

Bird breeding Avian Data: Bayesian Statistical Updating Prediction for the Size of Closed Population on the Species Richness Model Using Generalized Binomial Model with New Mixture Unit Interval Type Prior for Animal Ecology

Jamal A. Al-Saleh¹, and Satish K. Agarwal²

¹Department of Statistics and Operation Research, Faculty of Science,
Kuwait University, P.O. Box 5969, Safat, Kuwait

²Department of Mathematics, College of Science,
University of Bahrain, P.O. Box 32038, Bahrain

Abstract In this article a new way of statistical updating prediction and measuring behavior in the field of animal population dynamics and ecological data science is studied and examined on the species richness model. To estimate the size of closed population, with individual heterogeneity in detection probability, we have introduced and used generalized binomial model with five parameters $GBM(n,p,\pi,\rho,\tau)$ [[1] Al-Saleh et al. (2016)], as an alternative model to the standard Binomial model $SBM(n,p)$. The generalized model is developed by introducing new parameters called indicator parameters. The main advantage of an indicator parameter, is that, it will equip the new generalized model accession, more approaches, and options in statistical estimation for close fitting and measuring behavior of the data. To help with the examination of the data more precisely, we also consider a new mixture unit interval type prior on $(0,1)$ with two parameters $UITP(\alpha,\beta)$, as an alternative model to the uniform prior on $(0,1)$. To be a contrivance, and intermediary updating prediction model to existing model [[2] Royle and Dorazio (2008), Chapter 6], and reproduce more effective, generalized, and propagated model of particular importance for species richness populations, using capture–recapture model. The estimate of the total population size could be affected by the selection of the model suggested to fit the data. Therefore, we propose an expanded, generalized, and extended model with the advantage that the model parameters can be estimated using Bayesian methodology which serves as a subtle resource for model identification and classification. An illustration is provided using species richness model with the application to bird breeding survey (BBS) data set. The generalized posterior summaries using Markov Chain Monte Carlo (MCMC) Gibbs sampling approach, are presented for the new models. The study of the parameters of the new models would help the users to have more clarity and understanding about the role of the existing model. It is found that proposed new models is more resilience, liveness, and is fully adaptive to the available data and gives animal ecologist another option for modelling the data.

Keywords generalized species richness model, Bayesian data analysis, generalized binomial distribution, unit interval type prior

I. INTRODUCTION

Many estimation and prediction problems in statistical data science research inhibit the application of standard statistical models, and preferably expand and/or proclaim to a more generalized and extended model setting [see [3] Al-Saleh and Agarwal (2006), [1] Al-Saleh et al. (2016), and [4] Al-Saleh and Agarwal (2017)]. This presumption and proposition should be fully considered and examined for intelligible animal population ecological data analysis intention and motive. Because estimation and prediction process can be very much affected by the selection of the model considered to fit the data, and more over sensitive to the chosen pattern and paradigm.

In this article we will address the above proposition by introducing an efficient statistical estimation process, and procedure, with a computational updating prediction techniques for the first time in animal population ecology data science. It is done by proposing a new generalized Binomial statistical model, with new mixture unit interval type prior for species richness model for estimating the size of a closed population with individual heterogeneity in detection probability. With this generalization setting in mind, we can find proper and suitable solutions for the data.

Generally, problems involving new generalized and extended statistical models based prediction, are well suited for the Bayesian methodology [see [5] King and Brooks (2001), [6] Royle et al. (2007), [1] Al-Saleh et al. (2016), [4] Al-Saleh and Agarwal (2017), and [7] Lima et al. (2018)]. The motivation of this work is to explore these new generalized statistical models which can be used to provide a righteous, veracious, and adequate fit for the real data, than the usage of standard models. The Markov Chain Monte Carlo (MCMC) Gibbs sampling methods are used to simulate direct draws from the new generalized statistical model of interest. In section 2, we have proposed and studied new Bayesian approach for modelling species richness model, estimating the size of closed population with individual heterogeneity. In section 3, we have developed the generalized procedure to estimate the parameters of new models using Bayesian methodology. The Bayesian estimates of the parameters are obtained using Markov Chain Monte Carlo (MCMC) simulation technique based on the assumption that the new priors are independent, The generalized posterior analysis is performed and estimated. We have examine the issue of model compatibility using predictive results. A real data set of bird breeding survey (BBS) are analysed for illustrating the application and the proposed Bayesian approach.

II. THE MODEL

To establish and promote an efficient statistical estimation process, and procedure, for the problem of estimating the size of closed population. We first, introduce the five parameters generalized binomial model $GBM(n,p,\pi,\rho,\tau)$ [see [1] Al-Saleh et al. (2016)] with the following probability function

$$(2.1) \quad f(y) = \binom{n}{y} p^{\pi y} (1 - (1 - p^\tau)^\rho)^n \left[\frac{1}{1 - (1 - p^\tau)^\rho} - p^\pi \right]^{n-y}, \quad \pi > 0, \rho > 0, \tau \geq 0, y=0,1,\dots,n.$$

The model in equation (2.1) is developed by introducing three more parameters called indicator parameters. The main advantage of an indicator parameter is to give the generalized binomial model accession, more approaches, and options for fitting the data. When $\pi=1, \rho=1,$ and $\tau=0,$ model (2.1) is reduced to standard binomial model [i.e., $GBM(n,p,1,1,0) \equiv SBM(n,p)$]. Also, to help with the examination of the data more precisely, we introduce new mixture unit interval type prior $UITP(\alpha,\beta)$ on $(0,1)$ with two parameters $\alpha > 0, \beta > 0,$ and its pdf is given by,

$$(2.2) \quad f(x) = \frac{(1 - (1 - x^\alpha)^2)^\beta}{(2 - x^\alpha)^\beta} \left[\left(1 + \frac{\alpha \beta x^\alpha}{(2 - x^\alpha)} \right) + \frac{2 \alpha x^\alpha (1 - x^\alpha)}{(1 - (1 - x^\alpha)^2)} \right], \quad x \in (0,1), \quad \alpha > 0, \beta > 0.$$

The distribution function of model (2.2) is $F(x) = \frac{x(1 - (1 - x^\alpha)^2)^\beta}{(2 - x^\alpha)^\beta},$

When $\alpha=0,$ and $\beta=1,$ the model (2.2) reduces to standard uniform model [$UITP(0,1) = SUM(0,1)$]. In some particular cases the parameters $\alpha,$ and β of model (2.2) can be seen as providing not only an extra flexibility to the probability function, but also helps to express probability distribution as an exact form of mixture of probability distributions under certain conditions.

The model considered by [[2] Royle and Dorazio (2008), Chapter 6] is extended by using the generalized model (2.1) and the mixture model (2.2). For distinctness, we assume that there is no birth, death, or migration so that the population size is a constant over ambuscade times. The demographic closure assumption is usually valid for the birds breeding survey data collected in a comparatively short time during a nonbreeding season. The model is explained through the data set [see Table A], concerning bird breeding survey (BBS) in Maryland. This issue of estimating the size of closed population gives specification for each bird to have its own detection probability. Let k be the number of sample opportunities, the readings are the number of possible perusal (out of the k samples), on each of n_1 birds, with y_i for $i=1, 2, \dots, n_1.$ Assigning, for species specific detection probabilities, is an important part of the model due to, quandary and difficulty in detecting birds by the viewers and examiners, the biology, and the behavior of each bird. Here we adopt an analysis of the model by data augmentation. The

procedure and data analysis study here follows that of [[2] Royle and Dorazio (2008), Chapter 6]. The specific data set consists of observed detections for $n_1 = 71$ bird in a Breeding Bird Survey study in Maryland. There were $k=50$ samples of the population. In this study we deal with unknown N by data augmentation, therefore we introduce zero readings, $y_{n_1+1} = 0, \dots, y_M = 0$, and a set of implicit variables $z_i, i = 1, 2, \dots, M$ with z_i have a standard Bernoulli model. If $z_i = 1$, then element i of the list blend with a member of the population of size N , and if $z_i = 0$, it is an excess 0 relative to the binomial model. M is set arbitrarily large. It can be motivated as the upper limit of a discrete uniform type prior for N . That is, $N \sim \text{DiscreteUniform}(0, M)$. The population size parameter is $N = \sum z_i$. In the following data analysis, we have augmented the data set with 250 $y_i = 0$ subjects [see Table A].

(i) Model I: Standard Binomial model with uniform prior [see [2] Royle and Dorazio (2008), Chapter 6]

- (1) $z_i \sim \text{GBM}(1, \psi, 1, 1, 0)$ # Bernoulli
- (2) $y_i \sim \text{GBM}(k, z_i * p_i, 1, 1, 0)$ # Binomial
- (3) $\text{logit}(p_i) \sim \text{Normal}(\mu, \tau)$ for $i = 1, 2, \dots, M$

Prior distributions are:

- (1) $\psi \sim \text{SUM}(0, 1)$
- (2) $\mu \sim \text{Norm}(0, 0.001)$
- (3) $\tau \sim \text{Gamma}(0.001, 0.001)$; $\sigma = \sqrt{\tau}$

(ii) Model II: Standard Binomial model with mixture unit interval prior

- (1) $z_i \sim \text{GBM}(1, \psi, 1, 1, 0)$ # Bernoulli
- (2) $y_i \sim \text{GBM}(k, z_i * p_i, 1, 1, 0)$ # Binomial
- (3) $\text{logit}(p_i) \sim \text{Normal}(\mu, \tau)$ for $i = 1, 2, \dots, M$

Prior distributions are:

- (1) $\psi \sim \text{UITP}(\alpha, \beta)$ # Mixture Unit Interval Prior
- (2) $\mu \sim \text{Normal}(0, 0.001)$
- (3) $\tau \sim \text{Gamma}(0.001, 0.001)$; $\sigma = \sqrt{\tau}$

Hyper-Prior distributions are:

- (1) $\alpha \sim \text{Gamma}(1.00, 1.00)$
- (2) $\beta \sim \text{Gamma}(1.00, 1.00)$

(iii) Model III: Generalized Binomial model with mixture unit interval prior

- (1) $z_i \sim \text{GBM}(1, \psi, 1, 1, 0)$ # Bernoulli
- (2) $y_i \sim \text{GBM}(K, z_i * p_i, \pi, \rho, \tau)$ # Generalized Binomial
- (3) $\text{logit}(p_i) \sim \text{Normal}(\mu, \tau)$ for $i = 1, 2, \dots, M$

Prior distributions are:

- (1) $\psi \sim \text{UITP}(\alpha, \beta)$ # Mixture Unit Interval Prior
- (2) $\mu \sim \text{Norm}(0, 0.001)$
- (3) $\tau \sim \text{Gamma}(0.001, 0.001)$; $\sigma = \sqrt{\tau}$
- (4) $\pi \sim \text{Gamma}(1.00, 1.00)$
- (5) $\rho \sim \text{Gamma}(1.00, 1.00)$
- (6) $\tau \sim \text{Gamma}(1.00, 1.00)$

Hyper-Prior distributions are:

- (1) $\alpha \sim \text{Gamma}(1.00, 1.00)$
- (2) $\beta \sim \text{Gamma}(1.00, 1.00)$

III. BAYESIAN UPDATING PREDICTION DATA ANALYSIS

A realistic Bayesian model for the bird breeding survey data is to suggest a hierarchical model. A Markov Chain Monte Carlo (MCMC) Gibbs sampling approach implemented in using OPENBUGS[®] computer software can give an analysis of estimates of each parameter. A burn in of 1000 updates followed by a further 100k updates is implemented. The table 3.1, represent the estimates for model I, the table 3.2, represent the estimates for model II, where the table 3.3, represent the estimates for model III, along with standard deviation, mean and MC error.

Table 3.1 Bayesian summary of estimates for model I

	Mean	SD	MC error	2.5%	Median	97.5%
N	88.92	10.57	0.7905	75.0	87.0	118.0
mu	-2.658	0.4078	0.03107	-3.678	-2.595	-2.046
psi	0.2783	0.04101	0.002505	0.2127	0.2744	0.3751
Sigma	1.649	0.2793	0.02167	1.188	1.628	2.287

Table 3.2 Bayesian summary of estimates for model II

	Mean	SD	MC error	2.5%	Median	97.5%
N	90.82	11.82	0.9928	76.0	88.0	122.0
α	1.069	1.112	0.0216	0.02423	0.7092	4.181
β	0.7629	0.8244	0.01668	0.009912	0.4965	3.032
mu	-2.736	0.4404	0.03818	-3.82	-2.649	-2.088
psi	0.2838	0.04488	0.003163	0.2138	0.2778	0.3897
sigma	1.702	0.3048	0.02594	1.233	1.666	2.422

Table 3.3 Bayesian summary of estimates for model III

	Mean	SD	MC error	2.5%	Median	97.5%
N	108.4	9.74	0.6063	92.0	107.0	130.0
α	0.9735	0.9788	0.01048	0.02589	0.6685	3.681
β	1.01	1.002	0.009612	0.02574	0.6994	3.716
mu	-1.363	0.1718	0.0161	-1.78	-1.359	-1.071
psi	0.3382	0.04004	0.00191	0.2646	0.3356	0.4252
sigma	0.2757	0.1283	0.01217	0.1156	0.251	0.6041

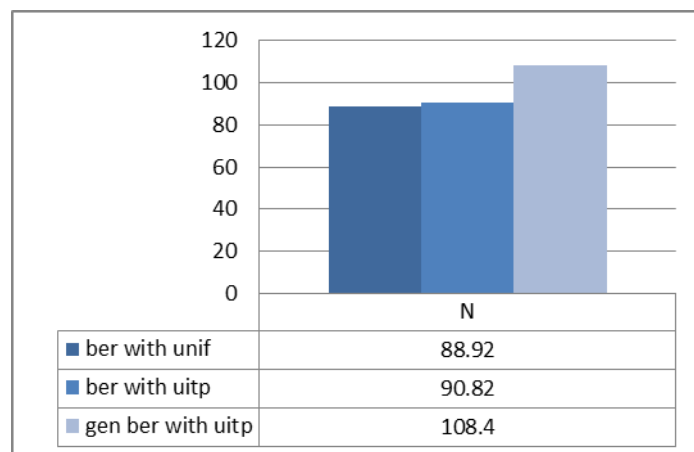


Fig 1 Comparisons of bird breeding data estimates between models I, II, and III

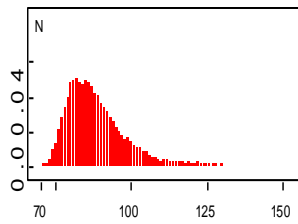


Fig 2a Model I

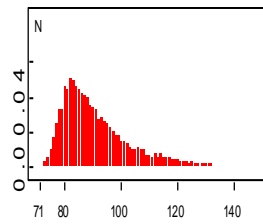


Fig 2b Model II

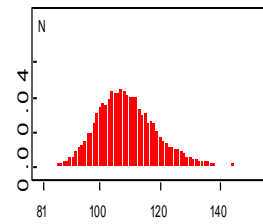


Fig 2c Model III

Fig 2 Comparisons of the size of the closed population estimate N between models I, II, and III

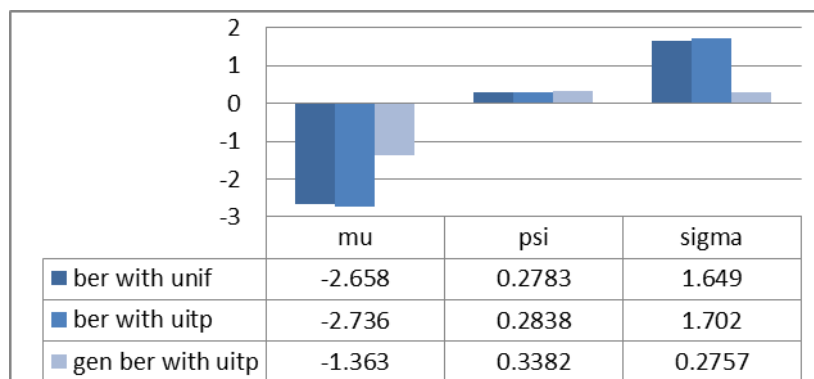


Fig 3 Comparisons of bird breeding data estimates between models I, II, and III

Examination of the above simulations (Tables 3.1–3.6 and Figures 1-3) yields the following observations:

1. The posterior mean of the estimate N of models I, II, and III are 88.92, 90.82, and 108.4, respectively. There is a clear and substantial shift of the posterior mean to the right. The posterior standard deviation (SD) is 10.57, 11.82 and 9.74, respectively, and hence a slight change in posterior SD. Comparison of the MC error for model I, II and III shows also a slight change in MC error.
2. The posterior mean of the estimate mu of models I, II, and III are -2.658, -2.736 and -1.363, respectively. There is a slight shift of the posterior mean to the right. The posterior standard deviation (SD) is 0.4078, 0.4404 and 0.1718, respectively, and hence a slight shift to the left in posterior SD. Comparison of the MC error for model I, II and III shows a slight change in MC error.
3. The posterior mean of the estimate psi of models I, II, and III are 0.2783, 0.2838 and 0.3382, respectively. There is a slight shift of the posterior mean to the right. The posterior standard deviation (SD) is 0.04101, 0.04488 and 0.04004, respectively, and hence it shows that they are about the same in posterior SD. Comparison of the MC error for model I, II and III shows a slight change in MC error.
4. The posterior mean of the estimate sigma of models I, II, and III are 1.649, 1.702 and 0.2757, respectively. There is a clear and substantial shift of the posterior mean to the left. The posterior standard deviation (SD) is 0.2793, 0.3048 and 0.1283, respectively, and hence a decrease in posterior SD. Comparison of the MC error for I, II and III shows that they are about the same.

Table 3.4 Bayesian summary estimates for π , when $\pi > 0$, for model III

No.	$\hat{\pi}$	No.	$\hat{\pi}$	No.	$\hat{\pi}$	No.	$\hat{\pi}$	No.	$\hat{\pi}$	No.	$\hat{\pi}$	No.	$\hat{\pi}$
1	1.057	51	0.5299	101	1.102	151	1.068	201	1.112	251	1.112	301	1.079
2	1.075	52	0.5033	102	1.115	152	1.135	202	1.09	252	1.09	302	1.066
3	1.119	53	0.4621	103	1.081	153	1.089	203	1.096	253	1.096	303	1.108
4	1.085	54	0.4593	104	1.093	154	1.065	204	1.105	254	1.105	304	1.085
5	1.113	55	0.4743	105	1.106	155	1.089	205	1.082	255	1.082	305	1.106
6	1.092	56	0.4779	106	1.109	156	1.112	206	1.051	256	1.051	306	1.102
7	1.087	57	0.4456	107	1.067	157	1.094	207	1.098	257	1.098	307	1.094
8	1.112	58	0.3959	108	1.075	158	1.105	208	1.088	258	1.088	308	1.086
9	1.046	59	0.3842	109	1.1	159	1.088	209	1.085	259	1.085	309	1.105
10	1.072	60	0.3861	110	1.096	160	1.095	210	1.109	260	1.109	310	1.097
11	1.08	61	0.3827	111	1.079	161	1.089	211	1.095	261	1.095	311	1.078
12	1.09	62	0.3678	112	1.099	162	1.084	212	1.098	262	1.098	312	1.062
13	1.101	63	0.3706	113	1.097	163	1.109	213	1.099	263	1.099	313	1.085
14	1.086	64	0.3435	114	1.085	164	1.093	214	1.093	264	1.093	314	1.088
15	1.082	65	0.3367	115	1.074	165	1.084	215	1.098	265	1.098	315	1.073
16	0.9165	66	0.3229	116	1.103	166	1.107	216	1.072	266	1.072	316	1.092
17	0.9356	67	0.2791	117	1.074	167	1.109	217	1.135	267	1.135	317	1.09
18	0.9278	68	0.2549	118	1.079	168	1.094	218	1.09	268	1.09	318	1.109
19	0.9359	69	0.2277	119	1.129	169	1.098	219	1.08	269	1.08	319	1.092
20	0.9046	70	0.2082	120	1.066	170	1.105	220	1.095	270	1.095	320	1.07
21	0.9154	71	0.1473	121	1.063	171	1.107	221	1.095	271	1.095	321	1.067
22	0.9312	72	1.078	122	1.085	172	1.075	222	1.12	272	1.12		
23	0.847	73	1.076	123	1.059	173	1.094	223	1.08	273	1.08		
24	0.8329	74	1.102	124	1.049	174	1.086	224	1.082	274	1.082		
25	0.8121	75	1.114	125	1.046	175	1.07	225	1.109	275	1.109		
26	0.8311	76	1.072	126	1.04	176	1.094	226	1.072	276	1.072		
27	0.8136	77	1.117	127	1.081	177	1.085	227	1.095	277	1.095		
28	0.7723	78	1.086	128	1.097	178	1.049	228	1.116	278	1.116		
29	0.772	79	1.084	129	1.107	179	1.127	229	1.096	279	1.096		
30	0.7124	80	1.084	130	1.088	180	1.083	230	1.07	280	1.07		
31	0.6943	81	1.107	131	1.121	181	1.089	231	1.098	281	1.098		
32	0.709	82	1.077	132	1.089	182	1.04	232	1.081	282	1.081		
33	0.7104	83	1.08	133	1.076	183	1.08	233	1.105	283	1.105		
34	0.7019	84	1.088	134	1.08	184	1.089	234	1.064	284	1.064		
35	0.6528	85	1.074	135	1.079	185	1.104	235	1.091	285	1.091		
36	0.6743	86	1.099	136	1.117	186	1.102	236	1.093	286	1.093		
37	0.6669	87	1.083	137	1.109	187	1.099	237	1.09	287	1.09		
38	0.6737	88	1.041	138	1.065	188	1.07	238	1.085	288	1.085		
39	0.6646	89	1.099	139	1.102	189	1.112	239	1.101	289	1.101		
40	0.6624	90	1.095	140	1.097	190	1.082	240	1.085	290	1.085		
41	0.6008	91	1.093	141	1.092	191	1.108	241	1.085	291	1.085		
42	0.6273	92	1.123	142	1.095	192	1.105	242	1.089	292	1.089		
43	0.6259	93	1.114	143	1.087	193	1.079	243	1.069	293	1.069		
44	0.5965	94	1.064	144	1.13	194	1.119	244	1.091	294	1.091		
45	0.5727	95	1.083	145	1.087	195	1.091	245	1.119	295	1.119		
46	0.5614	96	1.111	146	1.123	196	1.074	246	1.076	296	1.076		
47	0.519	97	1.114	147	1.068	197	1.075	247	1.122	297	1.122		
48	0.5366	98	1.076	148	1.123	198	1.112	248	1.06	298	1.06		
49	0.5009	99	1.137	149	1.067	199	1.105	249	1.095	299	1.095		
50	0.5038	100	1.105	150	1.113	200	1.067	250	1.115	300	1.115		

- The posterior mean of the estimate α of models II, and III are 1.069, and 0.9735, respectively. There is a clear shift of the posterior mean to the left. The posterior standard deviation (SD) is 1.112, and 0.9788, respectively, and hence a decrease in posterior SD. Comparison of the MC error for II, III shows that they are about the same.
- The posterior mean of the estimate β of models II, and III are 0.7629, and 1.01, respectively. There is a clear shift of the posterior mean to the left. The posterior standard deviation (SD) is 0.8244, and 1.002, respectively, and hence an increase in posterior SD. Comparison of the MC error for II, III shows that they are about the same.

Table 3.5 Bayesian summary estimates for ρ , when $\rho > 0$, for model III

No.	$\hat{\rho}$	No.	$\hat{\rho}$	No.	$\hat{\rho}$	No.	$\hat{\rho}$	No.	$\hat{\rho}$	No.	$\hat{\rho}$	No.	$\hat{\rho}$
1	0.8161	51	1.282	101	0.9369	151	0.939	201	0.9821	251	0.9356	301	0.9637
2	0.8097	52	1.219	102	0.9376	152	0.9217	202	0.9648	252	0.9741	302	0.922
3	0.8336	53	1.29	103	0.9759	153	0.9738	203	0.9558	253	0.9453	303	0.957
4	0.8107	54	1.317	104	0.9541	154	0.9575	204	0.9456	254	0.9347	304	0.9306
5	0.8065	55	1.269	105	0.9581	155	0.9558	205	0.9707	255	0.9515	305	0.9454
6	0.7734	56	1.279	106	0.9487	156	0.955	206	0.9551	256	0.9282	306	0.9582
7	0.8291	57	1.404	107	0.9486	157	0.9704	207	0.9399	257	0.971	307	0.9358
8	0.8682	58	1.383	108	0.9485	158	0.9427	208	0.9448	258	0.9627	308	0.9339
9	0.7944	59	1.392	109	0.9506	159	0.9436	209	0.9349	259	0.957	309	0.9231
10	0.8007	60	1.427	110	0.9644	160	0.9279	210	0.9662	260	0.9245	310	0.9508
11	0.8201	61	1.412	111	0.9505	161	0.9545	211	0.9395	261	0.9413	311	0.9529
12	0.8101	62	1.425	112	0.9463	162	0.9668	212	0.9416	262	0.9447	312	0.9242
13	0.8156	63	1.401	113	0.9717	163	0.9725	213	0.9285	263	0.943	313	0.9476
14	0.7972	64	1.474	114	0.9606	164	0.9556	214	0.9266	264	0.926	314	0.9603
15	0.8054	65	1.529	115	0.9675	165	0.9302	215	0.9277	265	0.9681	315	0.9649
16	0.8831	66	1.545	116	0.9126	166	0.9597	216	0.9645	266	0.9238	316	0.9746
17	0.8839	67	1.652	117	0.9555	167	0.9457	217	0.9337	267	0.9414	317	0.9213
18	0.8958	68	1.634	118	0.9723	168	0.9438	218	0.9195	268	0.9708	318	0.9581
19	0.9218	69	1.676	119	0.9546	169	0.9594	219	0.9446	269	0.9252	319	0.9316
20	0.9011	70	1.779	120	0.9511	170	0.9418	220	0.9519	270	0.9425	320	0.9291
21	0.8822	71	1.958	121	0.9745	171	0.934	221	0.9578	271	0.9427	321	0.9525
22	0.8685	72	0.9342	122	0.9379	172	0.941	222	0.9785	272	0.9729		
23	0.9858	73	0.9458	123	0.9564	173	0.9471	223	0.951	273	0.9464		
24	0.9511	74	0.9534	124	0.969	174	0.9493	224	0.9167	274	0.9598		
25	0.9844	75	0.9471	125	0.9409	175	0.9458	225	0.9774	275	0.9547		
26	0.9707	76	0.9492	126	0.9891	176	0.9648	226	0.9635	276	0.9689		
27	0.9761	77	0.9323	127	0.9342	177	0.9415	227	0.9341	277	0.9676		
28	1.011	78	0.9559	128	0.9421	178	0.9791	228	0.967	278	0.9539		
29	1.046	79	0.943	129	0.9399	179	0.9549	229	0.9532	279	0.9642		
30	1.027	80	0.9423	130	0.9721	180	0.9327	230	0.9728	280	0.9371		
31	1.068	81	0.9274	131	0.9324	181	0.9346	231	0.9489	281	0.9365		
32	1.076	82	0.9382	132	0.9626	182	0.9434	232	0.9588	282	0.9306		
33	1.079	83	0.94	133	0.943	183	0.9297	233	0.9661	283	0.9629		
34	1.052	84	0.9515	134	0.979	184	0.9375	234	0.9226	284	0.9594		
35	1.098	85	0.9584	135	0.9318	185	0.9546	235	0.9592	285	0.9616		
36	1.137	86	0.9605	136	0.953	186	0.9317	236	0.9785	286	0.9375		
37	1.133	87	0.9162	137	0.9403	187	0.9744	237	0.986	287	0.938		
38	1.112	88	0.9414	138	0.9658	188	0.947	238	0.9645	288	0.9608		
39	1.079	89	0.964	139	0.9482	189	0.9329	239	0.998	289	0.931		
40	1.145	90	0.9575	140	0.9463	190	0.9671	240	0.9558	290	0.9604		
41	1.094	91	0.9723	141	0.9519	191	0.9402	241	0.9597	291	0.9089		
42	1.124	92	0.9593	142	0.9667	192	0.9399	242	0.9518	292	0.9497		
43	1.099	93	0.9443	143	0.9317	193	0.937	243	0.9708	293	0.9467		
44	1.208	94	0.9464	144	0.9522	194	0.9502	244	0.9455	294	0.9165		
45	1.165	95	0.9676	145	0.9463	195	0.9433	245	0.986	295	0.9542		
46	1.204	96	0.9232	146	0.9419	196	0.9575	246	0.962	296	0.953		
47	1.268	97	0.9342	147	0.9614	197	0.952	247	0.9145	297	0.9307		
48	1.266	98	0.9351	148	0.9782	198	0.9829	248	0.9043	298	0.9413		
49	1.233	99	0.9382	149	0.9262	199	0.9322	249	0.9598	299	0.9347		
50	1.276	100	0.9464	150	0.9588	200	0.9582	250	0.9618	300	0.9504		

- The posterior mean of the estimate π of models III (table 3.4) vary between (0.1473, 1.37). This indicate that the data are effected by the generalized model III. These findings do not sport the work done by [[2] Royle and Dorazio (2008), Chapter 6]. Who use SBM(n,p). We also note the following interesting readings that bird (No. 71) has the lowest estimated value α at 0.1473, with highest number of observed detection at 36 readings. This maybe, related to the factor effect of behavior and/or biology of the bird.
- The posterior mean of the estimate ρ of models III (table 3.5) vary between (0.7734,1.958). This indicate that the data are effected by the generalized model III. These findings do not sport the work done by [[2] Royle and Dorazio (2008), Chapter 6]. Who use SBM(n,p). We also note the following interesting reading that bird (No. 71) has the highest estimated value α at 1.958, with highest number of observed detection at 36 readings.

Table 3.6 Bayesian summary estimates for τ , when $\tau \geq 0$, for model III

No.	$\hat{\tau}$	No.	$\hat{\tau}$	No.	$\hat{\tau}$	No.	$\hat{\tau}$	No.	$\hat{\tau}$	No.	$\hat{\tau}$	No.	$\hat{\tau}$
1	1.094	51	0.4792	101	1.058	151	1.12	201	1.082	251	1.092	301	1.073
2	1.107	52	0.4712	102	1.089	152	1.081	202	1.089	252	1.095	302	1.12
3	1.049	53	0.469	103	1.089	153	1.064	203	1.067	253	1.1	303	1.07
4	1.062	54	0.4974	104	1.073	154	1.058	204	1.111	254	1.09	304	1.08
5	1.056	55	0.473	105	1.057	155	1.113	205	1.086	255	1.102	305	1.104
6	1.032	56	0.4776	106	1.103	156	1.094	206	1.068	256	1.117	306	1.111
7	1.071	57	0.4419	107	1.103	157	1.105	207	1.071	257	1.099	307	1.09
8	1.094	58	0.3994	108	1.083	158	1.104	208	1.078	258	1.113	308	1.089
9	1.09	59	0.4168	109	1.131	159	1.129	209	1.085	259	1.151	309	1.112
10	1.073	60	0.3754	110	1.113	160	1.094	210	1.069	260	1.094	310	1.075
11	1.092	61	0.3774	111	1.081	161	1.069	211	1.081	261	1.082	311	1.073
12	1.061	62	0.4018	112	1.084	162	1.104	212	1.089	262	1.093	312	1.106
13	1.071	63	0.3707	113	1.069	163	1.094	213	1.097	263	1.108	313	1.09
14	1.059	64	0.3635	114	1.101	164	1.098	214	1.099	264	1.085	314	1.096
15	1.079	65	0.3677	115	1.083	165	1.084	215	1.094	265	1.079	315	1.105
16	0.878	66	0.3459	116	1.097	166	1.093	216	1.093	266	1.074	316	1.074
17	0.8868	67	0.3	117	1.068	167	1.103	217	1.109	267	1.079	317	1.103
18	0.9047	68	0.2873	118	1.085	168	1.089	218	1.089	268	1.102	318	1.135
19	0.9043	69	0.2746	119	1.068	169	1.083	219	1.082	269	1.102	319	1.111
20	0.9167	70	0.2686	120	1.077	170	1.107	220	1.084	270	1.115	320	1.109
21	0.9005	71	0.2235	121	1.107	171	1.109	221	1.058	271	1.116	321	1.081
22	0.8803	72	1.086	122	1.11	172	1.08	222	1.114	272	1.076		
23	0.7933	73	1.134	123	1.107	173	1.081	223	1.118	273	1.115		
24	0.8018	74	1.096	124	1.088	174	1.101	224	1.071	274	1.094		
25	0.8243	75	1.092	125	1.096	175	1.08	225	1.092	275	1.125		
26	0.7889	76	1.105	126	1.077	176	1.074	226	1.088	276	1.024		
27	0.806	77	1.108	127	1.111	177	1.115	227	1.087	277	1.098		
28	0.7128	78	1.102	128	1.098	178	1.099	228	1.106	278	1.09		
29	0.7147	79	1.081	129	1.093	179	1.078	229	1.075	279	1.071		
30	0.6464	80	1.096	130	1.122	180	1.108	230	1.109	280	1.106		
31	0.6815	81	1.094	131	1.09	181	1.088	231	1.133	281	1.065		
32	0.6848	82	1.112	132	1.086	182	1.125	232	1.152	282	1.108		
33	0.6795	83	1.132	133	1.071	183	1.093	233	1.097	283	1.085		
34	0.6768	84	1.104	134	1.129	184	1.141	234	1.084	284	1.083		
35	0.6416	85	1.101	135	1.109	185	1.107	235	1.071	285	1.121		
36	0.6329	86	1.058	136	1.108	186	1.089	236	1.109	286	1.131		
37	0.6412	87	1.1	137	1.127	187	1.117	237	1.11	287	1.08		
38	0.6259	88	1.111	138	1.083	188	1.117	238	1.114	288	1.111		
39	0.6234	89	1.142	139	1.096	189	1.061	239	1.086	289	1.124		
40	0.6357	90	1.081	140	1.073	190	1.118	240	1.091	290	1.089		
41	0.5896	91	1.092	141	1.102	191	1.06	241	1.101	291	1.13		
42	0.5828	92	1.082	142	1.086	192	1.072	242	1.06	292	1.135		
43	0.5716	93	1.104	143	1.082	193	1.092	243	1.114	293	1.052		
44	0.5832	94	1.129	144	1.141	194	1.098	244	1.1	294	1.101		
45	0.5836	95	1.094	145	1.109	195	1.093	245	1.081	295	1.071		
46	0.5348	96	1.1	146	1.129	196	1.072	246	1.071	296	1.108		
47	0.5363	97	1.092	147	1.073	197	1.083	247	1.108	297	1.089		
48	0.5127	98	1.103	148	1.091	198	1.101	248	1.096	298	1.052		
49	0.4916	99	1.121	149	1.118	199	1.079	249	1.092	299	1.073		
50	0.4987	100	1.107	150	1.103	200	1.094	250	1.069	300	1.085		

9. The posterior mean of the estimate τ of models III (table 3.6) vary between (0.2235, 1.152). This indicate that the data are effected by the generalized model III. These findings do not sport the work done by [[2] Royle and Dorazio (2008), Chapter 6]. Who use SBM(n,p). We also note the following interesting reading that bird (No. 71) has the lowest estimated value α at 0.2235, with highest number of observed detection at 36 readings.

In brief, the values of the posterior means of estimates vary to some extent across the results for models I, II, and III. For estimator psi, the values are similar. However, the differences for N, mu, and sigma are dramatic. The difference is clearer and more efficient in the case when generalized models is used compared to standard models. Hence we think, in the above illustration, the analysis using the new generalized models for the bird breeding survey (BBS) data seems preferably more successful than the standard models considered by [[2]

