbe a po ternarysemigroup. If f is fuzzy regular ternary subsemigroup of T thenfofof = f.
Proof: Let f be a fuzzy subset of T.From [6] f is a fuzzy ternary subsemigroup of T if and only iffofof $\subseteq$ f.

Let $x \in T$

if $f(x) \neq 0$ then $\exists\, x' \in R_x$ such that $f(x) \leq f(x')$ since f is fuzzy regular.
Now $(fofof)(x) = {}_{x \leq pqr}{}^{\vee}[f(p) \wedge f(q) \wedge f(r))] ={}_{x \leq x\,x'\,xx'\,x}{}^{\vee}[f(x\,x'\,x) \wedge f(x'\,x)]$
if f(x)=0 then $(fofof)(x) \leq f(x) \Rightarrow (fofof)(x) = f(x) = 0$

$$\geq f(x\,x'x) \wedge f(x'x)$$
$$\geq f(x\,x') \wedge f(x') \wedge f(x) = f(x), \forall x \in T$$

$\Rightarrow f \subseteq fofof$. Therefore $fofof = f$ if f is fuzzy regular.

Corollary 3.9: Let T be a po ternary semigroup and f is fuzzy ideal ofT. If f is fuzzy regular ternary subsemigroup of T thenfis fuzzy idempotent**.**

Definition 3.10: Letfbe a fuzzy subset of a po ternarysemigroup T.

We define (**f**]by$(f](x) = \bigvee_{x \leq y} f(y), \forall x \in T$.

Note 3.11: Clearly$f \subseteq (f]$.

Definition 3.12: Let T be a poternarysemigroup, $a \in T$ and $\lambda \in [0,1]$. An ordered fuzzy point a_λ,$a_\lambda : T \to [0,1]$ defined by $a_\lambda(x) = \begin{cases} \lambda \text{ if } x \in (a] \\ 0 \text{ if } x \notin (a] \end{cases}$

clearlya_λ is a fuzzy subset of T.

Lemma 3.13: If a_λ is an Ordered Fuzzy Point Of T Then $a_\lambda=(a_\lambda]$.

Note 3.14: Let f be a fuzzy subset of an po ternarysemigroup T then $(f] = \bigcup_{y_\lambda \in (f]} y_\lambda$.

Definition 3.15: Let f be a fuzzy subset of a po ternarysemigroupT. Then$\forall x \in T$, thefuzzy subset xfxfx of T is defined by $\forall y \in T$, $(xfxfx)(y) = \begin{cases} \bigvee_{y \leq xtxtx} f(s) \text{ if } \exists y \leq xtxtx \\ 0 \text{ otherwise} \end{cases}$

Theorem 3.16: If f is a fuzzy ternary subsemigroup of poternarysemigroup S. Then f is fuzzy regular if and only if$\forall x \in T$, $(xfxfx)(x) \geq f(x)$ provided $f(x) \neq 0$

Proof: Suppose f is fuzzy regular.

Consider $(xfxfx)(x) = \bigvee_{x \leq xtxtx} f(T)$

$= \bigvee_{x \leq xx'x'x} f(x') \geq f(x') \geq f(x)$,

since f is fuzzy regular $\Rightarrow (xfxfx)(x) \geq f(x) \forall x \in T$

conversely assume that $\forall x \in T$, $(xfxfx)(x) \geq f(x)$.

Since $f(x) \leq (xfxfx)(x) = \bigvee_{x \leq xx'xx'x} f(x') = \max_{x \leq xx'xx'x} f(x')$ provided $f(x) \neq 0$

That is $\forall x \in T\ \exists$ at least one x' such that $x \leq xx'xx'x$ and $f(x) \leq f(x')$ since $f(x) \neq 0$

$\Rightarrow$ f is fuzzy regular.

Corollary 3.17: If an ordered fuzzy point a_λ of a po ternary semigroup T is regular if and onlyif $\forall x \in T$,$(xa_\lambda xa_\lambda x)(x) \geq a_\lambda(x)$provided $a_\lambda(x) \neq 0$.

Proof: Proof follows from Theorem 3.14.

Definition 3.18: LetT be a poternarysemigroupand $x \in T$, define $LR_x = \{x' \in T / x \leq x^3 x' x'\}$. Let f be a fuzzy ternary subsemigroup of T. For every $x \in T \exists\, x' \in LR_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is called fuzzy left regularternary subsemigroupof T.

Definition 3.19: Let T be a poternarysemigroup and $x \in T$, define $RR_x = \{x' \in T / x \leq x' x' x^3\}$. Let fbe a fuzzy ternary subsemigroup of T. For every $x \in T\ \exists\, x' \in RR_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is called fuzzy right regularternary subsemigroupof T.

Definition 3.20: Let T be a poternarysemigroup and $x \in T$, define $MR_x = \{x' \in T / x \leq x' x^3 x'\}$. Let fbe a fuzzy ternary subsemigroup of T. For every $x \in T \exists\, x' \in RR_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is called fuzzy lateral regular ternary subsemigroupof T.

Definition 3.21: Let T be a po ternarysemigroup and $x \in T$, define $IR_x = \{x' \in T / x \leq x_1 x^3 x_2\}$. Letf be a fuzzy ternary subsemigroup of T. For every $x \in T\ \exists\, (x_1, x_2) \in IR_x \ni f(x) \leq f(x_1) \wedge f(x_2)$ provided $f(x) \neq 0$ then f is called fuzzy Intraregular ternary subsemigroupof T.

Definition 3.22: LetT be a po ternarysemigroup and $x \in T$, define $R_x = \{x'/x' \in T, x \leq xx'xx'x\}$and $C_x = \{x' \in T / xx'x = x'xx = xxx'\}$.Let f be a fuzzy subsemigroup of T and for every $x \in T\ \exists\, x' \in R_x \cap C_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$

then f is calledfuzzy completely regularternary subsemigroupof T.

Theorem3.23: Let T be a po ternarysemigroupand f is a fuzzy ternary subsemigroup of T. If f is fuzzy completely regular then f is lateral regular, left regular and right regular.

Proof: Suppose that f is completely regular. Then for every $x \in S\ \exists x' \in R_x \cap C_x$ $\ni f(x) \leq f(x')$ provided $f(x) \neq 0$. Since $x' \in R_x \Rightarrow x \leq xx'xx'x$ and $f(x) \leq f(x')$, $f(x) \neq 0$

$\Rightarrow$ f is fuzzy regular.

Now for every $x \in T\ \exists x' \in R_x \cap C_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$.

$\Rightarrow\ x \leq xx'xx'x$ and $xx'x = x'xx = xxx'$.

Consider $x \leq xx'xx'x = xx'xxx' = xxxx'x' = x^3(x')^2 \Rightarrow x \leq x^3(x')^2$ and $f(x) \leq f(x')$

$\Rightarrow$ f is fuzzy left regular.

Also$x \leq xx'xxx' = x'xx'xx = (x')^2x^3$. Therefore f is fuzzy right regular.

Similarly it is fuzzy lateral regular.

Definition 3.24: Let$a \in T, \lambda \in (0,1]$. Define ideal generated by the ordered fuzzy point a_λ of Tby $< \mathbf{a_\lambda} > (\mathbf{x}) = \begin{cases} \lambda \text{ if } x \in (a) = (a \cup aaT \cup aTa \cup Taa \cup TaTaT] = (t'at'at'] \\ 0 \quad\quad \text{otherwise} \end{cases}$

Definition 3.25: An ordered fuzzy element $\mathbf{a_\lambda}$of a po ternarysemigroup T is said to be fuzzy semisimpleif $a_\lambda \subseteq < a_\lambda >^3$.

Note 3.26: Clearly $a_\lambda \subseteq < a_\lambda >$

Theorem 3.27: Let T be a poternary Semi group and a_λ is a fuzzy ternary subsemigroup of T. If a_λ is fuzzy regular then a_λ is fuzzy semisimple.

Proof: Suppose a_λ isfuzzy regular.

$$
\begin{aligned}
\text{Consider } < a_\lambda >^3(x) &= \vee_{x \leq pqr}[< a_\lambda(p) > \wedge a_\lambda(q) > \wedge < a_\lambda(r) >] \\
&\geq a_\lambda(p) \wedge a_\lambda(q) \wedge a_\lambda(r) \\
&\geq a_\lambda(xx'x) \wedge a_\lambda(x') \wedge a_\lambda(x) \\
&\geq a_\lambda(x) \wedge a_\lambda(x') \wedge a_\lambda(x) \wedge a_\lambda(x') \wedge a_\lambda(x) \\
&\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) = a_\lambda(x) \forall x \in T
\end{aligned}
$$

$\Rightarrow a_\lambda \subseteq < a_\lambda >^3$

Therefore a_λ is fuzzy semisimple.

Theorem 3.28: Let T be a po ternary Semi group. If an ordered fuzzy point a_λ ofT is left(right,lateral) regular ternary semigroup then a_λ is fuzzy semisimple.

Proof: Suppose $a_\lambda$is fuzzy left regular. Then $\forall\ x \in T\ \exists\ x' \in LR_x$ such that $a_\lambda(x) \leq a_\lambda(x')$ provided $a_\lambda(x) \neq 0$.

$$
\begin{aligned}
\text{Consider } < a_\lambda >^3(x) &= \vee_{x \leq pqr}[< a_\lambda(p) > \wedge a_\lambda(q) > \wedge < a_\lambda(r) >] \\
&\geq a_\lambda(p) \wedge a_\lambda(q) \wedge a_\lambda(r) \\
&\geq < a_\lambda > (x^3) \wedge < a_\lambda > (x') > \wedge < a_\lambda > (x') \\
&\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x') \wedge a_\lambda(x') \\
&\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) = a_\lambda(x)
\end{aligned}
$$

$\Rightarrow a_\lambda \subseteq < a_\lambda >^3$

Therefore a_λ is fuzzy semisimple.

Similarly if a_λ is fuzzy right (lateral) regular ternary semigroup then a_λ is fuzzy semisimple.

Theorem 3.29: Let a_λ be an ordered fuzzy point of a po ternarysemigroup T. If a_λ is fuzzy intra regular ternary semigroup then a_λ is fuzzy semisimple.

Proof: Let $a_\lambda$is fuzzy intra regular.

$\Rightarrow \forall\ x \in S\ \ \exists\ (x_1, x_2) \in IR_x \ni a_\lambda(x) \leq a_\lambda(x_1) \wedge a_\lambda(x_2)$ provided $a_\lambda(x) \neq 0$.

Consider $< a_\lambda >^3(x) = (< a_\lambda > o < a_\lambda > o < a_\lambda >)(x)$

$$
\begin{aligned}
&= \vee_{x \leq pqr}[< a_\lambda(p) > \wedge a_\lambda(q) > \wedge < a_\lambda(r) >] \\
&\geq < a_\lambda(p) > \wedge a_\lambda(q) > \wedge < a_\lambda(r) > \\
&\geq < a_\lambda(x_1) > \wedge < a_\lambda > (x^3) \wedge < a_\lambda > (x_2) \\
&\geq \mathbf{a_\lambda(x_1)} \wedge \mathbf{a_\lambda(x)} \wedge \mathbf{a_\lambda(x)} \wedge \mathbf{a_\lambda(x)} \wedge \mathbf{a_\lambda(x_2)}
\end{aligned}
$$

$=a_\lambda(x), \forall x \in S$

$$\Rightarrow a_\lambda \subseteq < a_\lambda >^3$$

Therefore a_λ is fuzzy semisimple.

IV. FUZZY SIMPLE PARTIALLY ORDERED TERNARY SEMIGROUPS

Definition 4.1: A po ternarysemigroup Tis said to be a left simple ternary semigroupif T is its only left ideal.
Definition 4.2: Let Tbe a po ternarysemigroup. T is called fuzzy left simple ternary semigroupif every fuzzy left ideal of T is a constant function.

Definition4.3: Let T be a po ternary semigroup and f be a fuzzy subset of T. Define

$$f_{(TTa]}(x) = \begin{cases} 1 \text{ if } x \in (TTa] \\ 0 \text{ otherwise} \end{cases}$$

Theorem 4.4: Let T be a po ternarysemigroup. Then $f_{(TTa]}$ is a fuzzy left ideal of T, for every $a \in T$
Proof: (i) Let $x, y, z \in T$ and $x \leq y, y \leq z$
If $y \in (TTa]$ since $x \leq y \Rightarrow x \in (TTa]$ then $f_{(TTa]}(x) = f_{(TTa]}(y) = f_{(TTa]}(z) = 1$
If $y \notin (TTa]$then $f_{(TTa]}(y) = 0 \leq f_{(TTa]}(x)$.
By summarizing the above $f_{(TTa]}(x) \geq f_{(TTa]}(y)$
(ii) If $y \notin (TTa]$then $f_{(TTa]}(y) = 0 \leq f_{(TTa]}(xyz)$.
If $y \in (TTa]$ then $f_{(TTa]}(y) = 1$
Since $y \in (TTa]$ and $(TTa]$ is a left ideal of S from [8]
then $xyz \in (TTa], \forall x \in T$.

$$\Rightarrow f_{(TTa]}(xyz) = 1 = f_{(TTa]}(z)$$

Therefore$(xyz) \geq f_{(TTa]}(z)$.
From (i) and (ii) $f_{(TTa]}$ is a fuzzy left ideal of T.

Theorem 4.5: For a po ternarysemigroup T, the following are equivalent.
a) T is a left simple po ternarysemigroup
b) T is a fuzzy left simple ternary semigroup**.**
Proof:(a) ⇒(b):
Suppose T is a left simple po ternarysemigroup.
Let f be any fuzzy left ideal ofT. Then it follows from [7] that there exist elements $x, y, z \in T$ such that b=xaa and a=ybb.
Since f is a fuzzy left ideal of T, $f(a) = f(ybb) \geq f(b) = f(xaa) \geq f(a)$
$\Rightarrow f(a) = f(b)\ \ \forall a, b \in T \Rightarrow$ f is a constant fuzzy ideal.
Therefore T is a fuzzy left simple po ternarysemigroup.
(b)⇒(a):
Assume thatT is a fuzzy left simple poternarysemigroup.
Let A be any po left ideal of T. Then from [9] C_A is a fuzzy left ideal of T.
$\Rightarrow C_A$ is a constant function.
Let $x \in T$. Since $A \neq \emptyset$, $C_A(x) = 1 \Rightarrow x \in A \Rightarrow T \subseteq A$
ThereforeA=T.
HenceT is a left simple po ternarysemigroup.

Theorem 4.6: Let T be a po ternarysemigroup. Tis a fuzzy left simple ternary semigroup if and only if

$$f_{(TTa]} = f_T = T\ \forall a \in T$$

Proof: Assume that Tis a fuzzy left simple po ternarysemigroup. By Theorem 4.5 S is a left simple po ternarysemigroup. Then by [5], (TTa]=T.

Therefore$f_{(TTa]} = f_S = T$.
Conversely assume that$f_{(TTa]} = f_T = T \Rightarrow f_{(TTa]}(x) = f_T(x)$
⇒ (TTa]=T. Then from [5] Tis a left simple po ternarysemigroup. Then by Theorem 4.5 T is a fuzzy left simple po Ternary semigroup.
Definition 4.7: A po ternarysemigroup T is said to be a right simple ternary semigroupifT is its only po right ideal.
Definition 4.8: Let T be a po ternarysemigroup. T is called fuzzy right simple ternary Semigroupif every fuzzy right ideal of T is a constant function.

Definition 4.9: Let T be a po ternarysemigroup and f be a fuzzy subset of T. Define

$$\mathbf{f}_{(\mathbf{aTT}]}(\mathbf{x}) = \begin{cases} 1 \text{ if } x \in (aTT] \\ 0 \text{ otherwise} \end{cases}$$

Definition 4.10: A po ternarysemigroup T is said to be a fuzzy simple ternary semigroupif every fuzzy ideal of T is a constant function.

Theorem 4.11: LetT be a po ternarysemigroup.Then $f_{(aTT]}$ is a fuzzy right ideal of S for every $a \in T$

Proof: (i) Let $x, y, z \in T$ and $x \leq y , y \leq z$

If $y \in (aTT]$ since $x \leq y \Rightarrow x \in (aTT]$ then $f_{(aTT]}(x) = f_{(aTT]}(y) = f_{(aTT]}(z) = 1$

If $y \notin (aTT\,]$then $f_{(aTT]}(y) = 0 \leq f_{(aTT]}(x)$.

By summarizing the above $f_{(aTT]}(x) \geq f_{(aTT]}(y)$

(ii) If $x \notin (aTT]$then $f_{(aTT]}(x) = 0 \leq f_{(aTT]}(xyz)$.

If $x \in (aTT]$ then $f_{(aTT]}(x) = 1$

Since $x \in (aTT]$ and (aTT] is a po right ideal of T, from Theorem [5]

then $xyz \in (aTT], \forall\, y \in T$.

$$\Rightarrow f_{(aSS]}(xyz) = 1 = f_{(aSS]}(x)$$

Therefore$f_{(aSS]}(xyz) \geq f_{(aTT]}(x)$.

From (i) and (ii) $f_{(aTT]}$ is a fuzzy right ideal of T.

Theorem 4.12: For a po ternarysemigroup T, the following are equivalent.

a) T is a right simple po ternarysemigroup

b) T is a fuzzy right simple ternary semigroup**.**

Proof:(a) ⇒(b):

Suppose T is a right simple po Ternary semigroup.

Let f be any fuzzy right ideal of t. Then it follows from[7] that there exist elements $x, y, z \in T$ such that aax=b and a=bby.

Since f is a fuzzy right ideal of T, $f(a) = f(bby) \geq f(b) = f(aax) \geq f(a)$

$\Rightarrow f(a) = f(b)\ \ \forall a, b \in T \Rightarrow$ f is a constant fuzzy ideal.

Therefore T is a fuzzy right simple po ternarysemigroup.

(b)⇒(a):

Assume that S is a fuzzy right simple poternarysemigroup.

Let A be any po right ideal of T. Then from [9] C_A is a fuzzy right ideal of T.

$\Rightarrow C_A$ is a constant function.

Let $x \in T$ Since $A \neq \emptyset$, $C_A(x) = 1 \Rightarrow x \in A \Rightarrow T \subseteq A$

ThereforeA=T

Hence T is a right simple po ternary semigroup.

Theorem 4.13: Let T be a po ternarysemigroup. T is a fuzzy right simple po ternarysemigroup if and only if$f_{(aTT]} = f_T = T \forall a \in T$

Proof: Assume that T is a fuzzy right simple po ternarysemigroup. By Theorem 4.12T is a right simple po ternarysemigroup. Then from [5], (aTT]=T. Therefore$f_{(aTT]} = f_T = T$.

Conversly assume that$f_{(aTT]} = f_T = T \Rightarrow f_{(aTT]}(x) = f_T(x)$

⇒ (aTT]=T. Then from [5], T is a right simple po ternarysemigroup. Then by Theorem 4.12T is a fuzzy right simpleternary semigroup.

Definition 4.14: Let f,g,h be fuzzy subsets of T, (fogoh]is defined by

$$(fogoh](x) = {}_{x \leq pqr}^{\vee}(fogoh)(pqr), \forall p, q, r \in T$$

Definition4.15: A fuzzy ideal f of a po ternarysemigroup T is said to be globally idempotentif $(f^n] = (f]$ **,** $\forall n$

Definition4.16: Let T be a po ternarysemigroup. T is said to be fuzzy globally idempotentif $(T^n] = T$ **,** $\forall n$.

Theorem 4.17: Let T be a po ternarysemigroup with unity e and f be a fuzzy ideal of T with $f(e) = 1$ then $f = T = f_T$

Proof: Let $x \in T$. Consider$f(x) = f(xee) \geq f(e)=1$

$\Rightarrow f(x) \geq 1 \Rightarrow f(x) = 1, \forall\, x \in T$. Therefore$f = f_T = T$.

Definition4.18: A non-zero fuzzy ideal f of a po ternarysemigroup T is called a proper fuzzy idealif $f \neq C_T = T$.

Definition4.19: A fuzzy ideal f of a poternarysemigroup T is called maximalif there doesn't exist any proper fuzzy ideal g of T∋ $f \subset g$**.**

Theorem 4.20: If$\{f_i\}$ is a fuzzy ideals of a poternary semigroup T then the arbitrary union of fuzzyideals isfuzzy ideal ofT**.**

Proof: let $\{f_i\}$ is a fuzzy ideals of a po ternary semigroup T.
Let $x, y, z \in T$ such that $x \le y, y \le z$.
Consider $\cup f_i(x) = \max\{f_1(x), f_2(x), f_3(x), \dots\dots\dots\dots\}$
$= f_1(x) \vee f_2(x) \vee f_3(x) \vee \dots\dots\dots$
$\ge f_1(y) \vee f_2(y) \vee f_3(y) \dots\dots\dots$ since each f_i is a fuzzy ideal.
$= \max\{f_1(y), f_2(y) \dots\dots\} = \cup f_i(y)$
Therefore $\cup f_i(x) \ge \cup f_i(y)$ if $x \le y$
Consider $\cup f_i(xyz) = f_1(xyz) \vee f_2(xyz) \vee f_3(xyz) \vee \dots\dots..$
$\ge f_1(y) \vee f_2(y) \vee f_3(y) \vee \dots\dots\dots$ since each f_i is a fuzzy lateral ideal.
$= \cup f_i(y)$
Therefore $\cup f_i(xyz) \ge \cup f_i(y)$, Similarly $\cup f_i(xyz) \ge \cup f_i(x)$ $\cup f_i(xyz) \ge \cup f_i(z)$..
Thus the arbitrary union of fuzzy ideals is a fuzzy ideal of T.

Theorem 4.21: Let T be a po ternarysemigroup with unity e then the union of all proper fuzzy ideals of T isthe unique fuzzy maximal ideal of T**.**
Proof: Let $f_M$be the union of all proper fuzzy ideals of T.
$\Rightarrow f_M$is a fuzzy ideal of T by theorem 4.20.
If f_M is not proper then $f_M = C_T \Rightarrow f_M(x) = 1, \forall x \in T$
$\Rightarrow f_i(x) = 1$ for some fuzzy ideal f_i since $\cup f_i = f_M$
$\Rightarrow f_i = f_T$ but f_i is proper.
Therefore f_M is a proper fuzzy ideal of T.
Since f_M contains all proper fuzzy ideals of T.
$\Rightarrow f_M$is maximal fuzzy ideal of T.
If $g_M$is any other maximal fuzzy ideals of Tthen$g_M \subseteq f_M \subseteq C_T$.
Therefore$f_M = g_M$. Hence $f_M$is the unique fuzzy maximal ideal of T.

Theorem 4.22: If T is a fuzzy left(right,ternary) simple ternary semigroup then T is a fuzzy simplesemigroup.
Proof: Suppose T is a fuzzy left(right,ternary) simple semigroup.
Let f be a fuzzy ideal of T⇒ f is a fuzzy left, lateral and right ideal of T.
⇒f is a constant function ⇒T is a fuzzy simple semigroup.

Theorem 4.23: let a_λ be a fuzzy ordered element of a po ternary semigroup T. If a_λ is semisimpleand idempotent then$a_\lambda \subseteq < a_\lambda >^n, \forall$ n.
Proof: If a_λ is semisimple and idempotent .
Let $a \in T$ and n is a natural number.
$\Rightarrow \mathbf{a_\lambda \subseteq < a_\lambda >^3}$is true for n=3 since $a_\lambda$is fuzzy semisimple.
Assume that the statement is true for n-2. That is $a_\lambda \subseteq < a_\lambda >^{n-2}$
Consider $< a_\lambda >^n = < a_\lambda >^{n-2}$ o $< a_\lambda > o < a_\lambda >$
$\supseteq a_\lambda o a_\lambda o a_\lambda = {a_\lambda}^3 = a_\lambda$. since a_λ is idempotent.Therefore $< a_\lambda >^n \supseteq$,

V. CONCLUSION

In this study of poternary subsemigroups of a po ternarysemigroup T we introduced the concepts of fuzzysubsemigroup, regular ternarysubsemigroup,fuzzy regular subsemigroup,fuzzy completely regular,fuzzy semisimple and fuzzysimple of T and proved some relations between them.Hopefully some new results in the these topics can be obtained in the forthcoming papers.

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